3.9 Modelling with Vectors

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.9 Modelling with Vectors
Difficulty	Medium

Time allowed: 90

Score: /68

Percentage: /100

Question 1a

Two ships A and B are travelling so that their position relative to a fixed point O at time t, in hours, can be defined by the position vectors $\boldsymbol{r}_{\mathrm{A}} = (2-t)\boldsymbol{i} + (4+3t)\boldsymbol{j}$ and $\boldsymbol{r}_{\mathrm{B}} = (t-8)\boldsymbol{i} + (29-2t)\boldsymbol{j}$.

The unit vectors i and j are a displacement of 1 km due East and North of O respectively.

a)
Find the coordinates of the initial position of the two ships.

[2 marks]

Question 1b

b)
Show that the two ships will collide and find the time at which this will occur.

[3 marks]

Question 1c

c)
Find the coordinates of the point of collision.

[2 marks]

Question 2a

A car, moving at constant speed, takes 4 minutes to drive in a straight line from point $\mathrm{A}(-3, 5)$ to point $\mathrm{B}(7, 11)$.

At time t, in minutes, the position vector of the car relative to the origin can be given in the form $\boldsymbol{p} = a + tb$.

a)
Find the vectors a and b.

[3 marks]

Question 2b

A cat has decided to take a nap at point $\mathrm{X}(4, 9)$.

b)
Show that the cat does not lie on the route along which the car drives.

[3 marks]

Question 2c

c)
Find the shortest distance between the car and the cat during the movement of the car.

[6 marks]

Question 3a

A bird takes off from a perch and flies at a constant speed in a straight line. The position of the bird in flight relative to its nest, (east, north and above/below the nest), can be described by the vector equation

$$r_1 = \begin{pmatrix} 18 \\ 4 \\ -2 \end{pmatrix} + t \begin{pmatrix} 272 \\ -360 \\ 225 \end{pmatrix}.$$

All displacements are given in metres and t is the time in minutes.

(a) Find the distance between the perch that the bird took flight from and its nest.

[2 marks]

Question 3b

(b) Find the speed at which the bird is travelling. Give your answer in kmh^{-1}.

[3 marks]

Question 3c

A second bird takes off at the same time as the first bird from a different perch and also flies in a straight line at a constant speed. The flight of the second bird, relative to the same nest, can be described by the vector equation

$$r_2 = \begin{pmatrix} 12 \\ -8 \\ -3 \end{pmatrix} + t \begin{pmatrix} -187 \\ -438 \\ 80 \end{pmatrix}.$$

(c) Find the distance between the two birds after 8 minutes of flying.

[4 marks]

Question 4a

A drone travels in a straight line and at a constant speed. It moves from an initial point $A(4, 5, -2)$ to a second point $B(7, -1, 0)$. The person controlling the drone is located at $C(2, 3, 2)$.

The x, y and z directions are due east, due north and vertically upwards respectively with all distances in metres.

(a) Write down an equation for the line along which the drone travels.

[2 marks]

Question 4b

At some point P on its flight, the drone is vertically level with the person controlling the drone.

(b) Find the coordinates of point P.

[3 marks]

Question 4c

(c) Find the distance between P and the person controlling the drone.

[2 marks]

Question 5a

Two snails are taking part in a snail race starting from the same point and moving in a straight line. The position of the first snail S_1 is given by the equation

$$r = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \end{pmatrix},$$

The displacement of the second snail S_2, relative to the finish point, is given by

$$s(t) = 8 - 3t^2.$$

All distances are in centimetres and time is in minutes.

(a) Write down the distance that the snails race.

[1 mark]

Question 5b

(b) Find an expression for the velocity of S_2 at time t.

[2 marks]

Question 5c

(c) Find the displacement of S_2 from the finishing point when the speed of the two snails is equal.

[5 marks]

Question 6a

A ball is pushed off the top of a 150 m tall skyscraper with an initial velocity of $\begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$ ms^{-1}.

The point at which the ball is pushed can be considered the origin of a Cartesian coordinate system. It is assumed that any effects of air resistance will be negligible and $g = 9.81\ ms^{-2}$.

(a) Find the velocity vector at time t.

[2 marks]

Question 6b

(b) displacement vector of the ball at time t.

[2 marks]

Question 6c

(c) Find the time at which the ball reaches the ground.

[2 marks]

Question 6d

(d) Find the total horizontal distance travelled by the ball.

[2 marks]

Question 7a

Two aeroplanes are observed flying in straight lines, with respect to the airport located at $(0, 0, 0)$. The flightpaths l_A and l_B, of aeroplanes A and B respectively, can be defined by:

$$l_A: \boldsymbol{r} = \begin{pmatrix} 6 \\ 3 \\ -2 \end{pmatrix} + \alpha \begin{pmatrix} -1 \\ -4 \\ 3 \end{pmatrix}$$

$$l_B: \boldsymbol{s} = \begin{pmatrix} -7 \\ -1 \\ 5 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

where α and β is the time elapsed in minutes since the start of the observation for each aeroplane. All distances are in kilometres.

The flightpaths intersect at point P.

(a) Find the values of α and β and hence show that the two planes will not collide.

[4 marks]

Question 7b

(b) Find

(i) the coordinates of the point at which the flightpaths intersect,

(ii) the distance between the airport and the point P.

[4 marks]

Question 8a

A particle starts from a position at $(0, 0)$ and moves such that its velocity at time t, in seconds, is given by $v = \begin{pmatrix} 2e^{3t} \\ e^{3t} - 4 \end{pmatrix}$. All distances are in metres.

(a) Find an expression for the acceleration of the particle at time t.

[2 marks]

Question 8b

(b) Find an expression for the position of the particle at time t.

[4 marks]

Question 8c

(c) Find the value of t such that the speed of the particle is $6\ \text{ms}^{-1}$.

[3 marks]