2.3 Modelling with Functions

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.3 Modelling with Functions
Difficulty	Medium

Time allowed: 90

Score: /70

Percentage: /100

Question 1a

The total cost, C, in New Zealand dollars (NZD), of a premium gym membership at Cityfitness can be modelled by the function

$$C = 16.99t + 49, \qquad t \geq 0$$

where t is the time in weeks.

(a) Calculate the cost of the gym membership for a year. Give your answer correct to 2 decimal places.

[1 mark]

Question 1b

(b) Find the number of weeks it takes for the total cost to exceed 2000 NZD.

[2 marks]

Question 1c

At Les Mills the initial payment is 20 NZD lower than Cityfitness, however the weekly cost is 8.51 NZD higher than Cityfitness

(c) Write a cost function for a gym membership at Les Mills using an appropriate model.

[1 mark]

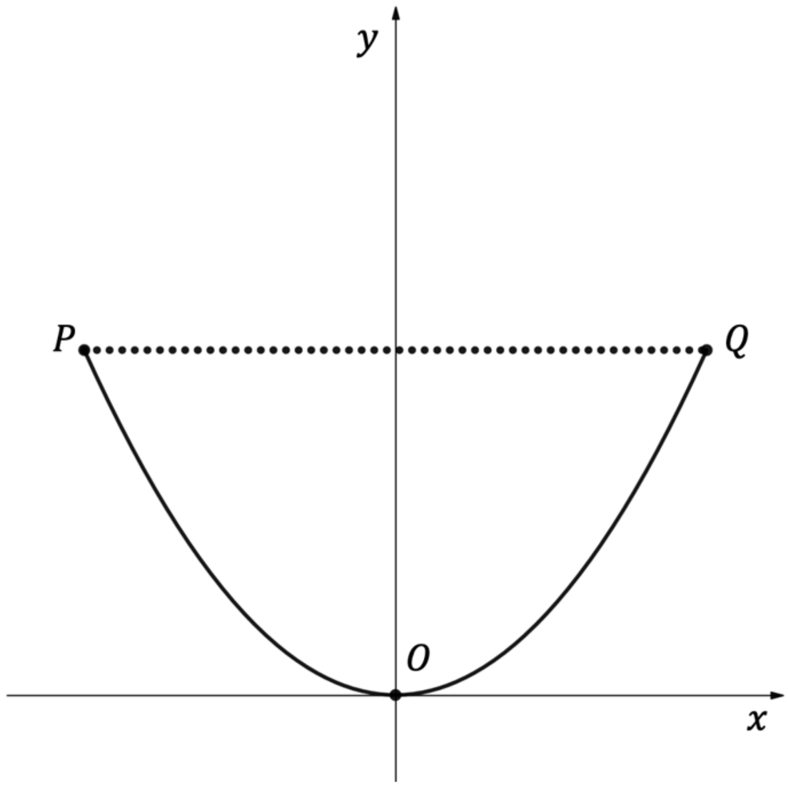
Question 1d

(d) Calculate how many weeks it will take for the cost of a Les Mills gym membership be more than the cost of a Cityfitness gym membership.

[3 marks]

Question 2a

The front view of the edge of a water tank is drawn on a set of axes below. The edge is modelled by $y = ax^2 + c$.



Point P has coordinates $(-4, 4)$, point O has coordinates $(0, 0)$ and point Q has coordinates $(4, 4)$.

(a) (i) Find the value of c.

(ii) Find the value of a.

(iii) Hence, write down the equation of the quadratic function which models the edge of the water tank.

[4 marks]

Question 2b

(b) Given that 1 unit represents 1 m, find the width of the water tank when its height is 2.25 m.

[2 marks]

Question 3a

The number of German words, W, that Helen remembers after completing a German language course decreases exponentially over time when she does not practice her German. This decrease can be modelled by the function

$$W(t) = a \times b^{-t} + 320, \qquad t \geq 0$$

Where a and b are positive constants and t is the time in years since Helen completed the German language course.

Helen can remember 2400 German words as soon as she completes the German language course.

(a) Find the value of a.

[2 marks]

Question 3b

After 2 years Helen has not practiced her German and can only remember 1020 German words.

(b) Find the value of b.

[3 marks]

Question 3c

The number of German words Helen remembers never decreases below a certain number of words, c.

(c) Find the value of c.

[1 mark]

Question 4a

In a trial for a new drug, scientists found that the amount of the drug in the bloodstream decreased over time, according to the model

$$D(t) = 1.4 \times 0.77^t, \qquad t \geq 0$$

where D is the amount of the drug in the bloodstream in mg per litre (mg L^{-1}) and t is the time in hours.

(a) Write down the amount of the drug in the bloodstream at $t = 0$.

[1 mark]

Question 4b

(b) Calculate the amount of the drug in the bloodstream after four hours.

[2 marks]

Question 4c

(c) Calculate the time, in hours, for the amount of the drug in the bloodstream to decrease to 0.22 mg L^{-1}.

[3 marks]

Question 4d

The scientists found that some of the test subjects had an elevated heart rate for 45 minutes after ingesting the drug.

(d) Find the amount of the drug in the bloodstream when the heart rates from the effected test subjects returned to normal.

