2.4 Further Functions & Graphs

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.4 Further Functions & Graphs
Difficulty	Hard

Time allowed: 70

Score: /54

Percentage: /100

Question 1a

Let $f(x) = \frac{7}{2(x-7)} - 5$, for $x \neq 7$.

(a) For the graph of f, find the:

(i) x-intercept

(ii) y-intercept.

[2 marks]

Question 1b

(b) For the graph of f, write down the equation of any asymptotes.

[2 marks]

Question 1c

Let $g(x) = 2(1 - 2x)$, for $x \in \mathbb{R}$. The graphs of f and g intersect at points P and Q.

(c) Write down the coordinates of P and Q.

[2 marks]

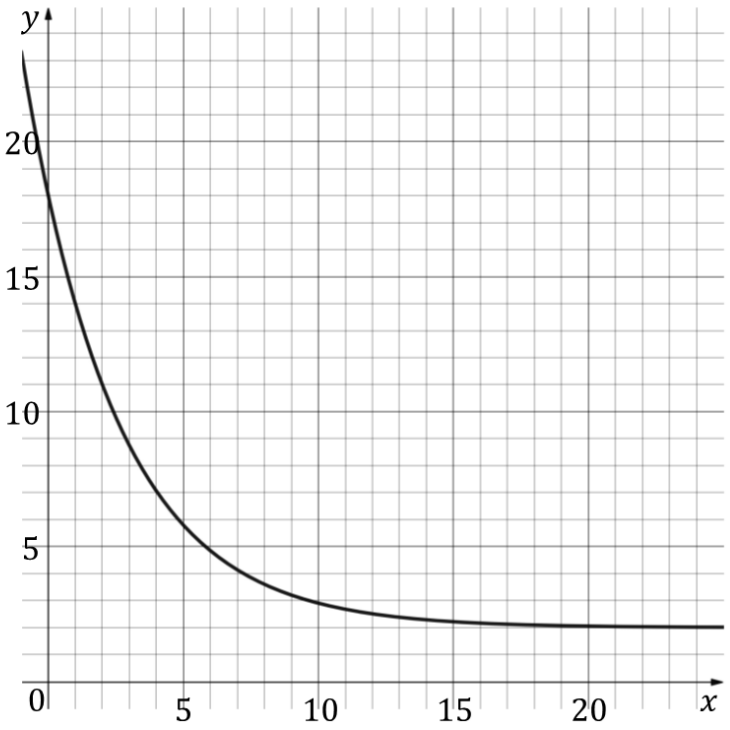
Question 1d

(d) Find the distance of PQ.

[2 marks]

Question 2a

Consider the function $f(x) = a(0.75)^x + b$ where a and b are constants. The graph of f passes through the points $(0, 18)$ and $(2, 11)$ and is shown below.



(a) Write down two equations relating a and b.

[2 marks]

Question 2b

(b) Find the value of a and b.

[2 marks]

Question 2c

(c) Write down the equation of the horizontal asymptote of the graph of f.

[2 marks]

Question 3a

A function is defined by $f(x) = \frac{1}{(x-3)^2} + 2\,,\ x \neq p$.

(a) Find the value of p.

[1 mark]

Question 3b

(b) For the graph of f write down the equation of any asymptotes.

[2 marks]

Question 3c

(c) Find the range of f.

[1 mark]

Question 3d

The line l intersects the graph of f when $x = 1$ and when $x = 4$.

(d) Find the equation of l. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers.

[4 marks]

Question 4a

A function is defined by $f(x) = 4 - \frac{12}{5x+9}$, $x \neq a$.

(a) Find the value of a. Give your answer as a fraction.

[2 marks]

Question 4b

(b) Find the range of f.

[3 marks]

Question 4c

(c) Find the value of $f^{-1}(2)$. Give your answer as a fraction.

[2 marks]

Question 5a

Let $f(x) = \frac{4}{6-x}$, for $x \neq 6$.

(a) For the graph of f, find

(i) the x-intercept

(ii) the y-intercept

(iii) The equation of the vertical asymptote.

[4 marks]

Question 5b

Let $g(x) = -\frac{x}{4}$, for $x \in \mathbb{R}$. The graphs of f and g intersect at points A and B.

(b) Find the coordinates of A and B.

[4 marks]

Question 6a

The average fat-free mass, M, in kg, of footballers as a function of their age, a, in years, can be given by the logarithmic function:

$$\mathrm{M}(a) = 10\log(a - 15) + 50, \qquad 16 \leq a \leq 25.$$

(a) Calculate the average fat free mass of players aged:

(i) 16 years

(ii) 25 years.

[2 marks]

Question 6b

(b) Find an expression for a linear model using your answers to part (a) (i) and (ii).

[3 marks]

Question 7a

The number of bacteria, n, in a dish, after t minutes is given by $n = 5231e^{0.12t}$.

(a) Find the initial amount of bacteria.

[2 marks]

Question 7b

(b) Find the amount of bacteria after 12 minutes. Give your answer in the form $a \times 10^k$, where $1 \leq a < 10, k \in \mathbb{Z}$.

[3 marks]

Question 7c

(c) Find the value of t when $n = 2.7 \times 10^4$.

[2 marks]

Question 8a

Let $f(x) = e^{-x} + 1$ and $g(x) = 2x - m$, for $x \in \mathbb{R}$, where m is a constant.

(a) Find $(g \circ f)(x)$.

[2 marks]

Question 8b

(b) Given that $\lim_{x \to \infty} (g \circ f)(x) = -1$, find the value of m.

[3 marks]