

IB Maths: Analysis & Approaches SL & HL

Formula sheet for use during the IB Maths course & examinations

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Prior Learning SL & HL

Area of a parallelogram	$A = bh$	b is the base, h is the height
Area of a triangle	$A = \frac{1}{2}(bh)$	b is the base, h is the height
Area of a trapezium	$A = \frac{1}{2}(a+b)h$	a and b are the parallel sides, h is the height
Area of a circle	$A = \pi r^2$	r is the radius
Circumference of a circle	$C = 2\pi r$	r is the radius
Volume of a cuboid	$V = lwh$	l is the length, w is the width, h is the height
Volume of a cylinder	$V = \pi r^2 h$	r is the radius, h is the height
Volume of a prism	$V = Ah$	A is the area of cross-section, h is the height
Area of the curved surface of a cylinder	$A = 2\pi rh$	r is the radius, h is the height
Distance between two points (x_1, y_1) & (x_2, y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$	
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1) & (x_2, y_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	

Topic 1: Number & Algebra – SL & HL

The n th term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d)$; $S_n = \frac{n}{2}(u_1 + u_n)$
The n th term of a geometric sequence	$u_n = u_1 r^{n-1}$
The sum of n terms of a finite geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}$, $r \neq 1$
Compound interest	$FV = PV \times \left(1 + \frac{r}{100}\right)^{kn}$ <i>FV</i> is the future value, <i>PV</i> is the present value, n is the number of years, k is the number of compounding periods per year, $r\%$ is the nominal annual rate of interest
Exponents & logarithms	$a^x = b \Leftrightarrow x = \log_a b$ $a > 0, b > 0, a \neq 1$
Exponents & logarithms	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$ $\log_a x = \frac{\log_b x}{\log_b a}$
The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1-r}$, $ r < 1$
Binomial theorem	$(a+b)^n = a^n + {}^n C_1 a^{n-1} b + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n$ ${}^n C_r = \frac{n!}{r!(n-r)!}$

Topic 1: Number & Algebra – HL only

Combinations	${}^n C_r = \frac{n!}{r!(n-r)!}$
Permutations	${}^n P_r = \frac{n!}{(n-r)!}$
Complex numbers	$z = a + bi$
Modulus-argument (polar) & exponential (Euler) form	$z = r(\cos \theta + i \sin \theta) = re^{i\theta} = r \operatorname{cis} \theta$
De Moivre's Theorem	$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta) = r^n e^{in\theta} = r^n \operatorname{cis} n\theta$

Topic 2: Functions – SL & HL

Equations of a straight line	$y = mx + c$; $ax + by + d = 0$; $y - y_1 = m(x - x_1)$
Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$
Solutions of a quadratic equation	$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$
Discriminant	$\Delta = b^2 - 4ac$
Exponential & logarithmic functions	$a^x = e^{x \ln a}$; $\log_a a^x = x = a^{\log_a x}$ $a, x > 0, a \neq 1$

Topic 2: Functions – HL only

Sum & product of the roots of polynomial equations of the form $\sum_{k=1}^n a_k x^k = 0$	Sum is $-\frac{a_{n-1}}{a_n}$; product is $\frac{(-1)^n a_0}{a_n}$
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Topic 3: Geometry & Trigonometry – SL & HL

Distance between two points (x_1, y_1, z_1) & (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$	
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1) & (x_2, y_2, z_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$	
Volume of a right-pyramid	$V = \frac{1}{3}Ah$	A is the area of the base, h is the height
Volume of a right cone	$V = \frac{1}{3}\pi r^2 h$	r is the radius, h is the height
Area of the curved surface of a cone	$A = \pi rl$	r is the radius, l is the slant height
Volume of a sphere	$V = \frac{4}{3}\pi r^3$	r is the radius
Surface area of a sphere	$A = 4\pi r^2$	r is the radius
Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	
Cosine rule	$c^2 = a^2 + b^2 - 2ab \cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$	
Area of a triangle	$A = \frac{1}{2}ab \sin C$	
Length of an arc	$l = r\theta$	r is the radius, θ is the angle measured in radians
Area of a sector	$A = \frac{1}{2}r^2\theta$	r is the radius, θ is the angle measured in radians
Identity for $\tan \theta$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$	
Pythagorean identity	$\cos^2 \theta + \sin^2 \theta = 1$	
Double angle identities	$\sin 2\theta = 2 \sin \theta \cos \theta$ $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$	

Topic 3: Geometry & Trigonometry – HL only

Reciprocal trigonometric identities	$\sec \theta = \frac{1}{\cos \theta}$ $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$
Pythagorean identities	$1 + \tan^2 \theta = \sec^2 \theta$ $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$
Compound angle identities	$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
Double angle identity for tan	$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$
Magnitude of a vector	$ v = \sqrt{v_1^2 + v_2^2 + v_3^2}$ $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$
Scalar product	$v \cdot w = v_1 w_1 + v_2 w_2 + v_3 w_3$ $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, $w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $v \cdot w = v w \cos \theta$, where θ is the angle between v and w
Angle between two vectors	$\cos \theta = \frac{v_1 w_1 + v_2 w_2 + v_3 w_3}{ v w }$
Vector equation of a line	$r = a + \lambda b$
Parametric form of the equation of a line	$x = x_0 + \lambda l$, $y = y_0 + \lambda m$, $z = z_0 + \lambda n$
Cartesian equations of a line	$\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}$
Vector product	$v \times w = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$, $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, $w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $ v \times w = v w \sin \theta$ θ is the angle between v and w
Area of a parallelogram	$A = v \times w $ v and w form two adjacent sides of a parallelogram
Vector equation of a plane	$r = a + \lambda b + \mu c$
Equation of a plane (using the normal vector)	$r \cdot n = a \cdot n$
Cartesian equation of a plane	$ax + by + cz = d$

