IB Physics DP

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2. Mechanics

CONTENTS

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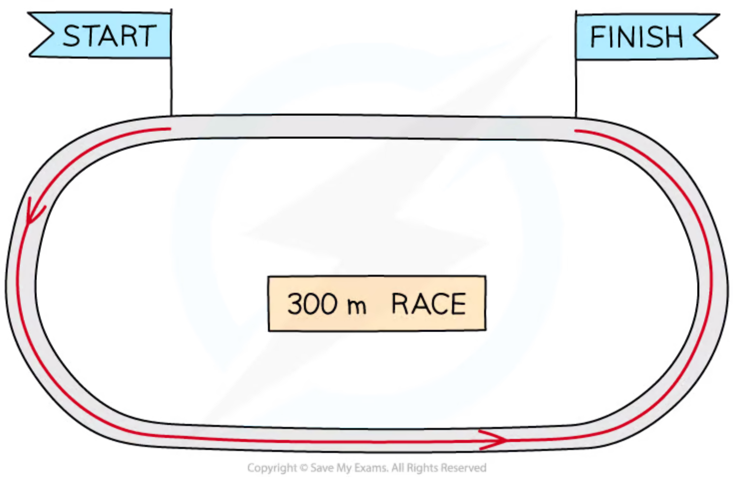
2.1 Motion

2.1.1 Distance & Displacement

Distance & Displacement

- **Distance** is a measure of how far an object travels
- It is a **scalar** quantity - in other words, the direction is not important

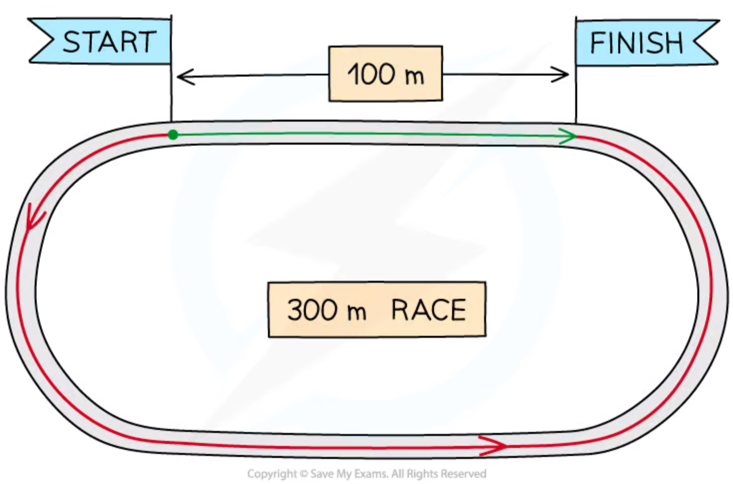


The athletes run a total distance of 300 m

- Consider a 300 m race
- From start to finish, the **distance** travelled by the athletes is **300 m**

Displacement

- **Displacement** is a measure of how far something is from its starting position, along with its direction
- It is a **vector** quantity - it describes both magnitude and direction



The athletes run a total distance of 300 m, but end up 100 m from where they started

- Consider the same 300 m race again
  - The athletes have still run a total **distance** of **300 m** (this is indicated by the arrow in red)
  - However, their **displacement** at the end of the race is **100 m to the right** (this is indicated by the arrow in green)
  - If they had run the full 400 m, their final displacement would be **zero**

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Speed & Velocity

Speed

- The **speed** of an object is the distance it travels every second
- Speed is a **scalar** quantity
  - This is because it only contains a magnitude (without a direction)
- The **average speed** of an object is given by the equation:

$$\text{Average speed} = \frac{\text{total distance}}{\text{time taken}}$$

- Speed is typically measured in **meters per second**, **m/s,** but can be measured in alternative units when it is more appropriate for the situation

?
Worked Example

Florence Griffith Joyner set the women's 100 m world record in 1988, with a time of 10.49 s.Calculate her average speed during the race.

