3.10 Vector Equations of Lines

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.10 Vector Equations of Lines
Difficulty	Very Hard

Time allowed: 110

Score: /87

Percentage: /100

Question 1

The line l has equation $r = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix}$ and point A has coordinates $(3,\ t,\ 2)$. Given that the shortest distance between point A and the line is $\frac{\sqrt{645}}{15}$ units, find t, where $t \in \mathbb{Z}$.

[7 marks]

Question 2a

A line l_1 has the equation $r_1 = (2+\lambda)\mathrm{i} + (6\lambda - 3)\mathrm{j} + (5+2\lambda)k$ and intersects the line l_2 with equation $r_2 = 5\mathrm{i} + (7-4\mu)\mathrm{j} + (-3-7\mu)k$ at point P, when $\lambda = 3$.

A third line l_3 runs parallel to l_1 and also intersects l_2 at point $X(t,\ t-2,\ -2t)$.

(a)
Find the parametric equations of l_3.

[6 marks]

Question 2b

(b)
Find the distance $|PX|$.

[2 marks]

Question 3a

Consider the two intersecting lines l_1 and l_2 defined by the equations:

$$l_1: r = \begin{pmatrix} 9 \\ 18 \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} -6 \\ -3 \\ k \end{pmatrix}$$

$$l_2: \frac{x+5}{2} = \frac{y+t}{-4} = \frac{z-20}{3}$$

(a)
Given that the angle between l_1 and l_2 is 1.281 rad, correct to 4 significant figures, find the value of k, where $k \in \mathbb{Z}$.

[4 marks]

Question 3b

(b)

Find the value of t, giving your answer correct to 3 significant figures.

[3 marks]

Question 4

Consider the two lines l_1 and l_2, where l_1 passes through the points $\mathrm{A}(11, -2, 3)$ and $\mathrm{B}(4, 4, -5)$ and l_2 is defined by the Cartesian equations $\frac{x+7}{3} = \frac{2y+9}{6} = \frac{z+4}{-4}$

Find the shortest distance between the two lines.

[8 marks]

Question 5a

Consider the line l_1 as defined by the equation $r_1 = \begin{pmatrix} -2 \\ 5 \\ -8 \end{pmatrix} + \alpha \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$.

A point $\mathrm{P}(r,\ t,\ -r)$ lies at a distance of $\sqrt{405}$ units perpendicular from a point $X(17,\ 15,\ -8)$ on l_1.

(a)
Find all possible coordinates of P.

[6 marks]

Question 5b

(b)
Given that $t > 0$, write down the set of parametric equations that defines the line l_2 that passes through points P and X.

[2 marks]

Question 5c

A third line l_3 is defined by the equations $\frac{-x-13}{5} = \frac{-y-9}{4} = \frac{z-4}{2}$.

(c)
Determine the relationship between lines l_2 and l_3.

[3 marks]

Question 6a

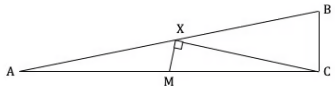
A wheelchair ramp is required to provide access to a building with a door that is located 22 cm above ground level. The maximum angle that a ramp must be from the horizontal is 4.8°.

(a)
Calculate the minimum horizontal distance that the ramp must extend out.

[2 marks]

Question 6b

The wheelchair ramp is supported by a steel frame. A cross section of the ramp can be seen in the diagram below. A metal strut joins M, the midpoint of [AC], to a point X on the line [AB]. [AB].XM=11.1 cm and $\widehat{MXC}$=90°.



(b)

Using the horizontal distance found in part (a) and assuming that point A is at the origin, use a vector method to calculate the length XB.

[8 marks]

Question 7a

Two drones X and Y are being flown over an area of rainforest to look for signs of illegal logging. Their positions relative to the observation centre, are given by

$$\mathbf{r}_x = \begin{pmatrix} -3 \\ 1.6 \\ 2.5 \end{pmatrix} + t\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \text{ and } \mathbf{r}_y = \begin{pmatrix} 2.5 \\ 0 \\ -2 \end{pmatrix} + t\begin{pmatrix} 1.5 \\ 6 \\ 4 \end{pmatrix}$$

at time t minutes after take-off, $0 \leq t < 20$. All distances are in metres.

(a)
Verify that the two drones will not collide.

[2 marks]

Question 7b

(b)
Find the shortest distance between the two drones and the time at which it occurs.

[6 marks]

Question 7c

A third drone Z begins its flight at $t = 8$ and its position relative to the observation centre is given by $r_Z = \begin{pmatrix} 2 \\ 1.5 \\ 4.5 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$

Each drone can observe a circular area of ground, A, such that $A = 1.8h^2$ where h is the height of the drone above the ground in metres.

(c)
Show that the area of ground that can be observed by drone Z five minutes after it takes off overlaps with the area of ground that can be observed by drone Y at that time.

[6 marks]

Question 8a

Consider the tetrahedron ABCD, where A(3, 5, 8), B(-2, 3, 2), C(5, -1, 3) and D(-3, 0, 1). M is the midpoint of the line BC and point P lies along the line DM.

(a)

Given that the volume of the tetrahedron ABCP is $\frac{1}{3}$ of the volume of the tetrahedron ABCD, find the Cartesian equations of the line going through points A and P.

[5 marks]

Question 8b

(b) X is the midpoint of [AD].

Find the coordinates of the point of intersection between the line found in part (a) and the line going through [MX].

[5 marks]

Question 9a

A car is moving at a constant speed of 15 ms^{-1} in the direction parallel to the vector $3\mathbf{i} - 6\mathbf{j}$. Two birds are perched at points $A(17, 28, 16)$ and $B(-48, 128, 26)$.

At $t = 0$, the car is located at $(2, 4, 0)$ and the bird at point A starts to fly at a constant velocity of $\frac{7\sqrt{365}}{10}$ ms^{-1}. The bird at point B begins to fly at a constant velocity in the direction of the vector $52\mathbf{i} - 60\mathbf{j} - 9k$ when $t = 1.2$.

When bird A reaches the position of $(44, -24, 4)$, both birds and the car lie in a straight line.

(a)
Find the equation of the line along which the birds and car lie.

[6 marks]

Question 9b

(b)
Find the speed at which bird B is travelling.

[6 marks]