a) By writing $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$, show that $sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

b) Find $cos\left(\frac{\pi}{12}\right)$

c) Hence, find the roots of the equation $z^4 = 2 + 2\sqrt{3}i$ giving your answers in the form $z = a + ib$

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a) $$sin\left(\frac{\pi}{12}\right) = sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

Use the compound angle identity

$$\sin(A - B) \equiv sinAcosB - cosAsinB$$

$$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = sin\frac{\pi}{3}cos\frac{\pi}{4} - cos\frac{\pi}{3}sin\frac{\pi}{4}$$

We know the exact values:

	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin\theta$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$

$$sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

b) $$cos\left(\frac{\pi}{12}\right) = cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

Use the compound angle identity

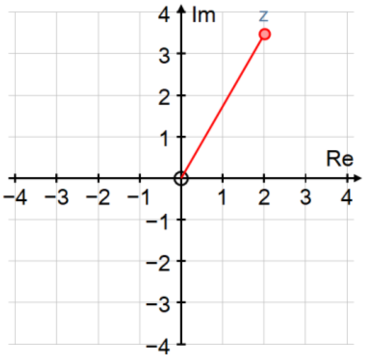
$$cos(A - B) \equiv cosAcosB + sinAsinB$$

$$cos\left(\frac{\pi}{12}\right) = cos\frac{\pi}{3}cos\frac{\pi}{4} + sin\frac{\pi}{3}sin\frac{\pi}{4}$$

$$\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

c) $z^4 = 2 + 2\sqrt{3}i$



$$z^4 = 4cis\left(\frac{\pi}{3} + 2k\pi\right) \qquad k = 0,1,2,3$$

$$z = 4^{\frac{1}{4}}cis\left(\frac{\pi}{12} + \frac{2k\pi}{4}\right) \qquad k = 0,1,2,3$$

$$z = \sqrt{2}cis\left(\frac{\pi}{12} + \frac{k\pi}{2}\right) \qquad k = 0,1,2,3$$

$$z = \sqrt{2}cis\left(\frac{\pi}{12}\right), \sqrt{2}cis\left(\frac{7\pi}{12}\right), \sqrt{2}cis\left(\frac{-11\pi}{12}\right), \sqrt{2}cis\left(\frac{-5\pi}{12}\right)$$

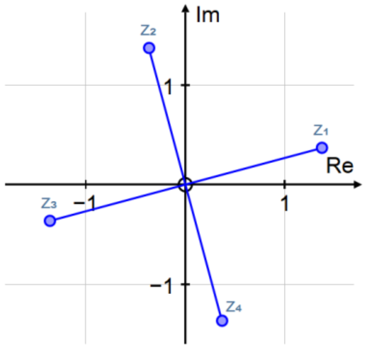
$$z_1 = \sqrt{2}\left(cos\left(\frac{\pi}{12}\right) + isin\left(\frac{\pi}{12}\right)\right)$$

$$z_1 = \sqrt{2}\left(\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} + i\left(\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right)\right)$$

$$z_1 = \frac{\sqrt{12}}{4} + \frac{2}{4} + i\left(\frac{\sqrt{12}}{4} - \frac{2}{4}\right)$$

$$z_1 = \frac{\sqrt{3}+1}{2} + i\left(\frac{\sqrt{3}-1}{2}\right)$$

We can use the symmetry of the roots to find the other ones!



$$z_2 = \frac{1-\sqrt{3}}{2} + i\left(\frac{\sqrt{3}+1}{2}\right)$$

$$z_3 = \frac{-\sqrt{3}-1}{2} + i\left(\frac{1-\sqrt{3}}{2}\right)$$

$$z_4 = \frac{\sqrt{3}-1}{2} + i\left(\frac{-\sqrt{3}-1}{2}\right)$$