Step 1: List the known quantities

- Distance, s = 100 m
- Time, t = 10.49 s

Step 2: Write the relevant equation

- Sprinters typically speed up out of the blocks up to some maximum speed
- Because Florence's speed changes over the race, we can calculate her average speed using the equation:

average speed = total distance ÷ time taken

Step 3: Check any unit conversions

- Check that all quantities given in the question are in standard units
- In this example, they are all in standard units

Step 4: Substitute the values for total distance and time

Average speed = 100 ÷ 10.49 = 9.53288... = 9.53 m/s

Velocity

- The **velocity** of a moving object is similar to its speed, except it also describes the object's direction
  - The speed of an object only contains a magnitude - it's a scalar quantity
  - The velocity of an object contains both magnitude and direction, e.g. '15 m/s south' or '250 km/h on a bearing of 030°'
- Velocity is, therefore, a **vector** quantity because it describes both magnitude and direction

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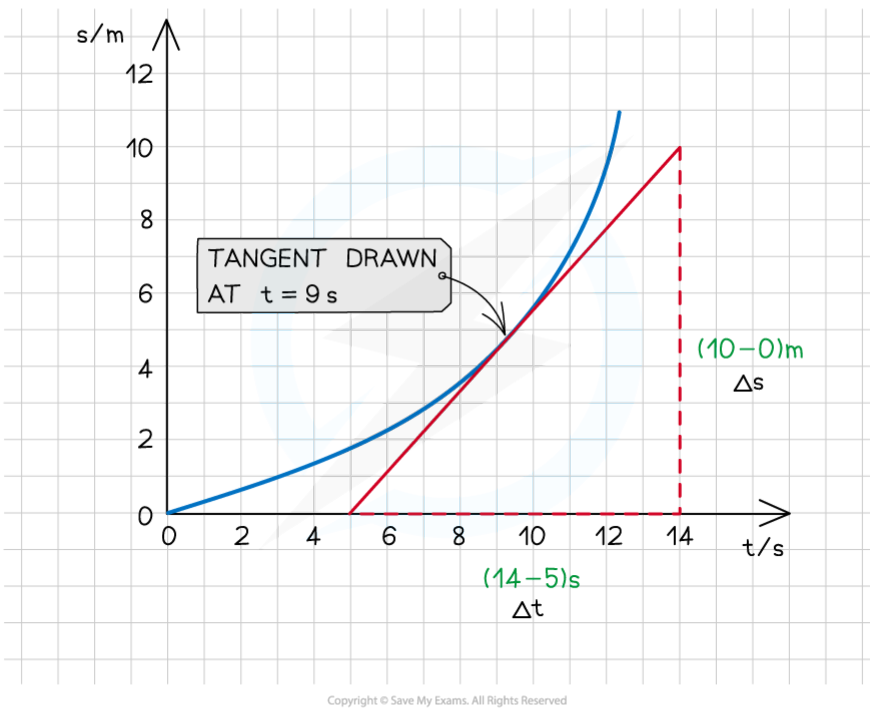
The cars in the diagram above have the same speed (a scalar quantity) but different velocities (a vector quantity). Fear not, they are in different lanes!

Instantaneous Speed / Velocity

- The **instantaneous** speed (or velocity) is the speed (or velocity) of an object **at any given point in time**
- This could be for an object moving at a constant velocity or accelerating
  - An object accelerating is shown by a **curved line** on a displacement – time graph
  - An accelerating object will have a changing velocity
- To find the instantaneous velocity on a displacement-time graph:
  - Draw a **tangent** at the required time
  - Calculate the **gradient** of that tangent



The instantaneous velocity is found by drawing a tangent on the displacement time graph

Average Speed / Velocity

- The average speed (or velocity) is the **total distance** (or displacement) divided by the **total time**

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- To find the average velocity on a displacement-time graph, divide the **total displacement** (on the y-axis) by the **total time** (on the x-axis)
  - This method can be used for both a curved or a straight line on a displacement-time graph

Exam Tip

When you draw a tangent to a curve, make sure it **just touches** the point at which you wish to calculate the gradient. The angle between the curve and the tangent line should be roughly equal on both sides of the point.

