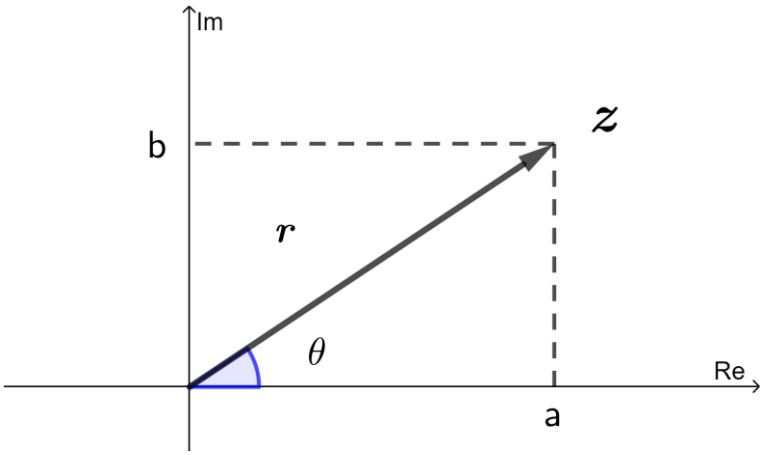
Complex Numbers – The Basics



$r = |z|$ = modulus of complex number = length of vector $\begin{bmatrix} a \\ b \end{bmatrix}$

$\theta = arg(z)$ is the angle created by the complex number and the $x\ axis$

$Arg(z)$ is the principal argument, $-\pi < Arg(z) \leq \pi$

$Re(z) = a$ = real part of z

$Im(z) = b$ = imaginary part of z

Useful results

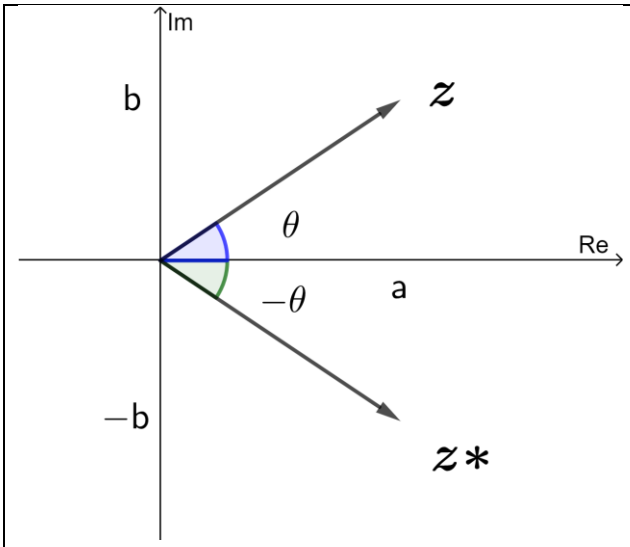
If $Arg(z) = 0, \pi$, then $Im(z) = 0$

If $Arg(z) = \pm\frac{\pi}{2}$, then $Re(z) = 0$

The 3 Forms

Cartesian Form	$z = a + bi$	
Polar Form	$z = r(cos\theta + isin\theta)$	$r = \|z\| = \sqrt{a^2 + b^2}$
Euler Form	$z = re^{i\theta}$	$\theta = \arg(z)$

The Complex Conjugate



If $z = a + bi$

then the complex conjugate of z is z^*

$z^* = a - bi$

Useful result

$z \cdot z^* = |z|^2 = a^2 + b^2$, a real number

e.g. $\frac{4}{3-2i} = \frac{4(3+2i)}{(3-2i)(3+2i)} = \frac{12+8i}{13}$

Properties

Modulus	argument
$\|z^*\| = \|z\|$	$arg(z^*) = -\arg(z)$
$z \cdot z^* = \|z\|^2$	
$\|zw\| = \|z\| \cdot \|w\|$	$\arg(zw) = \arg(z) + \arg(w)$
$\left\|\frac{z}{w}\right\| = \frac{\|z\|}{\|w\|}$	$arg\left(\frac{z}{w}\right) = \arg(z) - \arg(w)$
$\|z^n\| = \|z\|^n$	$arg(z^n) = n \cdot arg(z)$