2.3 Functions Toolkit

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.3 Functions Toolkit
Difficulty	Medium

Time allowed: 70

Score: /51

Percentage: /100

Question 1a

The functions f and g are defined such that $f(x) = 4x - 10$ and $g(x) = \frac{x+8}{2}$.

(a) Show that $(g \circ f)(x) = 2x - 1$.

[2 marks]

Question 1b

(b) Given that $(g \circ f)(a) = 27$, find the value of a.

[2 marks]

Question 1c

(c) Show that $(f \circ g)(x) = 2x + 6$.

[2 marks]

Question 1d

(d) Given that $(f \circ g)(b) = 44$, find the value of b.

[2 marks]

Question 2a

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = x^2 \qquad x \in \mathbb{R}$$
$$g(x) = 4x - 3 \qquad x \in \mathbb{R}$$

(a) Write down the range of $f(x)$.

[1 mark]

Question 2b

(b) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$

[4 marks]

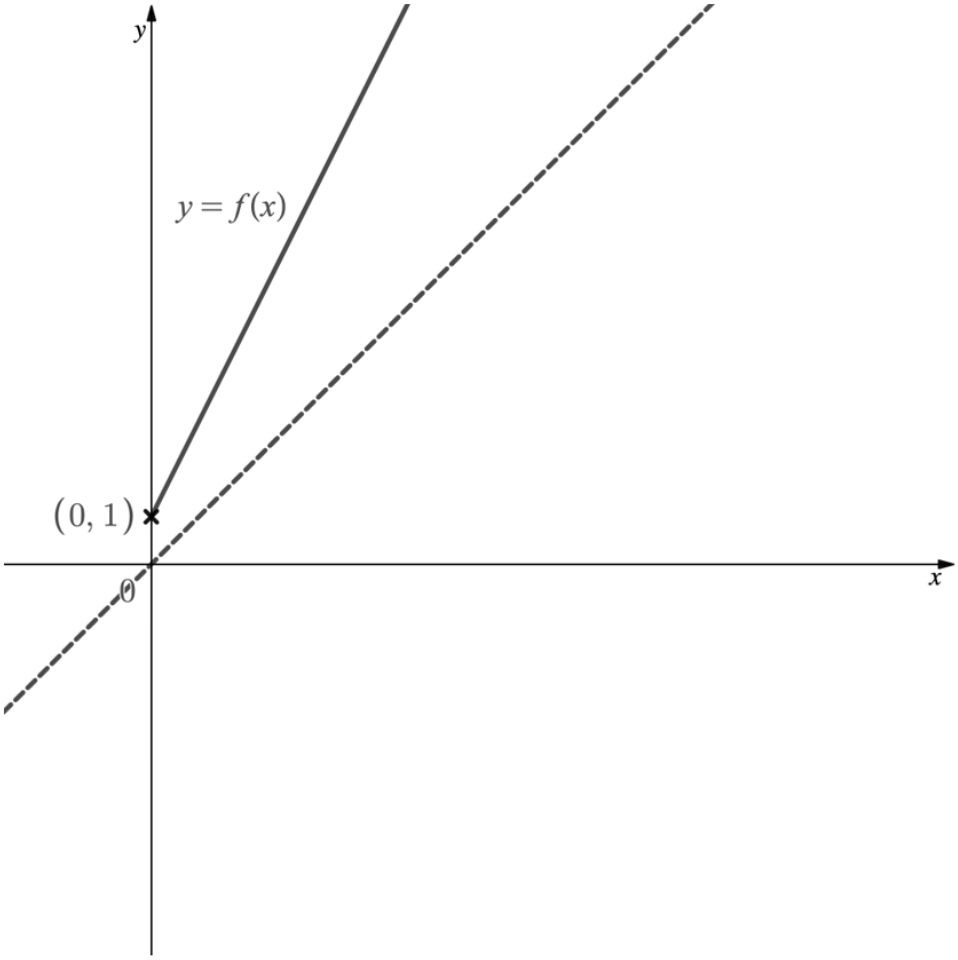
Question 2c

(c) Solve the equation $f(x) = g(x)$.

[2 marks]

Question 3a

The graph of $y = f(x)$ is shown below.



(a) (i) Use the graph to write down the domain and range of $f(x)$.

(ii) Given that the point (1, 1) lies on the dotted line, write down the equation of the line.

[3 marks]

Question 3b

(b) On the diagram above sketch the graph of $y = \mathrm{f}^{-1}(x)$.

