2.6 Transformations of Graphs

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.6 Transformations of Graphs
Difficulty	Very Hard

Time allowed: 100

Score: /82

Percentage: /100

Question 1a

Let $f(x) = \frac{4}{3}(x-5)^3 - 2$ and $g(x) = x^3$, for $x \in \mathbb{R}$.

(a) Give a full geometric description of two individual transformations that can be combined to obtain the graph of f from the graph of g, given that:

(i) a stretch is to be applied first, followed by a translation

(ii) a translation is to be applied first, followed by a stretch.

[4 marks]

Question 1b

The graph of f is translated by the vector $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$ to give the graph of h.

Now consider h as a transformation of g. The point where $g(x) = -27$ is translated to point A on the graph of h.

(b) Find the coordinates of A.

[4 marks]

Question 2a

Let f and g be functions defined for $x \in \mathbb{R}$ such that $g(x) = f(2x + 6) - 4$.

The graph of g is obtained from the graph of f after the follow transformations:

a horizontal stretch by a factor of v,

followed by

a translation by the vector $\begin{pmatrix} a \\ b \end{pmatrix}$.

(a) Write down the values of

(i) v

(ii) a

(iii) b.

[3 marks]

Question 2b

The point A$(2, 0)$ on the graph of f is mapped to point B on the graph of g.

(b) Find the distance between points A and B, giving your answer in the form $p\sqrt{q}$ where p and q are integers to be found, and where q has no square number factors.

[5 marks]

Question 3a

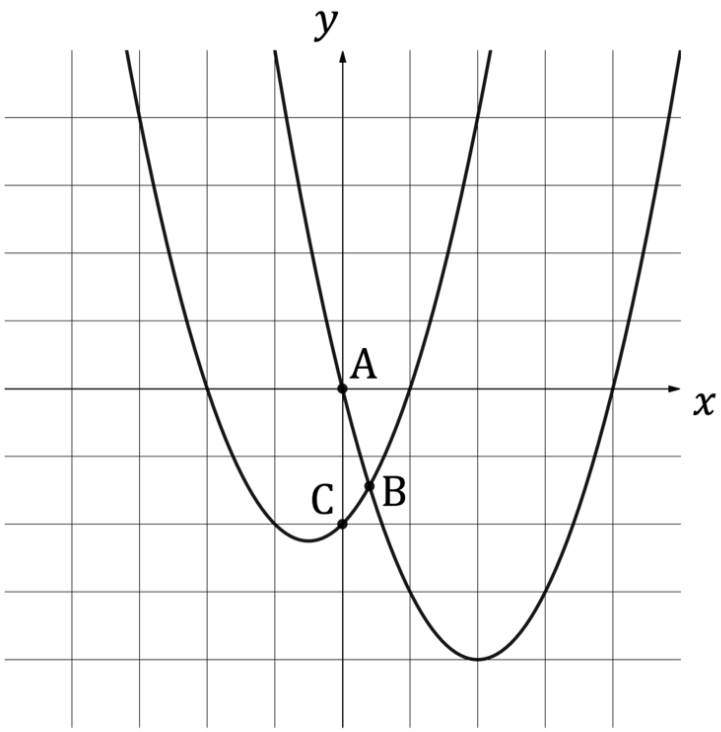
Let $f(x) = x^2 - 4x$ and $g(x) = x^2 + x - 2$.

(a) Show that the graph of g is a translation of the graph of f, and find the vector that translates the graph of f onto the graph of g.

[4 marks]

Question 3b

The diagram below shows parts of the graphs of f and g. Point A is the y-intercept of f, point B is the intersection between the graphs of f and g and point C is the y-intercept of g.



