

4.6 Random Variables

Question Paper

Course	DPIB Maths
Section	4. Statistics & Probability
Topic	4.6 Random Variables
Difficulty	Very Hard

Time allowed: 90
Score: /74
Percentage: /100

Question 1a

Two friends, Forrest and Gumpy, are planning to run a marathon together. The distributions $F \sim N(253, 95)$ and $G \sim N(281, 52)$ are used to model the times in minutes it takes Forrest and Gumpy to complete a marathon respectively. It can be assumed that their times are independent.

a)

Find the probability that Forrest completes the marathon quicker than Gumpy.

[4 marks]**Question 1b**

b)

Find the probability that Gumpy is still running the marathon one hour after Forrest has completed it.

[2 marks]**Question 1c**

c)

Find the probability that their times taken to complete the marathon differ by more than 5 minutes.

[3 marks]

Question 2a

Roger is considering buying a new pet. He has researched the prices, in €, of rabbits, chinchillas and degus. The information is shown in the table below. The prices of the three types of animals are normally distributed and independent of each other.

	Mean	Standard Deviation
Rabbit	30	9
Chinchilla	145	20
Degu	37	6

a)

Find the probability that the cost of two independently bought degus is less than €70.

[3 marks]

Question 2b

b)

Find the probability that a randomly selected degu is more expensive than a randomly selected rabbit.

[3 marks]

Question 2c

c)

Find the probability that a randomly selected chinchilla is more than five times as expensive as a randomly selected rabbit.

[4 marks]

Question 2d

Roger and his housemate Lucy have decided to buy one of each type of pet for their house. Roger loves rabbits so he will pay for the rabbit himself, he will pay 50% of the cost for the chinchilla and 10% of the cost for the degu.

d)

Find the probability that, in total, Roger pays less than €100 for the three pets.

[4 marks]

Question 3a

The random variables $X \sim N(50, 9^2)$ and $Y \sim N(400, 150)$ are independent.

a)

Find $P(Y < 7X + 40)$.

[3 marks]**Question 3b**

b)

There's a 99.95% chance that the sum of a random observation of X and a random observation of Y is bigger than k . Find the value of k .

[3 marks]**Question 3c**

c)

Find the probability that the sum of three independent observations of X is more than one third of one observation of Y .

[4 marks]

Question 4

In a video game a player gets points for completing a level and for defeating enemies, these points are independent of each other. The amount of points a player gets for completing the level and for defeating an enemy can be modelled as $L \sim N(500, 220)$ and $E \sim N(150, 85)$ respectively.

In a bonus stage, the points for completing the level are tripled and there are five enemies (points for defeating enemies are not tripled), the total score is the sum of the points for completing the level and defeating the enemies. The top 10% of scores make the leadership board.

Estimate the minimum score that would make the leadership board.

[5 marks]

Question 5a

Kate and Clint are working as a pair in an archery competition. They are both shooting arrows at a target. Kate shoots 20 arrows and Clint shoots 10. The number of times they each hit the target are added together to form the pair's final score, denoted by the random variable S . On average, Kate has a 95% chance of hitting the target and Clint has a 50% chance of hitting the target.

a)
Find $E(S)$. State any assumptions that are needed.

[2 marks]

Question 5b

b)

Find $\text{Var}(S)$. State an additional assumption that is needed.**[2 marks]****Question 5c**Kate claims that the pair's final score, S , follows a binomial distribution $B(30, p)$.

c)

By using the formulae for the mean and variance of a binomial distribution, show that Kate's claim is incorrect.

[3 marks]**Question 5d**

In the competition, pairs win a prize if their final score is at least 28.

d)

Find the probability that Kate and Clint win a prize.

[4 marks]

Question 6a

Viktor works for the emergency services and has found from previous data that the amount of call-outs per day can be modelled using a Poisson distribution with mean 14.9. The number of call-outs are independent of the day of the week. Viktor decides to monitor the number of call-outs each day over a seven-day period.

- a)
Find the mean and the standard deviation of the total number of call-outs during a seven-day period.

[2 marks]

Question 6b

- b)
Find the probability that the mean number of daily call-outs using Viktor's seven-day period is more than 16.

[3 marks]

Question 6c

After each call-out Viktor is required to complete three forms.

- c)
Find the mean and standard deviation of the number of forms that Viktor is required to complete in a day due to call-outs.

[3 marks]

Question 7a

Reuben works at a candy store and sells three types of sweets: chocolate, marshmallow and honeycomb. Reuben uses a scoop to measure a portion for each type of sweet and the price depends on the weight of each individual portion. The table below shows the mean and standard deviation of the masses of the portions for each type of sweet as well as the cost per unit weight.

Type of sweet	Mean (grams)	Standard deviation (grams)	Price (£ per kg)
Chocolate	167	5.2	3.50
Marshmallow	79	2.9	2.80
Honeycomb	125	8.1	4.20

Reuben offers a product called *Sugar Supreme* which contains 10 portions of sweets in total. Two portions are chocolate, x portions are marshmallow and y portions are honeycomb. The mean cost of a *Sugar Supreme* is £3.85.

a)

Find the values of x and y .

[3 marks]

Question 7b

b)

Find the standard deviation of the costs of the *Sugar Supreme* product.

[4 marks]

Question 8a

Danny and Mark use a biased four-sided dice to play a game. The number that the dice lands on, X , follows the probability distribution described in the table below.

x	0	2	6	8
$P(X = x)$	p	p	q	q

Danny calculates his score by multiplying the number on the dice by 15 and then adding 11. Mark calculates his score by adding 5 to the number on the dice and then multiplying by 8. They each roll the dice once and calculate their scores.

- a)
Given that Mark's expected score is 8 more than Danny's expected score, find the values of p and q .

[6 marks]

Question 8b

After they have rolled the dice once, each player is awarded a number of points which is calculated by subtracting their opponent's score from their own score. A player's number of points will be negative if their opponent's score is higher than their own.

b)

Given that the standard deviation for the number of points a player is awarded is 51, calculate the standard deviation of X .

[4 marks]