

Given that z is a complex number and $\frac{3z-4}{5} = \frac{p-2i}{3-i}$, where $p \in \mathbb{R}$

a) Express z in the form $a + bi$, $a, b \in \mathbb{R}$

Given that $\arg z = -\frac{\pi}{2}$

b) find the value of p

$$\begin{aligned} \text{a)} \quad \frac{3z-4}{5} &= \frac{p-2i}{3-i} \\ \frac{3z-4}{5} &= \frac{(p-2i)(3+i)}{(3-i)(3+i)} \\ \frac{3z-4}{5} &= \frac{3p+2+(p-6)i}{10} \\ 2(3z-4) &= 3p+2+(p-6)i \\ 6z-8 &= 3p+2+(p-6)i \\ 6z &= 3p+10+(p-6)i \\ z &= \frac{3p+10}{6} + \frac{(p-6)}{6}i \end{aligned}$$

b)

if $\arg z = -\frac{\pi}{2}$ then $\operatorname{Re}(z) = 0$

$$\frac{3p+10}{6} = 0$$

$$p = -\frac{10}{3}$$