5.5 Kinematics

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.5 Kinematics
Difficulty	Hard

Time allowed: 110

Score: /86

Percentage: /100

Question 1a

A golf ball is hit from a point O on a horizontal golf course, and travels at all times in a vertical plane that passes through O. The trajectory of the golf ball is modelled by the equation

$$y = -\frac{x^2}{180} + x$$

where x and y are respectively the horizontal and vertical displacements, in metres, of the golf ball relative to point O. Upwards is taken to be the positive direction for the vertical displacement.

(a) Write down the interval of x values for which the model is valid, explaining why the interval given is suitable within the context of the question.

[2 marks]

Question 1b

(b) Sketch the graph of y against x, labelling any intersections with the coordinate axes.

[2 marks]

Question 1c

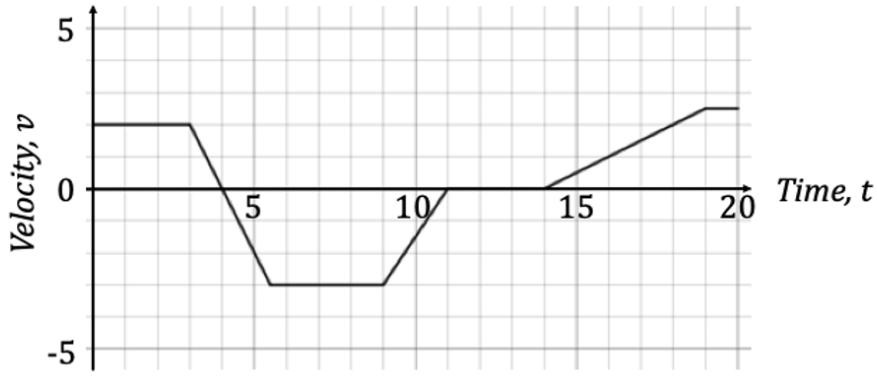
(c) Find the maximum height reached by the golf ball.

[1 mark]

Question 2a

A ball is placed at the midpoint, X, of a horizontal tube of length 120 cm. A player is positioned at each end of the tube and is required to pump air into the tube in order to move the ball.

The velocity-time graph for a 20 second game is shown below, where v is the velocity of the ball in cm s^{-1} and t is the time in seconds since the start of the game.



(a) Write down the maximum speed of the ball during the game and the time interval during which that maximum speed occurs.

[2 marks]

Question 2b

(b) There are two periods of time during which the ball is decelerating. Find the magnitude of the deceleration that the ball undergoes after it has changed direction for the first time.

[3 marks]

Question 2c

(c) Find the total distance that the ball travels in the game.

[3 marks]

Question 3a

A particle moves along a straight line relative to a fixed point, P. The motion of the particle can be modelled by the function

$$s(t) = \frac{t^3}{20} - \frac{17t^2}{8} + 18t - 4 \qquad 0 \leq t \leq 30$$

where s is the horizontal displacement in metres from point P and t is the time in seconds.

(a) Write down the initial distance of the particle from point P.

[1 mark]

Question 3b

(b) Find the final displacement of the particle from P.

[1 mark]

Question 3c

(c) Find an expression, in terms of t, for the velocity of the particle.

[2 marks]

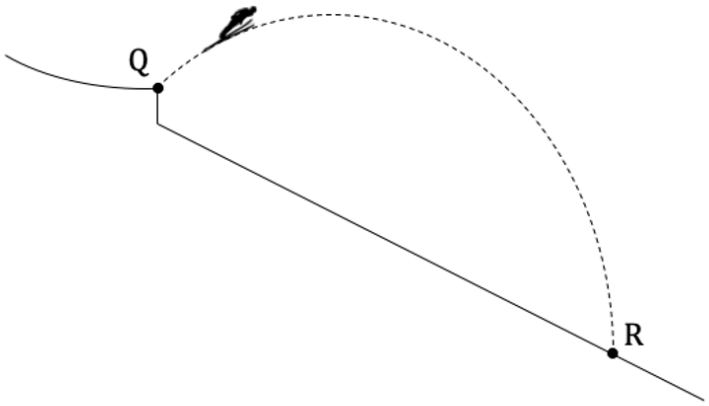
Question 3d

(d) Using your answer to part (c), find the times at which the particle is stationary.

[5 marks]

Question 4a

A skier performs a jump in a competition. She leaves the ground at point Q and travels in a vertical plane through Q, landing at point R. This can be seen in the diagram below.



