2.7 Polynomial Functions

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.7 Polynomial Functions
Difficulty	Medium

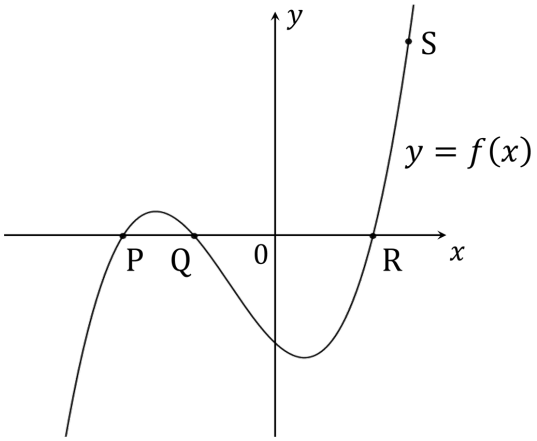
Time allowed: 120

Score: /95

Percentage: /100

Question 1a

Below is the graph of a function $f(x) = ax^3 + bx^2 + cx + d$, passing through the points P$(-3,0)$, Q$(-2,0)$, R$\left(\frac{1}{2}, 0\right)$ and S $(2, 60)$.



a)
Find the values of a, b, c and d.

[2 marks]

Question 1b

The function is translated vertically by the vector $\begin{pmatrix} 0 \\ k \end{pmatrix}$ so that it passes through the point $(3,190)$.

b)
Find the value of k.

[4 marks]

Question 2a

a)
Given that the equation $2x^2 + 4x - m = 0$ has two real solutions, find the set of possible values of m.

[2 marks]

Question 2b

b)
Given that the function $f(x) = x^2 - 5x + 2c$ has repeated roots, find c.

[2 marks]

Question 2c

c)
Given that the function $g(x) = 2x^2 + 2kx + \left(\frac{3}{2} - k\right)$ has no real roots, find the set of possible values of k.

[4 marks]

Question 3

Let a function f be defined by $f(x) = 2x^3 + 7x^2 - 3x - 18$.

(i)
Show that $(x + 3)$ is a factor of $f(x)$.

(ii)
Hence factorise $f(x)$ fully.

(iii)
Write down all the solutions to $2x^3 + 7x^2 - 3x - 18 = 0$.

[6 marks]

Question 4a

a)
Factorise fully $6x^3 + x^2 - 12x + 5$.

[4 marks]

Question 4b

b)
$f(x) = ax^3 + (5a-2)x^2 + (4a+2)x - 2a$

(i)
Given that $(x+3)$ is a factor of $f(x)$, find a.

(ii)
Hence factorise $f(x)$ fully.

[7 marks]

Question 5a

Consider the polynomial $g(x) = 3x^5 - 25x^4 + 72x^3 - 72x^2 - 16x + 48.$

a)
Show that 2 is a root of $g(x)$.

[2 marks]

Question 5b

b)
Given that 2 is a root of $g(x)$ with multiplicity 3, factorise $g(x)$ fully and hence state the other two roots.

[5 marks]

Question 6

Consider the function $f(x) = 4x^3 + 6x^2 - 7x + 2.$

(i)
Find the quotient and remainder when $4x^3 + 6x^2 - 7x + 2$ is divided by $(x - 2).$

(ii)
Hence write $4x^3 + 6x^2 - 7x + 2$ in the form $(x - 2)(ax^2 + bx + c) + d,$ where $a,\ b,\ c$ and d are constants to be determined.

[5 marks]

Question 7a

The function $f(x) = 2x^3 - 5x^2 + ax + b$ has $(2x + 3)$ as a factor, and when $f(x)$ is divided by $(x - 2)$ the remainder is 7.

a)
Show that a and b must satisfy the simultaneous equations:

$$2a + b = 11$$

$$3a - 2b = -36$$

[5 marks]

Question 7b

b)
Hence find a and b.

[2 marks]

Question 8

Given that $3+2i$ is one of the roots of the equation $x^3-3x^2-5x+39=0$, find the other two roots.

[5 marks]

Question 9a

a)

For each of the following polynomials, find the sum of the roots and the product of the roots.

(i)

$f(x)=9x^4+7x^3-3x+2$

(ii)

$g(x)=7x^5-x^4+2x^3+x^2-5x+14$

(iii)

$h(x)=2x^3-5x^2-3x$

(iv)

$j(x)=-3x^4+2x^2+5x-3$

[5 marks]

Question 9b

b)

Consider the equation $6x^3 - (4a)x^2 - (a+2)x = 0$.

Given that the sum of the roots is $\frac{8}{3}$, find the three roots of the equation.

[5 marks]

Question 10

For the function $f(x) = ax^4 + bx^3 - x^2 - 24x - (5b+1)$, the sum of the roots is $\frac{-7}{2}$ and the product of the roots is -18.

Find the values of a and b.

[4 marks]

Question 11a

The function $f(x) = (x-3)(x^2+3x-4)(ax^2+bx+c)$ has three real and two complex roots.

a)
Find the three real roots.

[2 marks]

Question 11b

It is given for $f(x)$ that the sum of the roots is $-\frac{3}{2}$ and the product of the roots is -60.

b)
Find the two complex roots, giving your answers in exact form.

[5 marks]

Question 11c

c)
Given that $f(2) = -144$, find the values of a, b and c.

[4 marks]

Question 12a

α and β are non-real roots of the equation $x^2 + 3kx + 2k + 1 = 0$, where $k > 0$ is a constant.

a)
Find $\alpha + \beta$ and $\alpha\beta$, in terms of k.

[2 marks]

Question 12b

b)
Given that $\alpha^2 + \beta^2 = 3$, show that $(\alpha + \beta)^2 = 4k + 5$.

[2 marks]

Question 12c

c)
Hence find the value of k.

[3 marks]

Question 13a

Consider the function $f(x) = kx^3 + 3x^2 + 11x + 3k$, where k is a constant.

It is given that $(2x - 1)$ is a factor of $f(x)$.

(a)
Find the value of k.

[2 marks]

Question 13b

(b)
Fully factorise $f(x)$.

[3 marks]

Question 13c

(c)

Hence sketch the graph of $y = f(x)$. Clearly label the coordinates of any points where the graph intersects the coordinate axes.

[3 marks]