2.1.3 Acceleration

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Acceleration

- **Acceleration** is defined as the **rate of change of velocity**
  - In other words, it describes how much an object's velocity **changes** every **second**
- The equation below is used to calculate the average acceleration of an object:

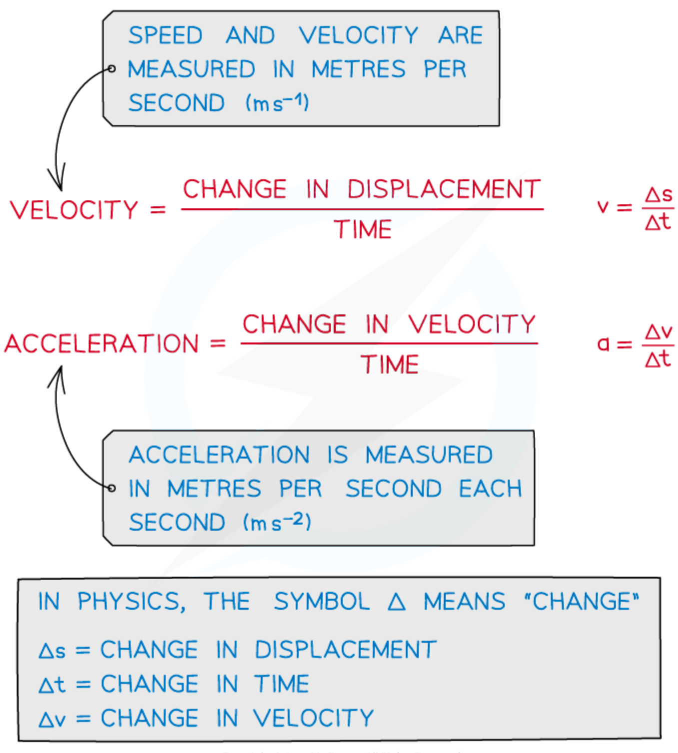
$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$a = \frac{\Delta v}{t}$$

- Where:
  - a = acceleration in metres per second squared (m s^{-2})
  - Δv = change in velocity in metres per second (m s^{-1})
  - t = time taken in seconds (s)
- The **change in velocity** is found by the **difference** between the initial and final velocity, as written below:

change in velocity = final velocity – initial velocity

Equations linking displacement, velocity, and acceleration



Speeding Up and Slowing Down

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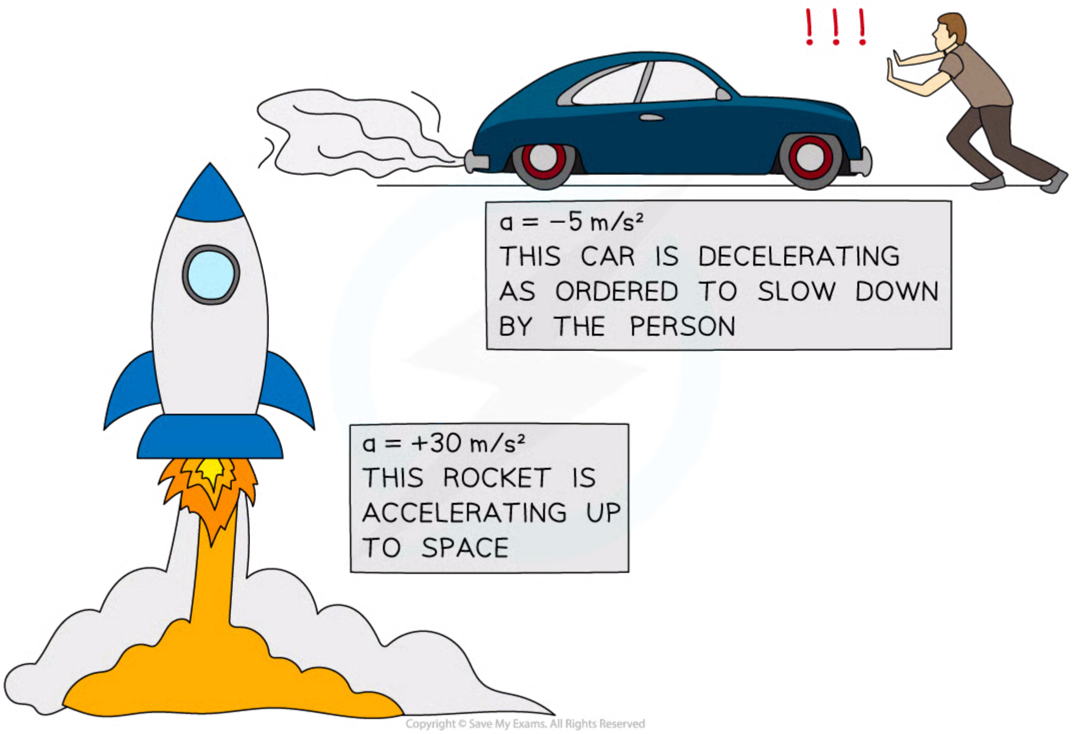
- An object that speeds up is **accelerating**
- An object that slows down is **decelerating**
- The **acceleration** of an object can be **positive** or **negative**, depending on whether the object is speeding up or slowing down
  - If an object is **speeding up**, its acceleration is **positive**
  - If an object is **slowing down**, its acceleration is **negative** (sometimes called **deceleration**)



