# 1.5 Further Proof & Reasoning

## Question Paper

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| Course | DP IB Maths |
| Section | 1. Number & Algebra |
| Topic | 1.5 Further Proof & Reasoning |
| Difficulty | Medium |

**Time allowed:** 80

**Score:** /59

**Percentage:** /100

## Question 1

Prove that there is no $x \in \mathbb{R}$ such that $-\frac{2}{x-2} = x-3$.

**[5 marks]**

## Question 2

Using the method of proof by contradiction, prove that $\sqrt{7}$ is irrational.

**[4 marks]**

## Question 3

Using mathematical induction, prove that $6^n - 1$ is divisible by 5 for $n \in \mathbb{Z},\ n \geq 1$.

**[4 marks]**

## Question 4a

The $n$th triangular number is given by the formula $u_n = \frac{1}{2}n(n+1)$.

(a)
Write down the first five triangular numbers.

**[1 mark]**

## Question 4b

(b)
Prove by exhaustion that the first five triangular numbers are all factors of 180.

**[2 marks]**

## Question 5

Determine, with appropriate reasoning, whether the following statements are true or false:

(i)
Given $n \in \mathbb{Z}$ and $n^2$ is divisible by 4, then $n$ is divisible by 4.

(ii)
Given $n \in \mathbb{Z}$ then $n^2 - 1$ is a prime number.

(iii)
Given $n \in \mathbb{Z}$ and $n^2$ is divisible by 3, then $n$ is divisible by 3.

(iv)
Given an integer is a multiple 8 and 6 then it is a multiple of 48.

**[8 marks]**

## Question 6

Prove that $x^2 - 3x + 3$ is positive for all real values of $x$.

**[3 marks]**

## Question 7a

(a)
Show that $(3n + 2)^2 - (n + 2)^2 = 8n^2 + 8n$, where $n \in \mathbb{Z}$.

**[2 marks]**

## Question 7b

(b)
Hence, or otherwise, prove that $(3n + 2)^2 - (n + 2)^2$ is a multiple of 8.

**[2 marks]**

## Question 8

Prove that $(a - b)^2 - (a + b)^2 = -4ab$.

**[3 marks]**

## Question 9

Prove that $(4x-1)(2x+3)-(2x+1)^2=2(2x-1)(x+2)$.

**[3 marks]**

## Question 10

Prove by mathematical induction $3^n \geq 1+2n$, given $n \geq 0$.

**[6 marks]**

## Question 11

Prove by mathematical induction that if $y = \frac{1}{1-x}$ then $\frac{d^n y}{dx^n} = \frac{n!}{(1-x)^{n+1}}$.

**[6 marks]**

## Question 12

Prove by induction that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

for all values of $n$, $n \in \mathbb{Z}^+$.

**[6 marks]**

## Question 13

Given $z = x + yi$

(i)
prove that $zz^* = |z||z^*|$,
(ii)
prove that, for $x \geq 0$, $\arg(z) + \arg(z^*) = 0$.

**[4 marks]**