The trajectory of the jump can be modelled as

$$h(x) = -\frac{x}{80}(x - 35) + 30, \qquad 0 \leq x < 70$$

where x is the horizontal displacement of the skier from point Q and h is the vertical displacement of the skier relative to point R.

(a) Write down the vertical distance between points Q and R.

[1 mark]

Question 4b

(b) Show that when the skier is again at the same vertical height from which she started, then her horizontal distance from point Q is 35 m.

[2 marks]

Question 4c

(c) Find an expression for $h'(x)$ and hence find the total distance in the vertical direction that the skier will travel. Show your working.

[6 marks]

Question 5a

A particle moves in a straight line with a velocity v ms^{-1} given by $v(t) = \sqrt{t^2 + 2t} - 3$, where t is measured in seconds such that $0 \leq t \leq 5$.

(a) Find the acceleration of the particle at time $t = 3.7$.

[2 marks]

Question 5b

(b) (i) Find the change in the particle's displacement between the times $t = 0$ and $t = 1$.

(ii) Explain what this change in displacement tells you about the particle's motion between those two times.

[3 marks]

Question 5c

(c) Find the total distance travelled by the particle.

[3 marks]

Question 6a

A marble is projected along a marble run that travels in a vertical plane through a fixed point O. The marble's vertical distance, h, in cm above point O can be modelled by

$$h(t) = 15 - 10t + 2t^2, \quad 0 \leq t \leq 6$$

where t is the time in seconds after the marble is projected.

(a) Write down the initial height of the marble relative to the fixed point O.

[1 mark]

Question 6b

(b) Find the time at which the marble reaches its lowest point, and find the total vertical distance that the marble has travelled up to that time.

[6 marks]

Question 6c

(c) Find the velocity and acceleration of the marble at the end of the given time period.

[4 marks]

Question 7a

A particle is found to have a velocity, v ms^{-1} that can be expressed by the function

$$v = t^3 \cos t, \text{ where } t \geq 0$$

(a) Find an expression for the acceleration, a, of the particle.

[4 marks]

Question 7b

(b) Hence, find the acceleration of the particle at the time $t = 5.2$.

[2 marks]

Question 8a

A particle moves along a straight line such that its displacement s in metres from a fixed point O is given by

$$s(t) = \frac{1}{2}t - \sin 2t\,, \qquad \text{for } 0 \leq t \leq 6$$

where t is the time in seconds.

(a) Write down the number of changes of direction that the particle makes.

[1 mark]

Question 8b

(b) Find an expression for the velocity of the particle at time t.

[3 marks]

Question 8c

(c) Find the maximum velocity and the time(s) at which it occurs.

[3 marks]

Question 8d

(d) Of the total distance travelled by the particle, calculate the percentage that the particle travels in the first 2 seconds of its movement.

[3 marks]

Question 9a

A particle is moving along a straight line. The position of the particle at time t seconds, measured in metres relative to a fixed origin point, is denoted by $x(t)$.

The particle starts at rest at the origin at time $t = 0$, and its motion over the next six seconds is described by the equation

$$\ddot{x}(t) = \frac{4\pi^2}{9}\sin\left(\frac{2\pi t}{3}\right) - \frac{\pi^2}{36}\cos\left(\frac{\pi t}{12}\right), \quad 0 \leq t \leq 6$$

(a) Find expressions for

(i) $\dot{x}(t)$

(ii) $x(t)$.

[7 marks]

Question 9b

(b) Find the maximum value that each of the following quantities takes on during the first six seconds of movement, as well as the time t at which those maximum values occur:

(i) the distance of the particle from the origin

(ii) the speed of the particle

(iii) the magnitude of the particle's acceleration.

[4 marks]

Question 9c

(c) Find the total distance travelled by the particle during the first six seconds of its movement.

[2 marks]

Question 10a

A particle is moving along a straight line. The position of the particle at any given time, measured in metres relative to a fixed origin point, is denoted by x.

It is known that the velocity, v ms^{-1}, of the particle is dependent on the particle's position, and that the velocity may be described by the function

$$v(x) = -\sqrt{4 - 9x^2}, \qquad -\frac{2}{3} \leq x \leq \frac{2}{3}$$

(a) Show that the acceleration, a ms^{-2}, of the particle may be expressed in the form

$$a = v\frac{\mathrm{d}v}{\mathrm{d}x}$$

[3 marks]

Question 10b

(b) Hence find a function giving the acceleration of the particle in terms of x.

[1 mark]

Question 10c

(c) Identify the minimum and maximum values of

(i) the speed of the particle

(ii) the magnitude of the particle's acceleration

along with the values of x for which those occur.

[3 marks]