6.1 Extended Questions (Paper 2, SL)

Question Paper

Course	DP IB Maths
Section	6. Extended Questions
Topic	6.1 Extended Questions (Paper 2, SL)
Difficulty	Hard

Time allowed: 120

Score: /94

Percentage: /100

Question 1a

Sharon set up an experiment to investigate the relationship between the mass of a mouse and the time the mouse takes to complete a mini assault course. She conducted the experiment with six mice and recorded her results in the table below.

Mouse mass, x (g)	19.9	18.3	21.1	19.8	17.5	16.3
Time, y (seconds)	18.0	17.7	20.9	18.6	15.0	14.2

(a)   (i)   Calculate Pearson's product-moment correlation coefficient, r.

(ii)   Describe the relationship between the mass of the mice and the time taken to complete the mini assault course.

[3 marks]

Question 1b

(b) Write down the equation of the regression line of y on x, in the form $y = mx + c$.

[2 marks]

Question 1c

(c) Find the coordinates of the point M$(\bar{x}, \bar{y})$.

[2 marks]

Question 1d

(d) Show that the point $M(\bar{x}, \bar{y})$ lies on the line of regression.

[2 marks]

Question 1e

The mass of a seventh mouse is found to be 20.6 g.

(e) (i)   Using your line of regression, estimate the time that the seventh mouse will take to complete the mini assault course.

(ii)   Justify whether it is valid to use the line of regression to estimate the result for the seventh mouse.

[4 marks]

Question 1f

In the actual experiment, it was found that the seventh mouse took 20.7 seconds to complete the mini assault course.

(f) Calculate the percentage error in the estimated value.

[2 marks]

Question 2a

Consider the functions $f(x) = -x^3 - 2x^2 + 4$ and $g(x) = 3(0.5^x) - 2$.

(a) Calculate $g(-2)$.

[2 marks]

Question 2b

(b) Find the value of x when $f(x) = 5$.

[2 marks]

Question 2c

(c) For $g(x)$

      (i)   write down the equation of the horizontal asymptote,

      (ii)   write down the coordinates of the y intercept.

[2 marks]

Question 2d

(d) Sketch the function $y = g^{-1}(x)$, labelling where the graph intersects with any axes.

[3 marks]

Question 2e

(e) Find the solutions of $f(x) = g(x)$.

[2 marks]

Question 2f

(f) The equation $f(x) = k$ has exactly two solutions. Find the possible values of k.

[2 marks]

Question 3a

A charity for premature babies asks for people to knit baby hats for newborns. Becky finds that she already has 5 baby hats that she knitted for her own child and decides to knit some more. At the end of the first week she has managed to knit only 1 new hat, but as her speed increases she knits 2 more each week than she did the previous week.

Becky knits this way for 7 weeks in total.

(a) (i) Find the number of hats she will make in the seventh week.

(ii) Find the total number of hats that she will have ready to donate by the end of the seventh week.

[5 marks]

Question 3b

In week 1, the charity starts off with 25 volunteers. The number of volunteer knitters increases at a rate of 20% each week. Each knitter averages 6 new hats per week.

(b) Determine the number of volunteers that the charity has knitting for them after 4 weeks.

[3 marks]

Question 3c

(c) Given that each hat requires 45 metres of yarn to make, find out how much yarn is used by the volunteers in the first 4 weeks.

[4 marks]

Question 3d

The charity sets a target of making 60, 000 hats, enough for each premature baby born in one year in the UK to receive one.

(d) Determine the first week in which this target is achieved.

[3 marks]

Question 4a

An area of sea alongside an island is infested with three man-eating sharks. Each shark has its own base located at the points marked A, B and C on the grid below. Each unit on the grid represents 1 km.
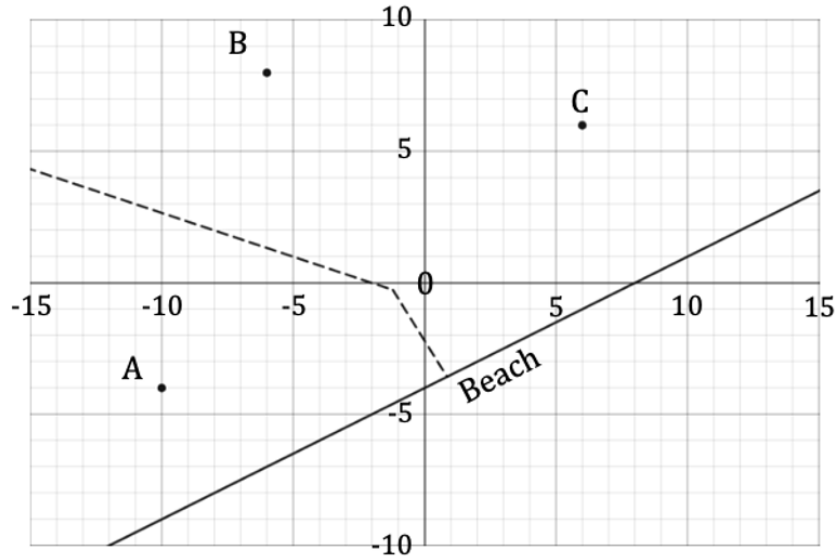
The sharks are very territorial and only hunt in their own section of the sea, the boundaries between which have naturally evolved so that they are equidistant from each shark's base. The boundaries between sharks A and B as well as sharks A and C are shown but the boundary between shark B and C is not indicated on the grid.

