

Find the possible values of a , given that $\left|\frac{z_1}{z_2}\right| = 2$ and $z_1 = a + 2i$ and $z_2 = 1 - 2i$

$$\frac{z_1}{z_2} = \frac{a + 2i}{1 - 2i}$$

Make the denominator real by multiplying numerator and denominator by the **complex conjugate**

$$\frac{z_1}{z_2} = \frac{a + 2i}{1 - 2i} \cdot \frac{1 + 2i}{1 + 2i}$$

$$\frac{z_1}{z_2} = \frac{a + 2ai + 2i + 4i^2}{1 + 2i - 2i - 4i^2}$$

$$i^2 = -1$$

$$\frac{z_1}{z_2} = \frac{a - 4 + (2a + 2)i}{5}$$

Find modulus of complex number using Pythagoras' Theorem

$$\left|\frac{z_1}{z_2}\right| = \sqrt{\frac{(a - 4)^2 + (2a + 2)^2}{5^2}} = 2$$

$$\frac{(a - 4)^2 + (2a + 2)^2}{25} = 2^2$$

$$a^2 - 8a + 16 + 4a^2 + 8a + 4 = 100$$

$$5a^2 + 20 = 100$$

$$5a^2 = 80$$

$$a^2 = 16$$

$$a = \pm 4$$



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