

# 2.2 Further Functions & Graphs

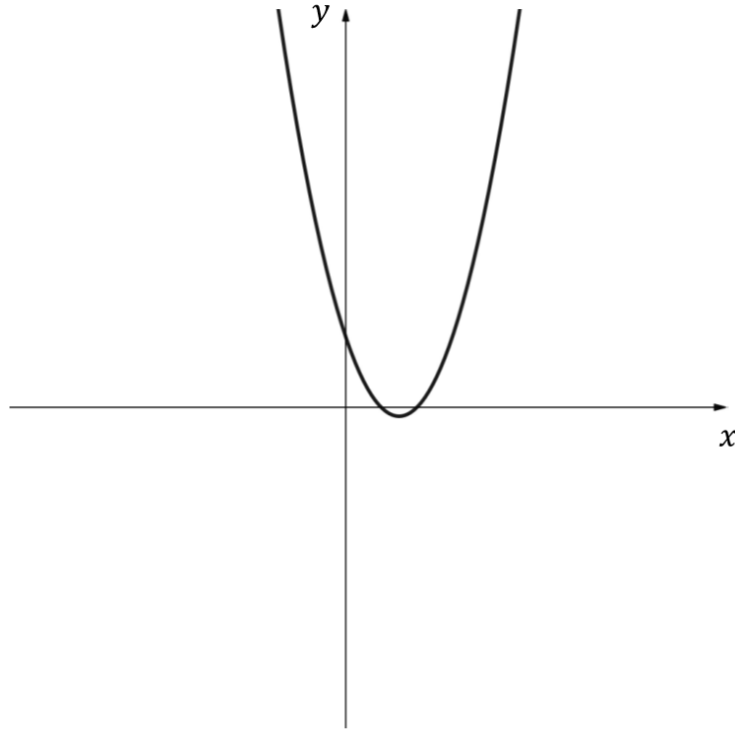
## Question Paper

Course	DPIB Maths
Section	2. Functions
Topic	2.2 Further Functions & Graphs
Difficulty	Hard

**Time allowed:** 100  
**Score:** /77  
**Percentage:** /100

**Question 1a**

Let  $f(x) = x^2 - 3x + 2$ . The diagram below shows part of the graph of  $f$ .



Another function is defined by  $g(x) = 2 - x$ .

(a) Sketch the graph of  $g$  on the axes above.

[1 mark]

**Question 1b**

The graph of  $f$  and  $g$  intersect at points A and B.

(b) Find the coordinates of A and B and label them on the diagram above.

[3 marks]

**Question 1c**

(c) Find the length of the line AB.

[2 marks]

**Question 2a**

Let  $f(x) = \frac{7}{2(x-7)} - 5$ , for  $x \neq 7$ .

(a) For the graph of  $f$ , find the:

- (i)  $x$ -intercept
- (ii)  $y$ -intercept.

[2 marks]

**Question 2b**

(b) For the graph of  $f$ , write down the equation of any asymptotes.

[2 marks]

**Question 2c**

Let  $g(x) = 2(1 - 2x)$ , for  $x \in \mathbb{R}$ . The graphs of  $f$  and  $g$  intersect at points P and Q.

(c) Write down the coordinates of P and Q.

[2 marks]

**Question 2d**

(d) Find the distance of PQ.

[2 marks]

**Question 3a**

The perimeter,  $P$ , and area,  $A$ , of a given square can be expressed by  $P = 4x$  and  $A = x^2$  respectively, where  $x$  is the length of the side of the square.

(a) Write down an expression for:

(i)  $P$  in terms of  $A$ ,  $P(A)$

(ii)  $A$  in terms of  $P$ ,  $A(P)$ .

[4 marks]

