**Mathematics: analysis and approaches**
**Standard level**
**Paper 1**

Save My Exams Practice Paper

1 hour 30 minutes

**Instructions to candidates**

- You are not permitted access to any calculator for this paper.
- Section A: answer all questions.
- Section B: answer all questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

## Section A

**1.** [Maximum mark: 5]

Let $A$ and $B$ be events such that $P(A) = 0.3$, $P(B) = 0.75$ and $P(A \cup B) = 0.9$.
Find $P(B \mid A)$.

**2.** [Maximum mark: 5]

Given that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 \cos\left(3x^3 + \dfrac{\pi}{2}\right)$ and that the graph of $y$ passes through the point $(0, -1)$, find an expression for $y$ in terms of $x$.

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**3.** [Maximum mark: 5]

The functions $f$ and $g$ are defined such that $f(x) = 6x + 7$ and $g(x) = \dfrac{x-5}{3}$.

(a)  Show that $(f \circ g)(x) = 2x - 3$. [2]

(b)  Given that $(f \circ g)^{-1}(a) = 6$, find the value of $a$. [3]

**4.** [Maximum mark: 5]
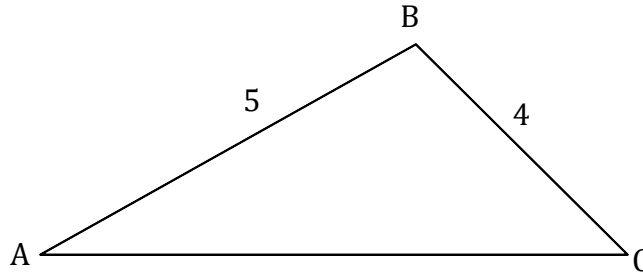
(a) (i) Expand $(2k - 1)^3$.

(ii) Hence, or otherwise, show that $(2k - 1)^3 - (2k - 1) = 8k^3 - 12k^2 + 4k$. [2]

(b) Thus prove, given $k > 1,\, k \in \mathbb{N},$ that the difference between an odd natural number greater than 1 and its cube is always even. [3]

**5.**   [Maximum mark: 5]

The following diagram shows triangle $ABC$, with $AB = 5$ and $BC = 4$.

**diagram not to scale**



(a)   (i)   Given that $\sin \widehat{B} = \dfrac{3}{5}$, find the possible values of $\cos \widehat{B}$.

(ii)   Given that $\widehat{B}$ is obtuse, find the precise value of $\cos \widehat{B}$.                                    [3]

(b)   Find the length of $AC$.                                    [2]

**6.** [Maximum mark: 8]

(a) Show that $\log_4(\cos 2x + 13) = \log_2 \sqrt{\cos 2x + 13}$. [3]

(b) Hence or otherwise solve $\log_2\left(3\sqrt{2}\cos x\right) = \log_4(\cos 2x + 13)$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$. [5]

## Section B

**7.**    [Maximum mark: 16]

Let $f(x) = \frac{1}{3}x^3 - 2x^2 - 21x - 24$.

(a)  Find $f'(x)$.                                                                                                 [2]

The graph of $f$ has horizontal tangents at the points where $x = a$ and $x = b$, $a < b$.

(b)  Find the value of $a$ and the value of $b$.                                              [3]

(c)  (i)     Find $f''(x)$.

     (ii)    Hence show that the graph of $f$ has a local maximum point at $x = a$.        [2]

(d)  (i)     Sketch the graph of $y = f'(x)$.

     (ii)    Hence, use your answer to part (d)(i) to explain why the graph of $f$ has
             a local minimum point at $x = b$.                                               [4]

The tangent to the graph of $f$ at $x = a$ and the normal to the graph of $f$ at $x = b$ intersect
At the point $(p, q)$.

(e)  Find the value of $p$ and the value of $q$.                                            [5]

**8.**   [Maximum mark: 16]

Let $f(x) = \dfrac{\ln px}{qx}$ where $x > 0, \; p, q \in \mathbb{R}^+$.

(a)   Show that $f'(x) = \dfrac{1 - \ln px}{qx^2}$.                                                                        [3]
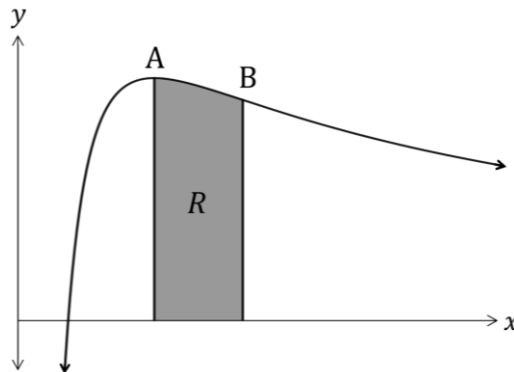
The graph of $f$ has exactly one maximum point A.

(b)   Find the $x$-coordinate of A.                                                                        [3]

The second derivative of $f$ is given by $f''(x) = \dfrac{2 \ln px - 3}{qx^3}$. The graph of $f$ has exactly one point of inflexion B.

(c)   Show that the $x$-coordinate of B is $\dfrac{e^{\frac{3}{2}}}{p}$.                                                  [3]

The region $R$ is enclosed by the graph of $f$ , the $x$-axis, and the vertical lines through the maximum point A and the point of inflexion B.