[2 marks]

Question 5a

The number of bacteria in a Petri dish is modelled by the function

$$N(t) = 75 \times 2^{0.5t}, \qquad t \geq 0$$

where N is the number of bacteria and t is the time in hours.

(a) Write down the number of bacteria in the Petri dish at $t = 0$.

[1 mark]

Question 5b

(b) Calculate the number of bacteria present after 10 hours.

[2 marks]

Question 5c

(c) Calculate the time, in hours, for the number of bacteria to reach 10 000.

[3 marks]

Question 6a

A remote-controlled sailboat's velocity is dependent on the wind speed. The sailboat's velocity is lower during very high and very low wind speeds.

The sailboat's velocity can be modelled by the function

$$V(w) = 0.0025w(2 - w)(w - 35), \qquad 2 \leq w \leq 35$$

where V is the sailboat's velocity, in km h^{-1}, and w is the wind speed, in km h^{-1}.

(a) Find the sailboat's velocity when the windspeed is 20 km h^{-1}.

[1 mark]

Question 6b

(b) Find the windspeed when the sailboat's velocity is 5.94 km h^{-1}.

[2 marks]

Question 6c

(c) Show that $V(w) = -0.0025w^3 + 0.0925w^2 - 0.175w$.

[2 marks]

Question 6d

(d) Using your graphics display calculator find the maximum velocity of the sailboat and the windspeed required for this.

[3 marks]

Question 7a

A Ferris wheel rotates at a constant speed, the height of a particular seat above the ground is modelled by the function

$$H(t) = -14\cos(10° \times t) + 16, \qquad t \geq 0$$

where H is the height of the seat above the ground, in metres, and t is the elapsed time, in seconds, since the start of the ride.

(a) Write down

(i) the minimum height of the seat

(ii) the maximum height of the seat.

[4 marks]

Question 7b

(b) Calculate the number of seconds it takes for the Ferris wheel to do one full rotation.

[2 marks]

Question 8a

The water depth, D, in metres, at a port can be modelled by the function

$$D(t) = 5\sin(30° \times t) + 15, \qquad 0 \le t \le 24$$

where t is the elapsed time, in hours, since midnight.

(a) Write down the depth of the water at midnight.

[1 mark]

Question 8b

(b) The cycle of water depths repeats every P hours. Find the value of P.

[2 marks]

Question 8c

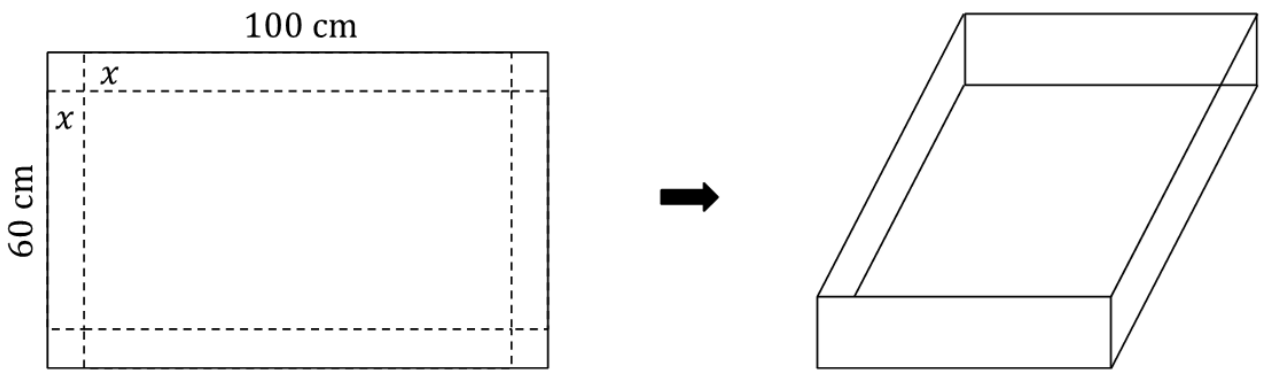
(c) (i) Calculate the maximum and minimum depths.

(ii) Find the times at which the maximum and minimum depths occur during the day.

[4 marks]

Question 9a

A rectangular sheet of cardboard 60 cm by 100 cm has square sides of x cm cut from each corner. It is folded to make an open box as shown.



(a) Show that the volume of the box can be modelled by the function

$$V = 4x^3 - 320x^2 + 6000x\ .$$

[3 marks]

Question 9b

(b) State the domain of V.

[2 marks]

Question 9c

(c) Using your graphics display calculator find the maximum value of V and the value of x which gives this volume.

[2 marks]

Question 10a

Grace leaves a cup of hot tea to cool and measures its temperature every minute. Her results are shown in the table below.

Time, t (minutes)	0	1	2	3	4
Temperature, y (°C)	88	58	43	35.5	k

(a) Write down the decrease in temperature of the tea

(i) during the first minute

(ii) during the second minute

(iii) during the third minute.

[3 marks]

Question 10b

(b) Assuming the pattern in the answers to part (a) continues, find the value of k. Leave your answer correct to 2 decimal places.

[2 marks]

Question 10c

The function that models the change in temperature of the tea is $y = a(2^{-t}) + b$, where b represents the temperature the tea tends towards and $a + b$ is the initial temperature.

(c) Write down two equations relating a and b.

[2 marks]

Question 10d

(d) Find the value of a and b.

[2 marks]