A rocket speeding up (accelerating) and a car slowing down (decelerating)

Worked Example

A Japanese bullet train decelerates at a constant rate in a straight line. The velocity of the train decreases from 50 m s^{-1} to 42 m s^{-1} in 30 seconds.

(a) Calculate the change in velocity of the train.

(b) Calculate the deceleration of the train, and explain how your answer shows the train is slowing down.

Part (a)

Step 1: List the known quantities

- Initial velocity = 50 m s^{-1}
- Final velocity = 42 m s^{-1}

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Step 2: Write the relevant equation

change in velocity = final velocity – initial velocity

Step 3: Substitute values for final and initial velocity

change in velocity = 42 – 50 = –8 m s^{-1}

Part (b)

Step 1: List the known quantities

- Change in velocity, Δv = –8 m s^{-1}
- Time taken, t = 30 s

Step 2: Write the relevant equation

$$a = \frac{\Delta v}{t}$$

Step 3: Substitute the values for change in velocity and time

a = –8 ÷ 30 = –0.27 m s^{-1}

Step 4: Interpret the value for deceleration

- The answer is **negative**, which indicates the train is **slowing down**

Exam Tip

Remember the units for acceleration are **metres per second squared**, m s^{-2}In other words, acceleration measures how much the velocity (in m s^{-1}) changes every second, m $s^{-1}s^{-1}$.

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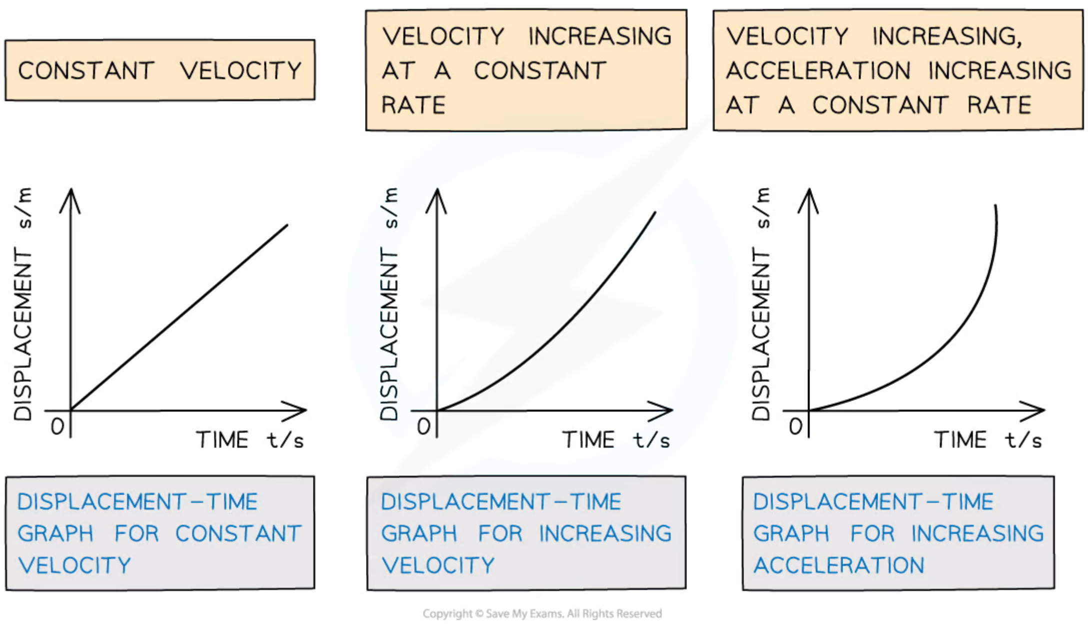
2.1.4 Graphs Describing Motion

