a) Find the roots of the equation $z^4 - 1 = 0$
b) Find the roots of the equation $z^4 + 1 = 0$
c) Show that roots of $z^4 - 1 = 0$ and $z^4 + 1 = 0$ together make the roots of $z^8 - 1 = 0$
d) Hence, find all the roots to $z^6 + z^4 + z^2 + 1 = 0$

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a) $z^4 - 1 = 0$

We can use de Moivre's Theorem, but it is easy to factorise...

$(z^2 - 1)(z^2 + 1) = 0$

$(z - 1)(z - 1)(z^2 + 1) = 0$

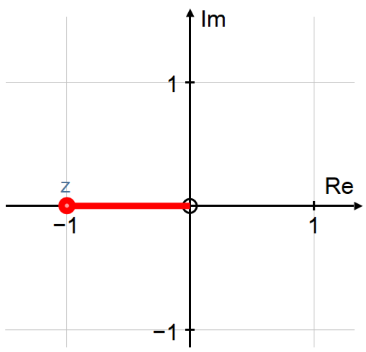
$z = \pm 1\,, \quad z^2 = -1$

$z = \pm 1\,, \quad z = \pm i$

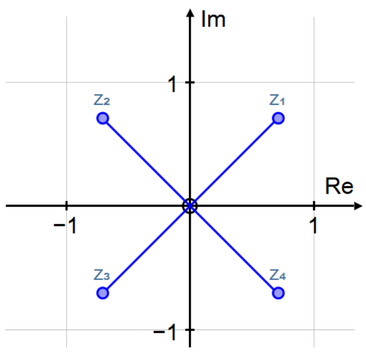
b) $z^4 + 1 = 0$

$z^4 = -1$

$z^4 = cis(\pi + 2k\pi), k = 0,1,2,3$



$z = cis\left(\frac{\pi}{4} + \frac{k\pi}{2}\right), k = 0,1,2,3$



$z = cis\left(\frac{\pi}{4} + \frac{k\pi}{2}\right)$

$z = cis\left(\frac{\pi}{4}\right), cis\left(\frac{3\pi}{4}\right), cis\left(-\frac{3\pi}{4}\right), cis\left(-\frac{\pi}{4}\right)$

$z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

c) $z^8 - 1 = 0$

$(z^4 - 1)(z^4 + 1) = 0$

Hence, the roots of $z^8 - 1 = 0$ are the roots of $(z^4 - 1) = 0, (z^4 + 1) = 0$

d) $(z^4 - 1)(z^4 + 1) = 0$

$(z^2 - 1)(z^2 + 1)(z^4 + 1) = 0$

$(z^2 + 1)(z^4 + 1) = z^6 + z^4 + z^2 + 1$

$z^6 + z^4 + z^2 + 1 = 0$

$(z^2 + 1)(z^4 + 1) = 0$

$z = -i, i, \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$