# 4.4 Probability Distributions

## Question Paper

| Course | DP IB Maths |
|---|---|
| Section | 4. Statistics & Probability |
| Topic | 4.4 Probability Distributions |
| Difficulty | Medium |

**Time allowed:** 70

**Score:** /51

**Percentage:** /100

## Question 1a

Three biased coins are tossed.

(a) Write down all the possible outcomes when the three coins are tossed.

[1 mark]

## Question 1b

For each coin the probability of getting heads is $\frac{2}{3}$. A random variable, $X$, is defined as the number of heads when the three coins are tossed.

(b) Complete the following probability distribution table for $X$:

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X = x)$ | | | | |

[3 marks]

## Question 1c

(c) Hence, by inserting the relevant probabilities, represent the probability distribution for $X$ as a piecewise function in the form

$$P(X = x) = f(x) = \begin{cases} & x = 0 \\ & x = 1 \\ & x = 2 \\ & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

[2 marks]

## Question 1d

(d) Represent the probability distribution for $X$ as a bar chart.

[2 marks]

## Question 2

The random variable $X$ has the probability function

$$P(X = x) = \begin{cases} \dfrac{x}{3k} & x = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

Show that $k = 5$.

[2 marks]

## Question 3a

The random variable $X$ has the probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3, 5, 7 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of $k$.

[2 marks]

## Question 3b

(b) Find $P(X > 3)$.

[2 marks]

## Question 3c

(c) State, with a reason, whether or not $X$ is a discrete random variable.

[1 mark]

## Question 4a

The random variable $X$ has the probability function

$$P(X = x) = \begin{cases} 0.23 & x = -1, 4 \\ k & x = 0, 2 \\ 0.13 & x = 1, 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of $k$.

[2 marks]

## Question 4b

(b) Construct a table giving the probability distribution of $X$.

[2 marks]

## Question 4c

(c) Find $P(0 \leq X < 3)$.

## Question 5

A discrete random variable $X$ has the probability distribution shown in the following table:

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | $\dfrac{5}{24}$ | $\dfrac{1}{3}$ | $\dfrac{1}{4}$ | $\dfrac{1}{12}$ | $\dfrac{1}{8}$ |

Find:

    (i)    $P(X < 4)$

    (ii)   $P(X > 1)$

    (iii)  $P(2 < X \leq 4)$

    (iv)  $P(0 < X < 4)$

## Question 6a

Leonardo has constructed a biased spinner with six sectors labelled 0, 1, 1, 2, 3 and 5. The probability of the spinner landing on each of the six sectors is shown in the following table:

| number on sector | 0 | 1 | 1 | 2 | 3 | 5 |
|---|---|---|---|---|---|---|
| probability | $\dfrac{6}{20}$ | $p$ | $\dfrac{3}{20}$ | $\dfrac{5}{20}$ | $\dfrac{3}{20}$ | $\dfrac{1}{20}$ |

(a) Find the value of $p$.

[1 mark]

## Question 6b

Leonardo is playing a game with his biased spinner. The score for the game, $X$, is the number which the spinner lands on after being spun.

(b) Leonardo plays the game twice and adds the two scores together. Find the probability that Leonardo has a *total* score of 5.

[3 marks]

## Question 6c

(c) Complete the following cumulative probability function table for $X$:

| Score $x$ | 0 | 1 | 2 | 3 | 5 |
|---|---|---|---|---|---|
| $P(X \leq x)$ | $\dfrac{6}{20}$ | | | | 1 |

[2 marks]

## Question 6d

(d) Find the probability that $X$ is

    (i)    no more than 1

    (ii)   at least 3.

[2 marks]

## Question 7a

A discrete random variable $X$ has the following probability distribution:

| $x$ | $-3$ | $-1$ | $0$ | $1$ | $3$ |
|-----|------|------|-----|-----|-----|
| $P(X = x)$ | $0.11$ | $k^2$ | $0.1$ | $2k$ | $0.1$ |

where $k$ is a positive constant.

(a) Show that $k^2 + 2k - 0.69 = 0$.

[2 marks]

## Question 7b

(b) Hence find the value of $k$.

[1 mark]

## Question 7c

(c) Find $E(X)$.

[3 marks]

## Question 8

A spinner is spun on a circle that is divided up into five sections, A, B, C, D and E.

The probability of the spinner landing on each section is given by the following table:

| Region | A | B | C | D | E |
|---|---|---|---|---|---|
| **Probability** | 0.55 | 0.15 | 0.15 | 0.1 | 0.05 |

A person who rotates the spinner scores points depending on which section the spinner lands on.  These points are shown below.

| Region | A | B | C | D | E |
|---|---|---|---|---|---|
| **Points** | $-5$ | 2 | 3 | 10 | $k$ |

Given that the game is fair, find the value of $k$.

[4 marks]

## Question 9a

A discrete random variable $X$ has the following probability distribution:

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | 0.1 | 0.05 | $a$ | $b$ | 0.1 |

The value of $E(X) = 2.3$.

(a) Show that $a$ and $b$ must satisfy the following two simultaneous equations:

$$a + b = 0.75$$
$$2a + 3b = 1.85$$

[3 marks]

## Question 9b

(b) Hence find the value of $a$ and the value of $b$.

[2 marks]

## Question 9c

(c) Find $P(1 \leq X < 4)$.

**[2 marks]**