[2 marks]

Question 4a

The functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are defined as follows

$$\mathrm{f}(x) = \frac{1}{2}(4x - 3) \qquad x \in \mathbb{R}$$
$$\mathrm{g}(x) = 0.5x + 0.75 \qquad x \in \mathbb{R}$$

(a) Find (i) $\mathrm{fg}(x)$
(ii) $\mathrm{gf}(x)$

[3 marks]

Question 4b

(b) Write down $f^{-1}(x)$ and state its domain and range.

[3 marks]

Question 5a

A function is defined by $f(x) = 54x - 13, \quad -2 < x < 20.$

(a) Find the value of $f\left(\frac{5}{2}\right)$.

[1 mark]

Question 5b

(b) Write down the range of $f(x)$.

[2 marks]

Question 5c

(c) Find the inverse function $f^{-1}(x)$.

[2 marks]

Question 5d

(d) Write down the range of the inverse function.

[1 mark]

Question 6a

Consider the function $f(x) = -6x - 3$. The domain of $f(x)$ is $-5 \leq x \leq 3$.

(a) Find

(i) $f(2)$

(ii) x when $f(x) = 15$.

[2 marks]

Question 6b

(b) Find the range of $f(x)$.

[3 marks]

Question 6c

(c) Write down the domain of the inverse function.

[1 mark]

Question 7a

The functions f and g are defined for $x \in R$ by $f(x) = 3x^2 + 10x + 7$ and $g(x) = x + d$, where $d \in R$.

(a) Find the range of f.

[2 marks]

Question 7b

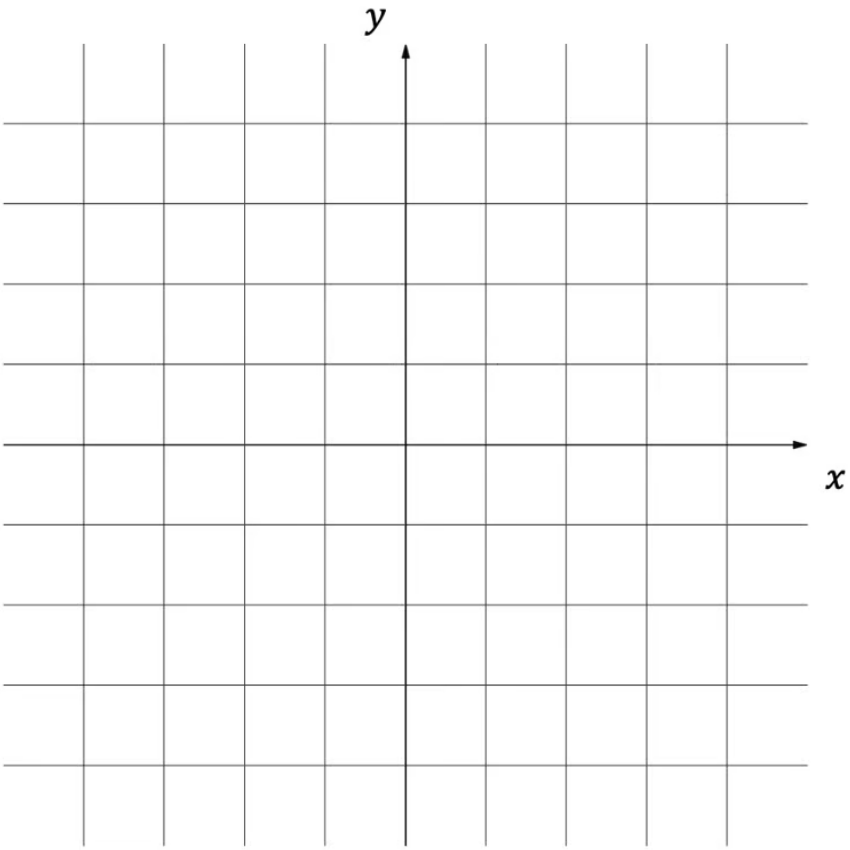
(b) Given that $(g \circ f)(x)$ is always positive for all x, determine the set of possible values for d.

[4 marks]

Question 8a

Consider the function $g(x) = \sqrt{4 - x}$.

(a) Sketch the graph of the function $g(x)$, labelling the x and y intercepts.



[3 marks]

Question 8b

(b) Find

(i) $g(-5)$

(ii) x when $g(x) = \frac{1}{2}$.

[2 marks]

Question 8c

(c) Find

(i) the maximum possible domain of the function $g(x)$

(ii) the range of the function $g(x)$ that corresponds to the domain found in part (c) (i).

[2 marks]