**Question 3b**

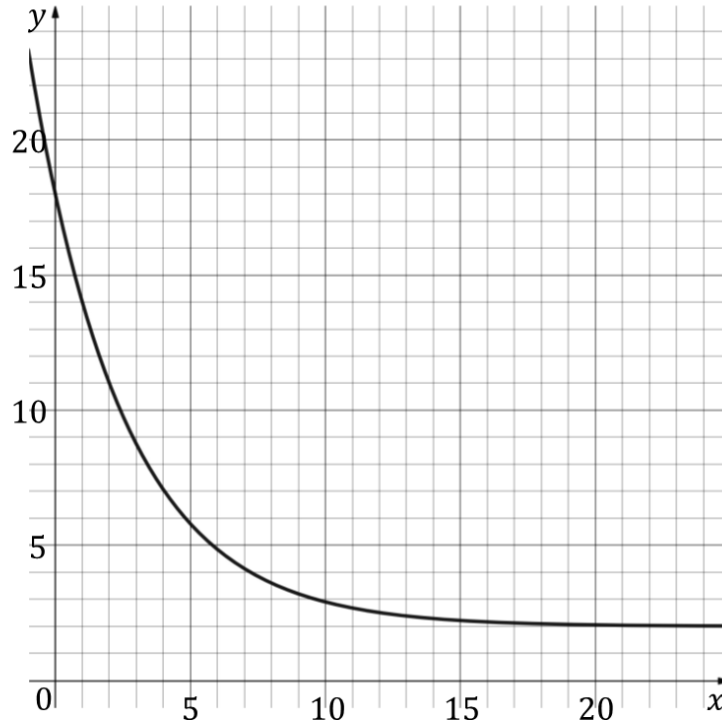
$$P^{-1}(40) = A(k).$$

(b) Find the value of  $k$  and  $A(k)$ .

[2 marks]

**Question 4a**

Consider the function  $f(x) = a(0.75)^x + b$  where  $a$  and  $b$  are constants. The graph of  $f$  passes through the points  $(0, 18)$  and  $(2, 11)$  and is shown below.



(a) Write down two equations relating  $a$  and  $b$ .

[2 marks]

**Question 4b**

(b) Find the value of  $a$  and  $b$ .

[2 marks]

**Question 4c**

(c) Write down the equation of the horizontal asymptote of the graph of  $f$ .

[2 marks]

**Question 5a**

The function  $f(x) = ax^2 + bx + c$  intercepts the  $y$ -axis at  $-12$  and has an  $x$ -intercept at  $x = 3$ . The function can be obtained by an appropriate shift of the graph  $y = -4x^2$ .

(a) Find the values of  $a$ ,  $b$  and  $c$ .

[4 marks]

**Question 5b**

(b) Find the other  $x$ -intercept of  $f(x)$ .

[1 mark]

**Question 5c**

(c) Determine the coordinates of the maximum value of  $f(x)$ .

[2 marks]

**Question 6a**

A function is defined by  $f(x) = \frac{1}{(x-3)^2} + 2$ ,  $x \neq p$ .

(a) Find the value of  $p$ .

[1 mark]

**Question 6b**

(b) For the graph of  $f$  write down the equation of any asymptotes.

[2 marks]

**Question 6c**

(c) Find the range of  $f$ .

[1 mark]



**Question 6d**

The line  $l$  intersects the graph of  $f$  when  $x = 1$  and when  $x = 4$ .

- (d) Find the equation of  $l$ . Give your answer in the form  $ax + by + d = 0$ , where  $a$ ,  $b$  and  $d$  are integers.

[4 marks]

**Question 7a**

A function is defined by  $f(x) = 4 - \frac{12}{5x+9}$ ,  $x \neq a$ .

- (a) Find the value of  $a$ . Give your answer as a fraction.

[2 marks]

**Question 7b**

- (b) Find the range of  $f$ .

[3 marks]

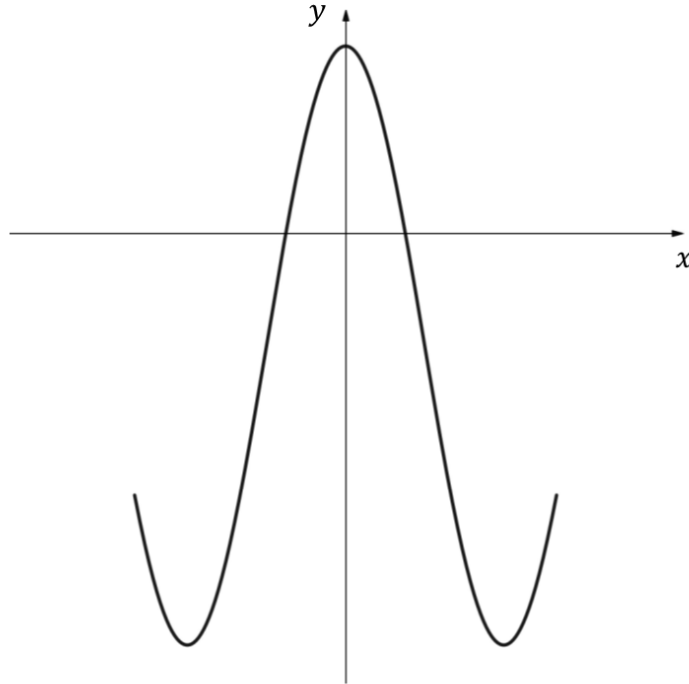
**Question 7c**

(c) Find the value of  $f^{-1}(2)$ . Give your answer as a fraction.