Topic 4: Statistics & Probability – SL & HL

Interquartile range	$\text{IQR} = Q_3 - Q_1$
Mean, \bar{x} , of a set of data	$\bar{x} = \frac{\sum_{i=1}^n f_i x_i}{n}$ $n = \sum_{i=1}^n f_i$
Probability of an event A	$P(A) = \frac{n(A)}{n(U)}$
Complementary events	$P(A) + P(\bar{A}) = 1$
Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
Conditional probability	$P(A B) = \frac{P(A \cap B)}{P(B)}$
Independent events	$P(A \cap B) = P(A)P(B)$
Expected value of a discrete random variable X	$E(X) = \sum x P(X = x)$
Binomial distribution $X \sim B(n, p)$	
Mean	$E(X) = np$
Variance	$\operatorname{Var}(X) = np(1-p)$
Standardized normal variable	$z = \frac{x - \mu}{\sigma}$

Topic 4: Statistics & Probability – HL only

Bayes' theorem	$P(B A) = \frac{P(B)P(A B)}{P(B)P(A B) + P(\bar{B})P(A \bar{B})}$ $P(B A) = \frac{P(B)P(A B)}{P(B)P(A B) + P(\bar{B})P(A \bar{B}) + P(\bar{B})P(A \bar{B})}$
Variance σ^2	$\sigma^2 = \frac{\sum_{i=1}^n f_i (x_i - \mu)^2}{n} = \frac{\sum_{i=1}^n f_i x_i^2}{n} - \mu^2$
Standard deviation σ	$\sigma = \sqrt{\frac{\sum_{i=1}^n f_i (x_i - \mu)^2}{n}}$
Linear transformation of a single random variable	$E(aX + b) = aE(X) + b$ $\operatorname{Var}(aX + b) = a^2 \operatorname{Var}(X)$
Expected value of a continuous random variable X	$E(X) = \mu = \int_{-\infty}^{\infty} x f(x) dx$
Variance	$\operatorname{Var}(X) = E(X - \mu)^2 = E(X^2) - [E(X)]^2$
Variance of a discrete random variable X	$\operatorname{Var}(X) = \sum (x - \mu)^2 P(X = x) = \sum x^2 P(X = x) - \mu^2$
Variance of a continuous random variable X	$\operatorname{Var}(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$

Topic 5: Calculus – SL & HL

Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
Integral of x^n	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$
Area between a curve $y = f(x)$ & the x -axis, where $f(x) > 0$	$A = \int_a^b y dx$
Derivative of $\sin x$	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
Derivative of $\cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
Derivative of e^x	$f(x) = e^x \Rightarrow f'(x) = e^x$
Derivative of $\ln x$	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
Chain rule	$y = g(u)$, where $u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
Product rule	$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$
Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
Acceleration	$a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$
Distance travelled from t_1 to t_2	distance = $\int_{t_1}^{t_2} v(t) dt$
Displacement from t_1 to t_2	displacement = $\int_{t_1}^{t_2} v(t) dt$
Standard integrals	$\int \frac{1}{x} dx = \ln x + C$ $\int \sin x dx = -\cos x + C$ $\int \cos x dx = \sin x + C$ $\int e^x dx = e^x + C$
Area of region enclosed by a curve and x -axis	$A = \int_a^b y dx$

Topic 5: Calculus – HL only

Derivative of $f(x)$ from first principles	$y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$
Standard derivatives	$\tan x$ $f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$ $\sec x$ $f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$ $\operatorname{cosec} x$ $f(x) = \operatorname{cosec} x \Rightarrow f'(x) = -\operatorname{cosec} x \cot x$ $\cot x$ $f(x) = \cot x \Rightarrow f'(x) = -\operatorname{cosec}^2 x$ a^x $f(x) = a^x \Rightarrow f'(x) = a^x (\ln a)$ $\log_a x$ $f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x \ln a}$ $\arcsin x$ $f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$ $\arccos x$ $f(x) = \arccos x \Rightarrow f'(x) = -\frac{1}{\sqrt{1-x^2}}$ $\arctan x$ $f(x) = \arctan x \Rightarrow f'(x) = \frac{1}{1+x^2}$
Standard integrals	$\int a^x dx = \frac{1}{\ln a} a^x + C$ $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$ $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C$, $ x < a$
Integration by parts	$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$ or $\int u dv = uv - \int v du$
Area of region enclosed by a curve & y -axis	$A = \int_a^b x dy$
Volume of revolution about the x or y -axes	$V = \int_a^b \pi y^2 dx$ or $V = \int_a^b \pi x^2 dy$
Euler's method	$y_{n+1} = y_n + h \times f(x_n, y_n)$; $x_{n+1} = x_n + h$ h is a constant (step length)
Integrating factor for $y' + P(x)y = Q(x)$	$e^{\int P(x) dx}$
Maclaurin series	$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots$
Maclaurin series for special functions	$e^x = 1 + x + \frac{x^2}{2!} + \dots$ $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$ $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$



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Level	Paper	Length	Marks
SL	1	90 mins	80
SL	2	90 mins	80
HL	1	2 hours	110
HL	2	2 hours	110
HL	3	1 hour	55