2.3 Functions Toolkit

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.3 Functions Toolkit
Difficulty	Hard

Time allowed: 100

Score: /82

Percentage: /100

Question 1a

The functions f and g are defined such that $f(x) = 2x^2 - 4x$ and $g(x) = \dfrac{5x + 12}{2}$.

(a) Find $(g \circ f)(x)$, giving your answer in the form $(g \circ f)(x) = m(x - h)^2 + k$ where m, h and k are constants to be found.

[3 marks]

Question 1b

(b) Hence, or otherwise, find the coordinates of the vertex of the graph of $y = (g \circ f)(x)$.

[1 mark]

Question 1c

(c) Find $(f \circ g)(x)$, giving your answer in the form $(f \circ g)(x) = ax^2 + bx + c$ where a, b and c are constants to be found.

[3 marks]

Question 1d

(d) Hence, or otherwise, find the coordinates of the y-intercept of the graph of $y = (g \circ f)(x)$.

[1 mark]

Question 2a

Let $f(x) = \dfrac{5-x^2}{3}$ and $g(x) = 4 - \dfrac{3}{x}$, where each function has the largest possible valid domain.

(a) Write down the range of f.

[1 mark]

Question 2b

(b) Write down the domain and range of g.

[2 marks]

Question 2c

(c) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$.

[3 marks]

Question 2d

(d) Solve the equation $(f \circ g)(x) = (g \circ f)(x)$.

[2 marks]

Question 3a

The function f is defined by $f(x) = \sqrt[3]{4(1-x)}$, for $-1 \leq x \leq 17$.
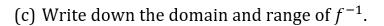
(a) Write down the range of f.

[2 marks]

Question 3b

(b) Write down an expression for f^{-1}.

[2 marks]

Question 3c

(c) Write down the domain and range of f^{-1}.

[2 marks]

Question 4a

The perimeter, P, and area, A, of a given square can be expressed by $P = 4x$ and $A = x^2$ respectively, where x is the length of the side of the square.

(a) Write down an expression for:

    (i)   P in terms of A, P(A)

    (ii)   A in terms of P, A(P).

[4 marks]

Question 4b

  $P^{-1}(40) = A(k)$.

(b) Find the value of k and $A(k)$.

[2 marks]

Question 5a

The values of two functions, f and g, for certain values of x are given in the following table:

x	-2	0	3
$f(x)$	-12	-4	8
$g(x)$	0	-12	30

(a) Find the value of $f^{-1}(8)$.

[1 mark]

Question 5b

(b) Find the value of $(f \circ g)(-2)$.

[2 marks]

Question 5c

(c) Given that $f(x)$ is a linear function, find $f(x)$.

[2 marks]

Question 6a

Let $f(x) = \sqrt{x - 14},$ for $x \geq 14$.

(a) Find $f^{-1}(2)$.

[3 marks]

Question 6b

Let g be a function such that g^{-1} exists for all real numbers.

(b) Given that $g(14) = 3$, find $(f \circ g^{-1})(3)$.

[3 marks]

Question 7a

Let the function f be defined by $f(x) = \sqrt{2x^2 - 16x + 41},$ where f has its largest possible valid domain.

(a) Find the domain and range of f.

[4 marks]

Question 7b

(b) (i)   Find the value(s) of x for which $f(x) = \sqrt{11}$.

(ii) Use your answer to part (b)(i) to explain why the inverse function f^{-1} does not exist.

[2 marks]

Question 8a

Let $f(x) = x^2 - 9$ and $g(x) = x^2 - 1$, both for $x \geq 0$.

(a) Find

(i) $f^{-1}(x)$

(ii) $g^{-1}(x)$.

[2 marks]

Question 8b

(b) Find $(f \circ g)(x)$ in the form $ax^4 + bx^2 + c$.

[2 marks]

Question 8c

(c) Solve the equation $(f \circ g)(x) = 0$.

[3 marks]

Question 9a

(a) Express $x^2 + 12x + 24$ in the form $a(x + b)^2 + c$, where $a, b, c \in \mathbb{Z}$.

[2 marks]

Question 9b

(b) Given that $g(x) = x + 6$ and $(f \circ g)(x) = x^2 + 12x + 24$, find $f(x)$.

[3 marks]

Question 10a

(a) Write $2x^2 + 8x - 3$ in the form $a(x + h)^2 + k$.

[2 marks]

Question 10b

(b) Explain why the function f defined by $f(x) = 2x^2 + 8x - 3, \ x \in \mathbb{R},$ does not have an inverse.

[1 mark]

Question 10c

The function g defined by $g(x) = 2x^2 + 8x - 3, \ x \geq p$ has an inverse.

(c) (i) Write down the smallest possible value of p.

Given that p takes its smallest possible value:

(ii) Find the domain and range of g^{-1}.

(iii) Find the inverse function g^{-1}.

[6 marks]

Question 10d

(d) Solve $(g \circ f)(x) = 21$.

[3 marks]

Question 11a

Let $f(x)$ be an even function and let $g(x)$ be an odd function. Both functions are defined for all real values of x.

(a) Prove the following statements:

  (i) $p(x) = f(x)g(x)$ is an odd function.

  (ii) $q(x) = (f \circ g)(x)$ is an even function.

[4 marks]

Question 11b

(b) Determine whether or not it is possible for the function r defined by

$$r(x) = f(x) + g(x)$$

to be even or odd, being sure to state clearly any conditions that apply.

[3 marks]

Question 12

The function f is defined by $f(x) = \dfrac{ax+b}{cx+d}$, where a, b, c and d are real constants with $c \neq 0$.

Given that f is a self-inverse function, find the value of $a + d$.

[6 marks]