[2 marks]

**Question 8a**

The diagram below shows the graph of the function  $f(x) = 8\cos(48x) - 3$ , for  $-5 \leq x \leq 5$ .



(a) For the graph of  $f$ , find the:

- (i)  $x$ -intercepts
- (ii)  $y$ -intercept.

[2 marks]

**Question 8b**

(b) Write down the range of  $f$ .

[1 mark]

**Question 8c**

- (c) Three lines are drawn connecting the two local minimum points and the local maximum point forming a triangle. Calculate the area of the triangle.

[4 marks]

**Question 9a**

Let  $f(x) = 2 - 5 \cos(30(3x - 1))$ , for  $x > 0$ .

The  $n$ th maximum point on the graph of  $f$  has  $x$  coordinate  $x_n$ , where  $n \in \mathbb{Z}^+$ .

- (a) Given that  $x_n = x_1 + (n - 1)d$ , find  $x_1$  and  $d$ .

[4 marks]

**Question 9b**

- (b) (i) Using sigma notation, write down an expression for  $x_1 + x_2 + x_3 + \dots + x_8$ .
- (ii) Find the value of the sum from part (b) (i). Give your answer to 2 decimal places.

**[4 marks]****Question 10a**

The average fat-free mass,  $M$ , in kg, of footballers as a function of their age,  $a$ , in years, can be given by the logarithmic function:

$$M(a) = 10 \log(a - 15) + 50, \quad 16 \leq a \leq 25.$$

- (a) Calculate the average fat free mass of players aged:
- (i) 16 years
- (ii) 25 years.

**[2 marks]**

### Question 10b

(b) Find an expression for a linear model using your answers to part (a) (i) and (ii).

[3 marks]

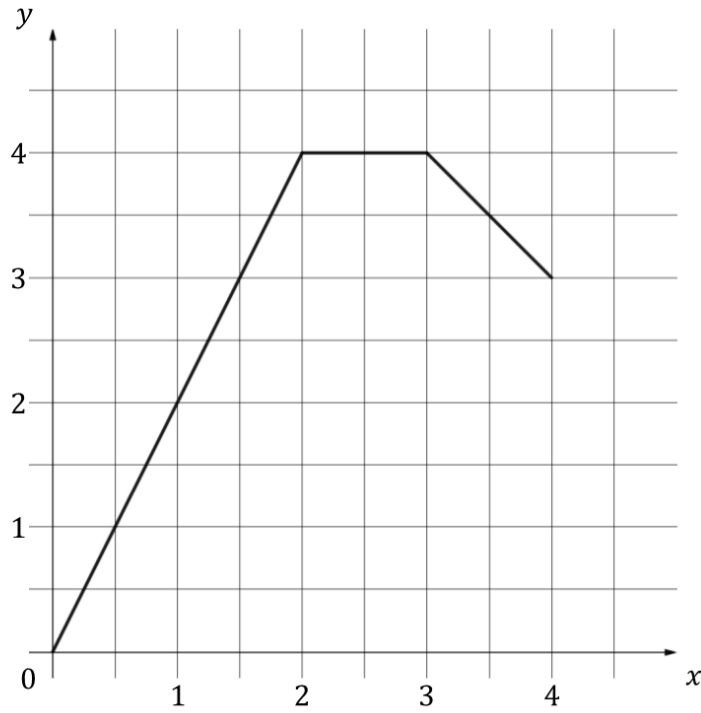
### Question 10c

(c) Calculate the percentage error from using the linear model found in part (b) to approximate the average fat free mass of a player aged 20 years old.

[3 marks]

**Question 11a**

The axes below shows the graph of the piecewise function,  $f$



The gradient of the graph of  $f$  is 0 for  $a \leq x < b$ .

(a) Find the values of  $a$  and  $b$ .

[2 marks]

**Question 11b**

(b) Complete the following piecewise function for  $f$

$$f(x) = \begin{cases} & 0 \leq x < 2 \\ & 2 \leq x < 3 \\ & 3 \leq x \leq 4 \end{cases}$$

[4 marks]