4.3 Probability

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.3 Probability
Difficulty	Hard

Time allowed: 110

Score: /89

Percentage: /100

Question 1a

A game is played using a fair spinner with four sectors numbered 1 to 4, as well as a fair dice with its six sides numbered 1 to 6.

(a) Using an appropriate representation, describe the sample space of possible outcomes when the spinner is spun and the dice is rolled at the same time.

[3 marks]

Question 1b

When the game is played, the spinner is spun and the dice is rolled at the same time, and the player's score is defined to be the (positive) difference between the two results.

(b) Find the probability of the score in the game being

(i) exactly 0

(ii) 3 or more

(iii) a prime number

[4 marks]

Question 1c

The game is played 150 times.

(c) Find the expected number of times that a prime number score will occur.

[2 marks]

Question 2a

A survey was carried out of residents of a particular town, to find out what their preferred activity was when coronavirus lockdown restrictions were in place. Five hundred residents were surveyed, and the results are shown in the table below:

		Preferred activity				
		Daydreaming	**Staring at phone**	**Exercising**	**Playing chess**	**Other**
Age	**13-17**	11	37	33	1	2
	18-30	2	45	40	1	1
	31-54	33	8	31	21	8
	55-70	31	35	30	11	10
	>70	34	17	38	13	7

(a) One of the surveyed residents is selected at random. Given that the resident did not give a response of 'Other' to the survey, find the probability that this resident

(i) preferred playing chess during lockdown

(ii) was less than 55 years old and did not prefer daydreaming during lockdown.

[3 marks]

Question 2b

The town has a total population of 23681.

(b) Assuming that the survey figures are representative of the town as a whole, estimate the number of residents of the town who

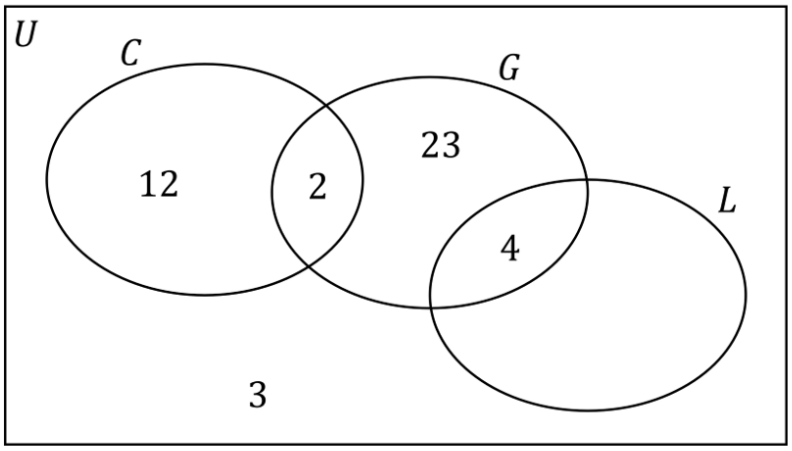
(i) preferred daydreaming, exercising, staring at their phone or playing chess during lockdown

(ii) were between 31 and 70 years old and did not prefer exercising during lockdown.

[3 marks]

Question 3a

The Venn diagram displays information about the number of students taking each of three languages: Mandarin Chinese (C), German (G) and Latin (L).



There are fifty students in total.

(a) Determine the number of students who take only Latin.

[2 marks]

Question 3b

A student is randomly chosen from the group.

(b) Find the probability that

(i) the student studies German or Latin

(ii) the student studies neither Mandarin Chinese nor Latin

(iii) the student studies Mandarin Chinese, given that they study German

(iv) the student studies Latin, given that they study Mandarin Chinese

(v) the student studies Latin, given that they do not study German.

[10 marks]

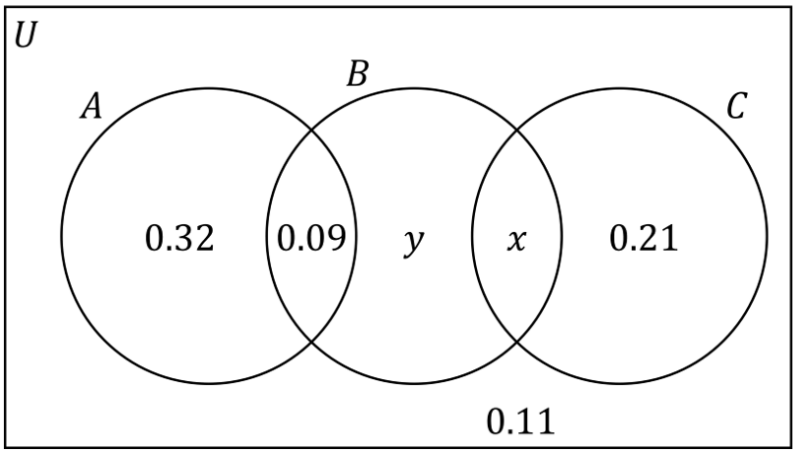
Question 4a

The Venn diagram illustrates the probabilities of members of a costumed performers' union having dressed as one or another superhero during a performance.

A represents the event that the member has dressed as Aquaman.

B represents the event that the member has dressed as Batman.

C represents the event that the member has dressed as Captain Marvel.



Given that the probability of a member having dressed as Captain Marvel is 0.44,

(a) determine the values of

(i) x

(ii) y.

[3 marks]

Question 4b

264 of the union's members have dressed as exactly two of the three superheroes.

(b) Use this information to determine the total number of members of the union.

