4.6 Normal Distributions

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.6 Normal Distributions
Difficulty	Very Hard

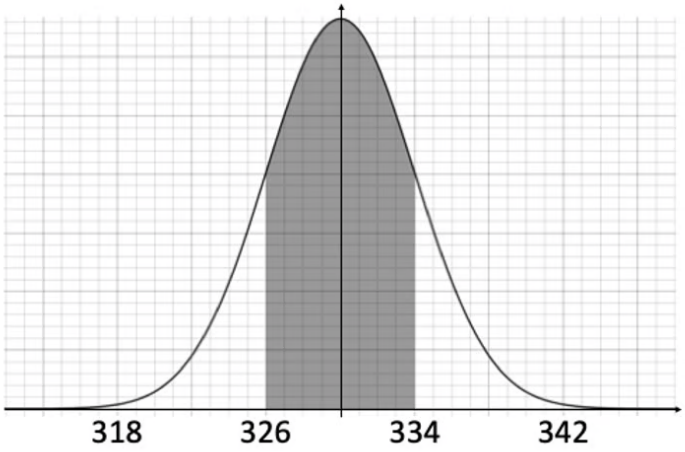
Time allowed: 120

Score: /93

Percentage: /100

Question 1a

The graph below shows the normal distribution of the volume, in ml, of drink in a can provided by a manufacturer, with the central 68% of the distribution shaded.



(a) State the mean volume of ml found in a can and the standard deviation.

[2 marks]

Question 1b

(b) The probability that a can picked at random has a volume that is greater than v is 0.19. Find v.

[2 marks]

Question 1c

A sample of cans of drink were analysed and 27 were found to have a volume of less than 320 ml.

(c) Find an estimate for the number of cans of drink that were in the sample.

[3 marks]

Question 2a

It is known that the time in minutes, T, that a customer spends on hold when calling a customer service line follows a normal distribution, where $T \sim \mathrm{N}(\mu, 4^2)$. The probability that a customer spends more than 25 minutes on hold is 0.0228.

(a) Find the mean length of time that a customer spends on hold.

[3 marks]

Question 2b

(b) Find the interquartile range of the hold times.

[2 marks]

Question 2c

(c) Find the time spent on hold that is exceeded by 6% of the population.

[2 marks]

Question 2d

(d) From a sample of 200 customers find the probability that exactly 25% of the customers in the sample would spend less than 15 minutes on hold.

[3 marks]

Question 3a

The stem heights, H in cm, of a particular variety of tulip follow a distribution where $H \sim \mathrm{N}(60.1, 57.76)$.

The probability that a randomly selected tulip will have a stem height that is greater than h is 0.72.

(a) Find h.

[3 marks]

Question 3b

A tulip is selected at random. It is known that the height of the stem is more than 62 cm.

(b) Find the probability that the stem height is taller than 64 cm.

[4 marks]

Question 3c

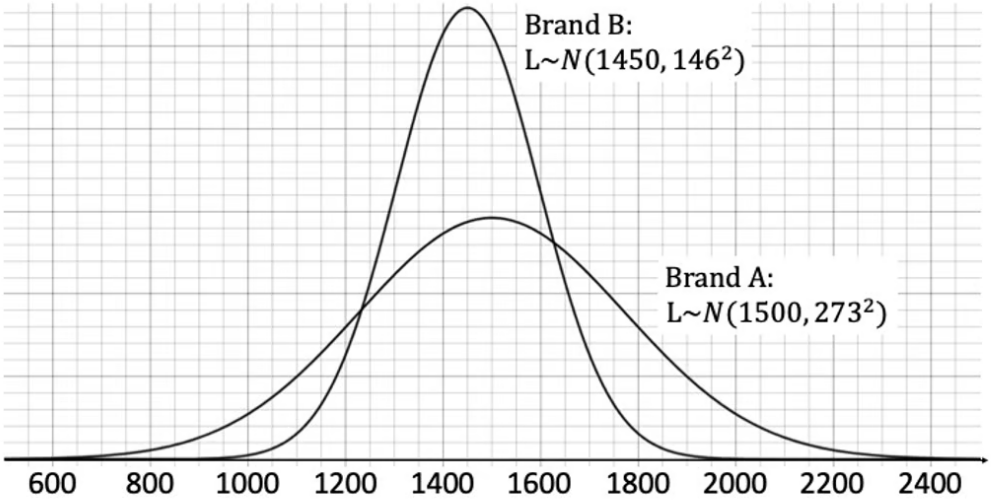
Leila buys a bunch of 24 tulips.

(c) Calculate the probability that at least 10 of them have a stem height of less than 57 cm.

[4 marks]

Question 4a

The diagram below shows the normal distributions of the life expectancy, L, in hours for two different brands of vacuum cleaner.



(a) State which vacuum cleaner brand is more reliable. Give a reason for your answer.

[2 marks]

Question 4b

(b) Using the information for the brand of vacuum cleaner than you stated in part (a), find the probability that the life expectancy of the vacuum will be between 1550 hours and 1700 hours.

[2 marks]

Question 4c

When sold, each brand B vacuum cleaner comes with a 5-year warranty, where the manufacturer will replace the vacuum in the event of failure. It is assumed that a typical customer uses the vacuum cleaner for approximately 4 hours each week.

(c) Find the probability that from a batch of 620 brand B vacuum cleaners, more than 0.5% of them could be returned to the manufacturer within the warranty period.