(d)   Calculate the area of $R$ in terms of $q$ and show that the value of the area is independent of $p$.                                                                        [7]

**9.**     [Maximum mark: 15]

A school surveyed 80 of its final year students to find out how much time they spent reading the news on a given day. The results of the survey are shown in the following cumulative frequency diagram.

time spent reading the news (minutes)
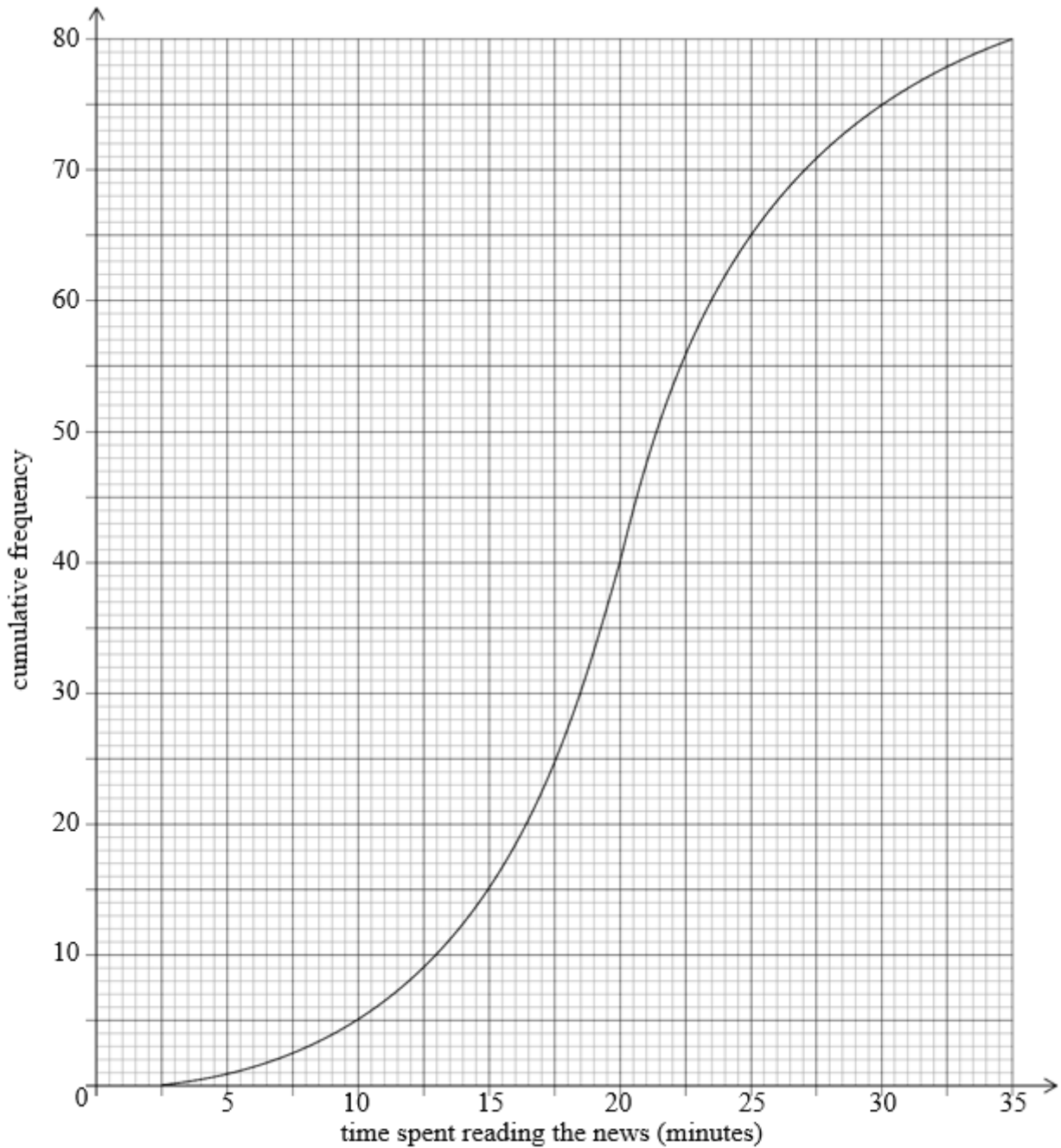
**(This question continues on the following page)**

**(Question 9 continued)**

(a) Find the median number of minutes spent reading the news. [2]

(b) Find the number of students whose reading time is within 2.5 minutes of the median. [3]

Only $15\%$ of students spent more than $k$ minutes reading.

(c) Find the value of $k$. [3]

The results of the survey can also be displayed on the following box-and-whisker diagram.

time spent reading the news (minutes)
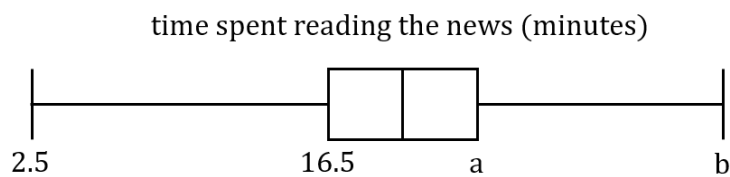


(d) Write down the value of $b$. [1]

(e) (i) Find the value of $a$.

  (ii) Hence, find the interquartile range. [4]

(f) Determine whether someone who spends 30 minutes reading the news would be an outlier. [2]