Motion Graphs

- Three types of graphs that can represent motion are **displacement-time** graphs, **velocity-time** graphs, and **acceleration-time** graphs

Displacement-Time Graph

- On a **displacement-time graph…**
  - **Slope** equals **velocity**
  - The **y-intercept** equals the **initial displacement**
  - A **straight** (diagonal) line represents a **constant** velocity
  - A **curved** line represents an **acceleration**
  - A **positive slope** represents motion in the **positive direction**
  - A **negative slope** represents motion in the **negative direction**
  - A **zero** slope (horizontal line) represents a state of **rest**
  - The area under the curve is meaningless



Displacement-time graphs displacing difference velocities

Velocity-Time Graph

- On a **velocity-time graph…**
  - **Slope** equals **acceleration**
  - The **y-intercept** equals the **initial velocity**
  - A **straight** line represents **uniform** acceleration
  - A **curved** line represents **non-uniform acceleration**
  - A **positive** slope represents an **increase** in **velocity** in the **positive direction**
  - A **negative** slope represents an **increase** in **velocity** in the **negative direction**
  - A **zero** slope (horizontal line) represents motion with **constant velocity**
  - The **area** under the curve equals the **change** in **displacement**

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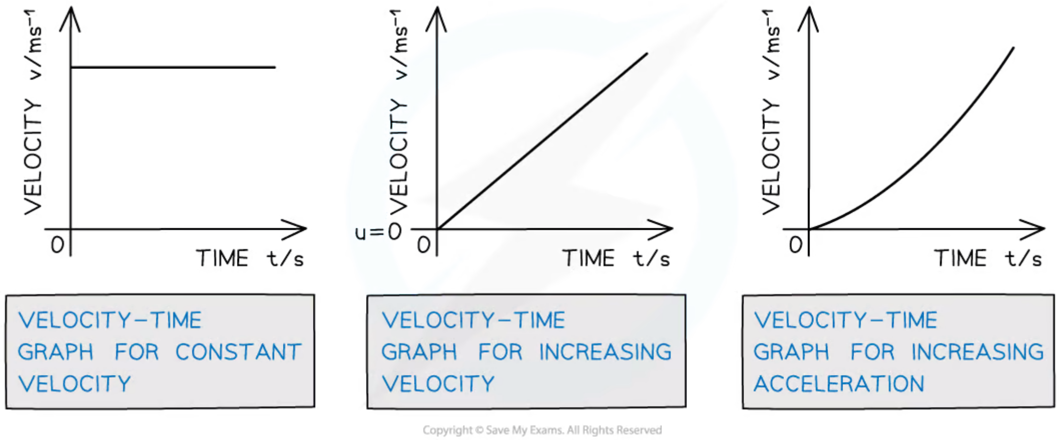
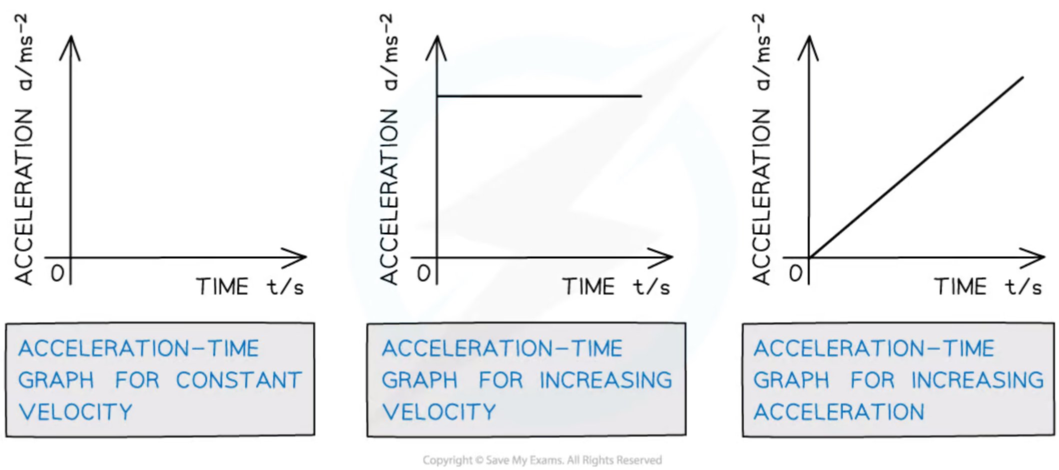