(a) (i)   Find the midpoint between B and C.

  (ii) Find the gradient of the line connecting B and C.

  (iii) Hence, find the equation of boundary line between sharks B and C.

  (iv) Draw the boundary on the grid.

[5 marks]

Question 4b

A person takes a boat out on the water and decides to go for a swim between the shark bases.

(b) (i)   Find the equation of the boundary line between A and B.

(ii) Hence, or otherwise, find the coordinates of the safest point in the water, furthest from each of the shark bases, in which the person should swim.

[5 marks]

Question 4c

(c) Calculate the distance between the swimmer and the nearest shark when the swimmer is at the point identified in part (b)(ii).

[2 marks]

Question 4d

After the deadly shark attack on the swimmer, it is decided to install a shark protection barrier parallel to the beach line. The barrier is built along the line with equation $y = \frac{1}{2}x - 2$.

(d) Calculate the area of the sea in the region shown in the diagram that it is now safe to swim in.

[4 marks]

Question 5a

An IB student, Fred, decides to survey students in his school to find out their preferred area of study out of the arts, humanities and sciences. He also wants to know what their preferred part of the CAS program is out of creativity, activity and services. Fred uses a list of all the students in the school and selects every 5th student to survey. The data is shown in the following table.

	Creativity	Activity	Service	Total
Arts	19	15	7	41
Humanities	9	22	11	42
Sciences	16	8	15	39
Total	44	45	33	122

(a) State the type of sampling that Fred uses.

[1 mark]

Question 5b

A student is picked at random from the data.

(b) Calculate the probability that the student picked

(i) prefers to study the arts,

(ii) prefers to complete service or creative options in their free time, given that their favourite area of study is the sciences.

[4 marks]

Question 5c

(c) Given that there are 612 students in Fred's school, find the expected number of students who prefer to study humanities or to take part in activity CAS options in their free time.

[3 marks]

Question 5d

Fred performs a χ^2 test on the data at a significance level of 5%. The critical value for this test is 9.488.

(d) Write down

    (i) H_0, the null hypothesis,

    (ii) H_1, the alternative hypothesis,

    (iii) the number of degrees of freedom.

[3 marks]

Question 5e

(e) Find

    (i)   the chi squared statistic,

    (ii)   the p-value.

[3 marks]

Question 5f

(e) Find

    (i)   the chi squared statistic,

    (ii)   the p-value.

[2 marks]

Question 5g

(f) State whether you would reject the null hypothesis. Give a reason for your answer.

[2 marks]

Question 6a

Chun-hee is creating some packaging in the shape of a square based pyramid where the base has length x cm and the perpendicular height of the pyramid is h cm. Chun-hee wants to keep the distance from the apex of the pyramid to the midpoint of the base edge fixed at 7 cm.

(a) Write down an equation for the volume, V, of the packaging in terms of x and h.

[1 mark]

Question 6b

(b) Show that V can be expressed by $\dfrac{196}{3}h - \dfrac{4}{3}h^3$.

[3 marks]

Question 6c

(c) Find $\dfrac{dV}{dh}$.

[2 marks]

Question 6d

(d) Find the value of h for which the volume of the pyramid is maximised.

[2 marks]

Question 6e

(e) Find the value of x when the volume of the pyramid is maximised.

[2 marks]

Question 6f

Chun-hee decides to make the packaging using the dimensions required to maximise the volume. The material for the packaging costs 4 KRW / cm^2.

(f) Calculate the number of units that Chun-hee can make given that she has 90, 000 KRW.

[4 marks]

Question 6g

Chun-hee takes out a 3 year loan for 90,000 KRW at a nominal annual interest rate of 2.3% compounded monthly. Repayments are made at the end of each month.

(g) Find the value of the repayments that Chun-hee must make to pay off the loan.

[3 marks]