

2.4 Functions Toolkit

Question Paper

Course	DPIB Maths
Section	2. Functions
Topic	2.4 Functions Toolkit
Difficulty	Hard

Time allowed: 90
Score: /68
Percentage: /100

Question 1a

The functions f and g are defined such that $f(x) = 2x^2 - 4x$ and $g(x) = \frac{5x + 12}{2}$.

(a) Find $(g \circ f)(x)$, giving your answer in the form $(g \circ f)(x) = m(x - h)^2 + k$ where m , h and k are constants to be found.

[3 marks]

Question 1b

(b) Hence, or otherwise, find the coordinates of the vertex of the graph of $y = (g \circ f)(x)$.

[1 mark]

Question 1c

(c) Find $(f \circ g)(x)$, giving your answer in the form $(f \circ g)(x) = ax^2 + bx + c$ where a , b and c are constants to be found.

[3 marks]

Question 1d

(d) Hence, or otherwise, find the coordinates of the y -intercept of the graph of $y = (g \circ f)(x)$.

[1 mark]

Question 2a

Let $f(x) = \frac{5-x^2}{3}$ and $g(x) = 4 - \frac{3}{x}$, where each function has the largest possible valid domain.

(a) Write down the range of f .

[1 mark]

Question 2b

(b) Write down the domain and range of g .

[2 marks]

Question 2c

(c) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$.

[3 marks]

Question 2d

(d) Solve the equation $(f \circ g)(x) = (g \circ f)(x)$.

[2 marks]

Question 3a

The function f is defined by $f(x) = \sqrt[3]{4(1-x)}$, for $-1 \leq x \leq 17$.

(a) Write down the range of f .

[2 marks]

Question 3b

(b) Write down an expression for f^{-1} .

[2 marks]

Question 3c

(c) Write down the domain and range of f^{-1} .

[2 marks]

Question 4a

The perimeter, P , and area, A , of a given square can be expressed by $P = 4x$ and $A = x^2$ respectively, where x is the length of the side of the square.

(a) Write down an expression for:

(i) P in terms of A , $P(A)$

(ii) A in terms of P , $A(P)$.

[4 marks]

Question 4b

$$P^{-1}(40) = A(k).$$

(b) Find the value of k and $A(k)$.

[2 marks]

Question 5a

The values of two functions, f and g , for certain values of x are given in the following table:

x	-2	0	3
$f(x)$	-12	-4	8
$g(x)$	0	-12	30

(a) Find the value of $f^{-1}(8)$.

[1 mark]

Question 5b

(b) Find the value of $(f \circ g)(-2)$.

[2 marks]

Question 5c

(c) Given that $f(x)$ is a linear function, find $f(x)$.

[2 marks]

Question 6a

Let $f(x) = \sqrt{x - 14}$, for $x \geq 14$.

(a) Find $f^{-1}(2)$.

[3 marks]

Question 6b

Let g be a function such that g^{-1} exists for all real numbers.

(b) Given that $g(14) = 3$, find $(f \circ g^{-1})(3)$.

[3 marks]

Question 7a

Let the function f be defined by $f(x) = \sqrt{2x^2 - 16x + 41}$, where f has its largest possible valid domain.

(a) Find the domain and range of f .

[4 marks]

Question 7b

- (b) (i) Find the value(s) of x for which $f(x) = \sqrt{11}$.
- (ii) Use your answer to part (b)(i) to explain why the inverse function f^{-1} does not exist.

[2 marks]

Question 8a

Let $f(x) = x^2 - 9$ and $g(x) = x^2 - 1$, both for $x \geq 0$.

(a) Find

- (i) $f^{-1}(x)$
- (ii) $g^{-1}(x)$.

[2 marks]

Question 8b

(b) Find $(f \circ g)(x)$ in the form $ax^4 + bx^2 + c$.

[2 marks]

Question 8c

(c) Solve the equation $(f \circ g)(x) = 0$.

[3 marks]

Question 9a

(a) (i) Show that $(x + a)^2 + b = x^2 + 2ax + (a^2 + b)$.

(ii) Hence show that $x^2 + 12x + 24 = (x + 6)^2 - 12$.

[3 marks]

Question 9b

(b) Given that $g(x) = x + 6$ and $(f \circ g)(x) = x^2 + 12x + 24$, find a possible expression for $f(x)$.

[3 marks]

Question 10a

(a) (i) Sketch the graph of the function f defined by $f(x) = 2x^2 + 8x - 3$, $x \in \mathbb{R}$, clearly labelling the minimum point with its coordinates.

(ii) Explain why the function f does not have an inverse.

[3 marks]

Question 10b

The function g defined by $g(x) = 2x^2 + 8x - 3$, $x \geq p$ has an inverse.

(b) (i) Write down the smallest possible value of p .

Given that p takes its smallest possible value:

(ii) Find the domain and range of g^{-1} .

(iii) Find the inverse function g^{-1} .

[3 marks]

Question 10c

(c) Solve $(g \circ f)(x) = 21$.

[3 marks]

Question 11

The function f is defined by $f(x) = \frac{x+2}{4x-1}$.

(i) Find $f^{-1}(x)$.

It is always true that the graphs of a function and its inverse will be reflections of each other in the line $y = x$.

(ii) Based on the answer to part (i), state what else will be true about the graphs of f and f^{-1} .

[1 mark]