[4 marks]

Question 5a

The distance that a honeybee will travel from the hive to collect pollen is normally distributed with an average distance of 1.1 miles. The probability that a honeybee will travel further than 1.6 miles from the hive is 0.00621.

(a) Find the variance of the distances that the honeybees will travel from the hive.

[4 marks]

Question 5b

83% of the honeybees travel a distance that is greater than h miles.

(b) Find the value of h.

[2 marks]

Question 5c

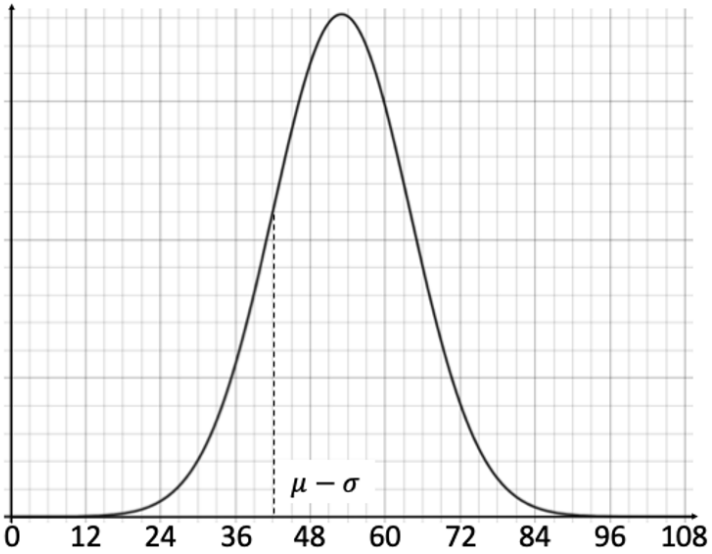
A colony consists of 60 000 honeybees.

(c) Calculate the probability that at least 28% but no more than 31% of the honeybees in this colony will stay within 1 mile of the hive.

[4 marks]

Question 6a

A teacher sets the same test every year for which the results follow a normal distribution with a mean score of 53 points. The teacher decides that any student who scores lower than one standard deviation below the mean will have to re-sit the test. The diagram below shows the distribution of test scores and the boundary for those that need to re-sit.



For a class that complete the test, four students are instructed that they will need to re-sit.

(a) Find:

(i) the standard deviation of the test scores.

(ii) an estimate for the total number of students in the class

[3 marks]

Question 6b

A second class of 28 students sit the same test. If they achieve a score of greater than 65, they will achieve a commendation.

(b) Find the probability that exactly 3 students in this class achieve a commendation.

[4 marks]

Question 7a

The masses, M, of cockroaches are normally distributed with an average mass of 28.8 g.

It is known that 0.15% of the population of cockroaches has a mass greater than 51.9 g.

(a) Find an approximate value for the variance of the population of cockroaches.

[3 marks]

Question 7b

38 cockroaches are caught in a trap.

(b) Using your value for the variance in part (a), find the probability that:

(i) exactly 15 of the cockroaches have a mass that is greater than 30 g

(ii) more than 26 cockroaches have a mass that is between 26 g and 56 g.

[6 marks]

Question 8a

Alannah cycles to school each day via one of two possible routes and the time taken to complete either journey follows a normal distribution.

The journey time for route A has a mean of 28 minutes and a standard deviation of 10 minutes.

The journey time for route B has a mean 33 of minutes and a standard deviation of 4 minutes.

(a) Identify an advantage of each route.

[2 marks]

Question 8b

The school day begins at 08:30 and Alannah leaves her house at 07:55.

(b) Determine which route is more likely to make her late.

[3 marks]

Question 8c

Route A is closed off for maintenance work so Alannah travels by route B every day from Monday to Friday.

(c) Find the probability that she is on time for school on 2 consecutive days and late for the other 3 days.

[5 marks]

Question 9a

Within a given population, the length of time that a person can hold their breath is normally distributed. 42% of the population can hold their breath for up to a maximum of 1.2 minutes but only 0.5% of the population can hold their breath for longer than 2.8 minutes.

(a) Find

(i) the mean,

(ii) the variance.

[4 marks]

Question 9b

In order to be able to access and swim through a particular underwater cave without breathing equipment a person must be able to hold their breath for longer than 3.5 minutes.

A sample of 1200 people from the population in part (a) are surveyed to determine how long they can hold their breath for.

(b) Find the probability that 2 or more people would be able to hold their breath long enough to be able to swim through the underwater cave.

[3 marks]

Question 10a

It is known that the time spent on a smart phone per day by teenage boys is normally distributed with a mean of 306 minutes and a variance of 56^2.

(a) Find the proportion of teenage boys that spend between 275 minutes and 375 minutes per day on a smart phone. Sketch a diagram to show this information.

[4 marks]

Question 10b

A sample of 50 teenage boys are selected from this population.

(b) Find the expected number of boys from the sample who spend more than 7 hours on their smart phone each day.

[3 marks]

Question 10c

The time spent on smart phones each day by teenage girls is also considered to follow a normal distribution with a mean of 286 minutes and a standard deviation of 74 minutes.

(c) 25 teenage girls are selected at random. Find the probability that exactly 3 of them spend less than 200 minutes on a smart phone each day.

[3 marks]

Question 10d

A teenager is selected at random from the group of 50 teenage boys in part (b) and the group of 25 teenage girls in part (c).

(d) Find the probability that a teenager picked at random is a boy, given that they spend less than 250 minutes on a smart phone each day.

[4 marks]