[2 marks]

Question 5

A and B are events such that $\mathrm{P}(A) = 0.24,\ \mathrm{P}(B) = 0.73,$ and $\mathrm{P}(A \cup B) = 0.84.$

By drawing a Venn diagram to illustrate these probabilities, find

(i) $\mathrm{P}(A' \cup B)$

(ii) $\mathrm{P}(A \cap B')$

(iii) $\mathrm{P}((A \cap B)')$

[6 marks]

Question 6

A and B are independent events, such that $\mathrm{P}(A) = 0.25$ and $\mathrm{P}(B) = 0.52$. C is another event, such that B and C are mutually exclusive and $\mathrm{P}(A \cap C) = 0.09$.

Given that $\mathrm{P}(A \cup B \cup C) = 0.95$, find

(i) $\mathrm{P}(A \cap B)$

(ii) $\mathrm{P}(C)$

(iii) $\mathrm{P}(A' \cap B')$

(iv) $\mathrm{P}(A|C')$

[9 marks]

Question 7a

A bag contains 12 red marbles, 7 green marbles and 1 black marble. Two marbles are drawn from the bag without replacement.

(a) Draw a tree diagram to illustrate the process described above, showing clearly the probabilities on each branch.

[3 marks]

Question 7b

(b) Find the probability that

(i) the two marbles drawn are not both the same colour

(ii) both marbles are green, given that both marbles drawn are the same colour.

[5 marks]

Question 7c

(c) In the context of the question, give an example of two mutually exclusive events. Be sure to justify that they are mutually exclusive.

[2 marks]

Question 8a

In a game of Unicorns Versus Zombies your unicorn is attempting to use the magic of its horn do dispel a cloud of zombie apocalypse flies. On the first attempt, the probability of the magic working is 0.7. If the magic works, then there is a probability of 0.2 that the flies will be turned into glitter pixies and join your rainbow army, otherwise the flies will simply be dispelled. If the magic does not work the first time you may try again, although the probability of your magic working the second time is only 0.6. Similarly, if your magic does not work the second time you may try a third time, but on the third attempt the probability of your magic working is reduced to 0.5. If your magic works on the second or third attempts the probabilities of dispelling the flies or turning them into glitter pixies are the same as for the magic working on the first attempt. If your magic does not work on the third attempt, however, then your unicorn is turned into an evil zombiecorn and joins the zombie horde. In all cases, the game ends when either the flies are turned into glitter pixies, or the flies are dispelled, or your unicorn is turned into a zombiecorn.

(a) Draw a tree diagram to illustrate the above question, showing clearly the probabilities on each branch.

[4 marks]

Question 8b

(b) Find the probability that

(i) the flies are turned into glitter pixies

(ii) the flies are dispelled

(iii) your unicorn is turned into a zombiecorn.

[3 marks]

Question 8c

(c) Explain why the events "the flies are turned into glitter pixies" and "the magic worked on the second attempt" are not independent events.

[3 marks]

Question 9

For two events, A and B, the conditional probability of A occurring given that B has occurred is given by

$$\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$$

If in addition events A and B are independent, then it will also be true that

$$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$$

Given that events C and D are independent, use the results above to show that

(i) $\mathrm{P}(C|D) = \mathrm{P}(C)$

(ii) $\mathrm{P}(D|C) = \mathrm{P}(D)$.

[4 marks]

Question 10

Sciura is a scientist who studies grey squirrel populations in Canada. Some grey squirrels actually have black fur, and Sciura believes that the probability of a grey squirrel having black fur is independent of whether the squirrel is male or female.

Because of coronavirus lockdown restrictions, Sciura and her team have been unable to get into the field to conduct research to test her belief. However Sciura has managed to locate scientific publications in which the results of past studies of grey squirrel populations in Canada are reported.

In one study, a random sample of 98 291 grey squirrels was looked at and the authors noted that 297 of the squirrels had black fur.

In another study, a random sample of 13 583 female grey squirrels was looked at and the authors noted that 41 of the squirrels had black fur.

Clearly stating any assumptions you make, state with a justification whether or not the results of these two studies support Sciura's belief.

[5 marks]

Question 11a

100 people are queuing to buy tickets for a live concert by a particular band. When asked, 90% said they had seen the band play live on a previous occasion. Of those 90%, 70% said they had been queueing longer than an hour. Of those who had not seen the band play live before, 80% said they had been queueing an hour at maximum.

(a) Draw a tree diagram to represent this situation.

[3 marks]

Question 11b

(b) Write down the probability that a randomly selected person from the queue has been

(i) queueing longer than an hour, given that they had previously seen the band play live,

(ii) queueing longer than an hour, given that they had not previously seen the band play live.

[2 marks]

Question 11c

(c) Find the probability that a randomly selected person from the queue has seen the band play live previously, given that they have been queueing more than an hour.

[2 marks]

Question 12

In a computer game, a player has to capture a variety of creatures and can offer different types of fruit to the creatures in order to capture them.
The decision a player makes regarding which fruit to feed a creature affects how likely they are to be captured. Players have a choice of three options; strawberries, raspberries or blueberries.

A particular player always feeds a creature fruit before attempting to capture it.
They randomly select a strawberry on 30% of their attempts at capturing a creature, 40% of such attempts result in the creature being successfully captured.
The player selects a raspberry 50% of the time with a 50% success rate whilst blueberries yield a 30% success rate.

(i) Given that the player successfully captures a creature, find the probability they fed it a raspberry.

(ii) Given that the player fails to capture a creature, find the probability they fed it a blueberry.

[6 marks]