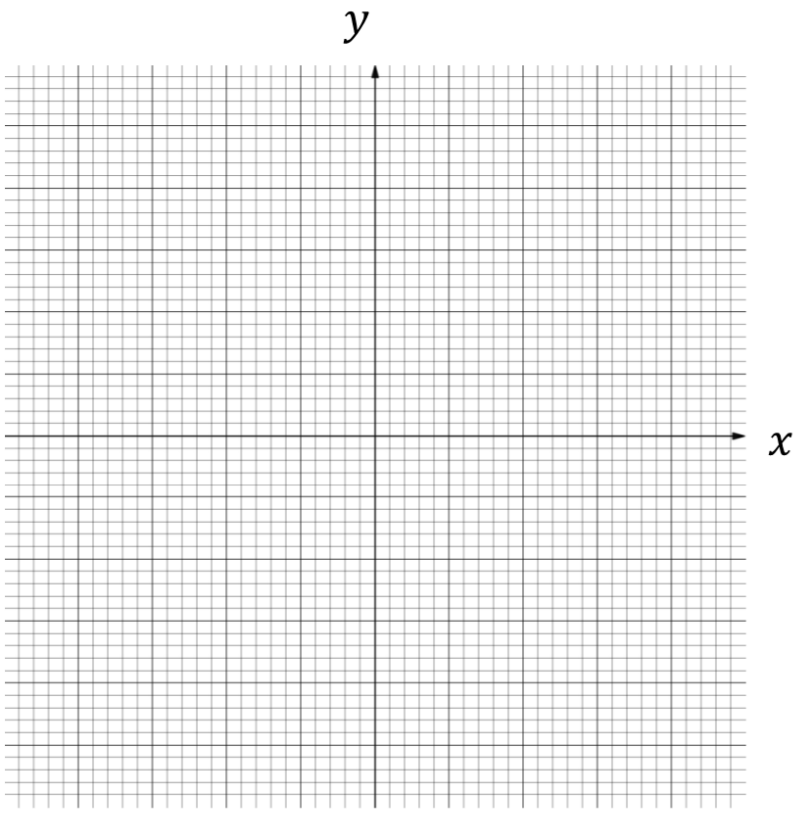
(b) Find the area of the triangle that has points A, B and C as its vertices.

[6 marks]

Question 4a

Let $f(x) = \frac{1}{3}e^{-x-2} - 1$, for $-4 \leq x \leq 4$.

(a) Sketch the graph of $y = f(x)$ on the grid below, clearly labelling any intersections the graph makes with the coordinate axes.



[4 marks]

Question 4b

The graph of f is reflected in the x-axis and then translated by the vector $\begin{pmatrix}-1\\-1\end{pmatrix}$ to obtain the graph of $y = g(x)$.

(b) Show that the equation $g(x) = 0$ has no solutions.

[3 marks]

Question 5

The function f is defined by

$$f(x) = \begin{cases} 4 - 3x & \text{if} \quad x \leq 1 \\ 2x^2 - 2x + 1 & \text{if} \quad x > 1 \end{cases}$$

The graph of the function g is obtained by applying the following transformations to the graph of f:

a reflection in the y-axis,

followed by

a translation by the vector $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$.

Find $g(x)$.

[4 marks]

Question 6a

Let $f(x) = -2x^3 + 54$, where $x < 0$.

The graph of a function g is obtained when the graph of f is transformed by

a reflection in the y-axis,

followed by

a vertical stretch by a factor of $\frac{5}{4}$.

(a) (i) Find $g(x)$, giving your answer in the form $ax^3 + b$.

(ii) Write down the domain of g.

[4 marks]

Question 6b

A particle moves along a straight line so that its velocity in ms^{-1} at time x seconds is given by $g(x)$.

(b) Find the value of x when the particle's velocity is $85\ \text{ms}^{-1}$.

[2 marks]

Question 7a

Let $f(x) = ax^2 - 12x + c,\ x \in \mathbb{R},$ where $a, c \in \mathbb{Z}^-$.

The equation $f(x) = 0$ has two equal roots.

(a) Given that $4a = c$, find the values of a and c.

[3 marks]

Question 7b

(b) Find the coordinates of the vertex of the graph of f.

[2 marks]

Question 7c

The graph of a function g is obtained from the graph of f by a reflection in the y-axis, followed by a horizontal stretch by a factor of $\frac{2}{3}$.

(c) Find an expression for $g(x)$, along with the coordinates of the y-intercept of the graph of g.

