Mathematics: analysis and approaches
Standard level
Paper 2

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1 hour 30 minutes


Instructions to candidates

- A graphic display calculator is required for this paper.
- Section A: answer all questions.
- Section B: answer all questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

1.   [Maximum mark: 6]

A circular pond with radius 0.8 m contains 16 lily pads. The diagram below shows the shape of each lily pad as part of a circle with centre O and radius 5 cm, $A\widehat{O}B = \theta$.
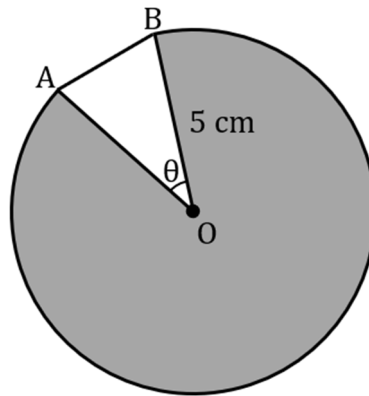
The lily pads cover 5 % of the pond's surface.

(a) Find the surface area of each lily pad.   [2]

(b) Find the value of θ, giving your answer in radians.   [2]

(c) Find the area of the triangle AOB.   [2]

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2. [Maximum mark: 6]

A six-sided biased die is weighted in such a way that the probability of obtaining a "one" is $\frac{3}{7}$.

The die is tossed 10 times. Find the probability of obtaining

(a) at most four "ones". [3]

(b) the fourth "one" on the tenth toss. [3]

3. [Maximum mark: 5]

A car safety expert is investigating a possible link between the tread depth of a car's tyres and the car's stopping distance.

Using the same car on the same track under the same weather conditions the expert records the average tread depth, (x mm), from the car's four tyres and the stopping distance, (y m), when the car's brakes are applied at a particular speed.

Tread depth (x)	6.8	1.4	4.1	0.9	5.7	1.9	3.5	2.6	2.9
Stopping distance (y)	29	45	33.5	49.5	31	42	34	36.5	36

(a) (i) Calculate the Pearson product moment correlation coefficient for these data.

(ii) State the type of linear correlation that is shown between tread depth and stopping distance. [2]

Let L be the regression line of y on x.

(b) (i) Find the equation of L in the form $y = a + bx$.

(ii) Give an interpretation of the values of a and b in the context of the investigation. [3]

4. [Maximum mark: 6]

On 1st January 2021, Nerys invests $\$P$ in an account that pays a nominal annual interest rate of $4.2\,\%$, compounded **monthly**.

The amount of money in Nerys' account **at the end of each year** follows a geometric sequence with common ratio, r.

(a) Find the value of r, giving your answer to four decimal places. [3]

Nerys makes no further deposits to or withdrawals from the account.

(b) Find the year in which the amount of money in Nerys' account will become double the amount she invested. [3]

5. [Maximum mark: 6]

Ali makes cone shaped candles which have a radius of 63 mm and a height of 122 mm.

(a) Find the volume of each candle expressing your answer in the form $a \times 10^k$, $1 \leq a \leq 10$ and $k \in \mathbb{Z}$. [3]

Ali melts three cones down and remoulds them to make one candle in the shape of a sphere.

(b) Find the radius of the sphere, correct to 2 significant figures. [3]

6. [Maximum mark: 7]

The velocity, v m s^{-1}, of a particle, at time t seconds, is given by $v(t) = 10e^{0.5t} \sin 2t$, $0 \leq t \leq \pi$.

(a) Find the maximum speed of the particle and at what time this occurs. [3]

(b) Find the initial acceleration of the particle. [2]

(c) Show that the distance travelled by the particle is 48.0 m to the nearest 0.1 m. [2]

Section B

7.   [Maximum mark: 15]

For cans of a particular brand of soft drink labelled as containing 330 ml, the actual volume, V ml, of soft drink in a can is normally distributed with mean 330 and variance σ^2.

The probability that V is greater than 336 is 0.1288.

(a)   Find $\mathrm{P}(330 < V < 336)$.   [2]

(b)   (i)   Find σ, the standard deviation of V.

   (ii)   Hence, find the probability that a can of soft drink selected at random will contain less than 320 ml of soft drink.   [5]

Tilly buys a pack of 24 cans of this soft drink. It may be assumed that those 24 cans represent a random sample. Let L represent the number of cans that contain less than 320 ml of soft drink.

(c)   Find $\mathrm{E}(L)$.   [3]

(d)   Find the probability that exactly two of the cans contain less than 320 ml of soft drink.   [2]

A can selected at random contains more than 320 ml of soft drink.

(e)   Find the probability that it contains between 330 ml and 335 ml of soft drink.   [3]

8.   [Maximum mark: 13]

Consider a function f such that $f(x) = 3\sin(2x + \alpha)$ where $0 \le x \le 2\pi$ and $0 \le \alpha \le \frac{\pi}{2}$.

(a)   Write down the amplitude of f.   [1]

The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{3}, \frac{3}{2}\right)$.

(b)   (i)   Find the value of α.

        (ii)   Find the x-coordinates of the three other points where the graph of $f(x) = \frac{3}{2}$.   [4]

The function g is given by $g(x) = pf(x) + q$ where $0 \le x \le 2\pi$ and $p, q \in \mathbb{R}$.
The graph of $y = g(x)$ passes through the origin and intercepts the graph of $y = f(x)$ at the points where $f(x)$ is at its maximum value.

(c)   Find the values of p and q.   [5]

(d)   In the case when $\alpha = 0$, show that g can be written in the form
        $g(x) = k(m\sin x \cos x + n)$, where k, m and n are integers to be found.   [3]

9. [Maximum mark: 16]

Lucy sets out for a bike ride from her house at point A to her friend Heather's house at point B. She rides at an average speed of 7.4 km/h for 15 minutes, travelling in a straight line on a bearing of 333° from her house.

(a) Find the distance from point A to point B. [2]
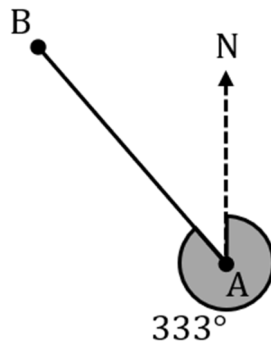
Lucy and Heather both leave point B on a bearing of 102° and continue to ride in that direction for a distance of 2.8 km until they reach a nature reserve at point C.
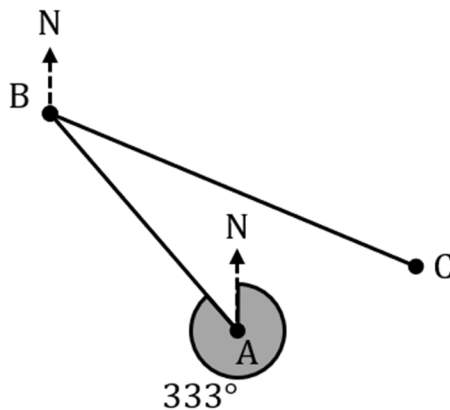
At the nature reserve, Lucy gets a puncture so decides to walk back to her house directly from point C. She is able to walk along a straight line the entire way from C to A.

(b) (i)   Show that $A\hat{B}C$ is 51°.

    (ii)   Find the distance from the nature reserve to Lucy's house at point A. [5]

(c) Find $B\hat{A}C$. [3]

(d) Find the bearing that Lucy must take to go home to point A. [3]

    It takes Lucy 48 minutes to walk home.

(e) Find the average speed Lucy must have walked on her journey home. [3]