[3 marks]

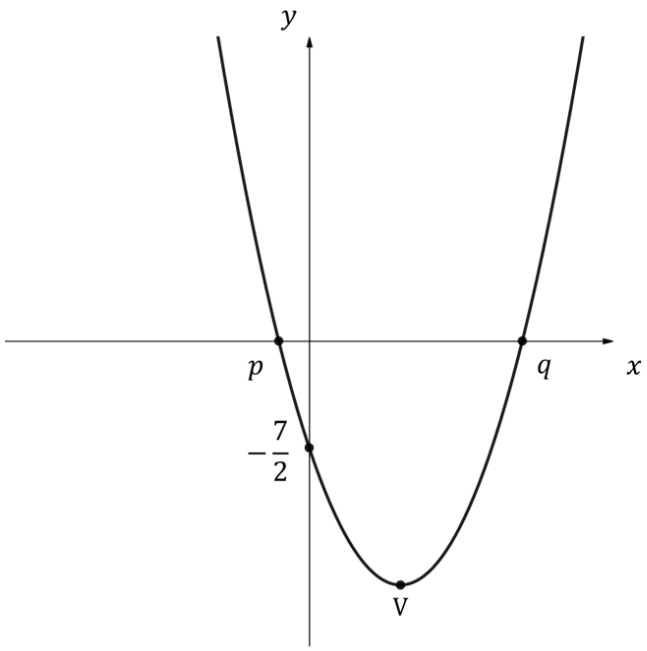
Question 7d

(d) Using the geometric nature of the two transformations by which the graph of g was obtained from the graph of f, explain why the graphs of f and g must have the same y-intercept.

[2 marks]

Question 8a

Let $f(x) = \frac{(x-p)(x-q)}{2}$, $x \in \mathbb{R}$, where $p, q \in \mathbb{Z}$. The graph of f has x-intercepts at $(p, 0)$ and $(q, 0)$, with $(p, 0)$ lying on the negative x-axis and $(q, 0)$ lying on the positive x-axis. The y-intercept of the graph is at $\left(0, -\frac{7}{2}\right)$, and the vertex of the graph lies in the fourth quadrant. This information is represented on the diagram below.



(a) (i) Find the values of p and q.

(ii) Find the coordinates of the vertex, V.

[6 marks]

Question 8b

The graph of a function g is obtained from the graph of f by a translation by the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$, followed by a reflection in the y-axis. Point A on the graph of f has an x-coordinate of 1 and is mapped to point B on the graph of g.

(b) Find the coordinates of B.

[4 marks]

Question 9a

Consider the function f defined by $f(x) = 8^{x-2}$.

The graph of f is stretched horizontally by scale factor of 0.75 and then translated by the vector $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$. The function corresponding to the transformed graph is denoted g.

(a) Write down an expression for the function g.

[4 marks]

Question 9b

Consider the function h defined by $h(x) = 2^x$.

The graph of f can be transformed onto the graph of h by a horizontal stretch and a vertical stretch.

(b) Describe the stretches needed to map the graph of f onto the graph of h.

[4 marks]

Question 10a

Let f be a function.

(a) For each of the functions below, describe two different sequences of transformations that map the graph of f onto the graph of the function. Each sequence should consist of exactly two basic transformations (translations, stretches, or reflections).

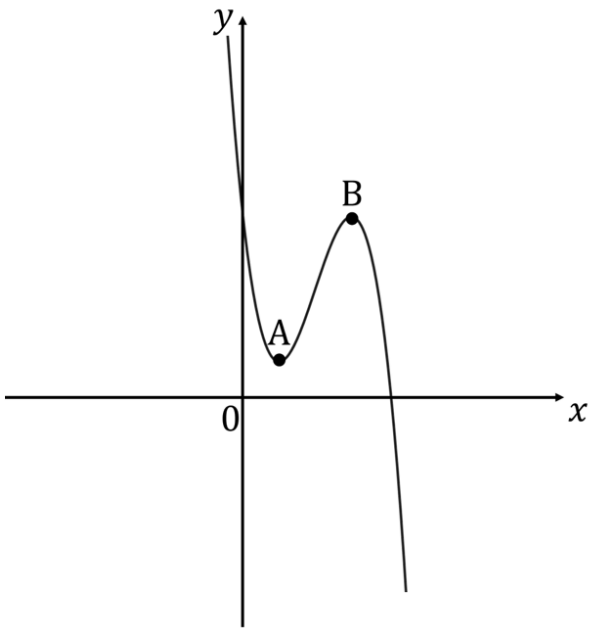
(i) $f(5-x)$.

(ii) $f\left(\frac{x+3}{5}\right)$.

[5 marks]

Question 10b

The graph of the function g defined by $g(x) = 1 - f(2x - 3)$ is shown below. There is a local minimum at $A(0.5, 1)$ and a local maximum at $B(1.5, 5)$.



(b) Sketch the graph of f. Clearly state the coordinates of the points corresponding to the points A and B.

[6 marks]