Velocity-time graphs displacing different acceleration

Acceleration-Time Graph

- On an **acceleration-time graph…**
  - Slope is meaningless
  - The **y-intercept** equals the **initial acceleration**
  - A **zero slope** (horizontal line) represents an object undergoing **constant acceleration**
  - The **area** under the curve equals the **change** in **velocity**



How displacement, velocity and acceleration graphs relate to each other

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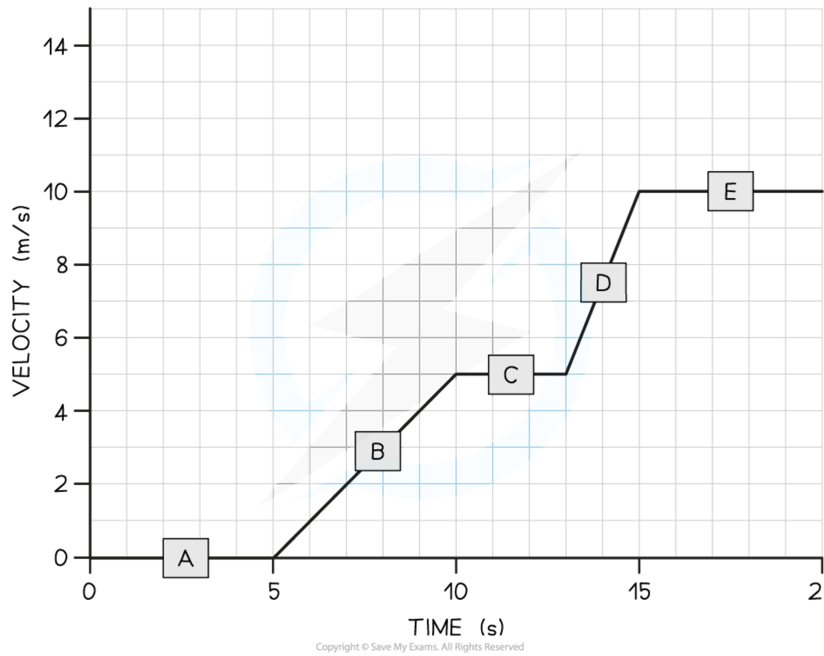
Worked Example

Tora is training for a cycling tournament.

The velocity-time graph below shows her motion as she cycles along a flat, straight road.



(a) In which section (A, B, C, D, or E) of the velocity-time graph is Tora's acceleration the largest?
(b) Calculate Tora's acceleration between 5 and 10 seconds.

Part (a)

Step 1: Recall that the slope of a velocity-time graph represents the magnitude of acceleration

- The slope of a velocity-time graph indicates the magnitude of acceleration Therefore, the only sections of the graph where Tora is accelerating is section B and section D
- Sections A, C, and E are flat – in other words, Tora is moving at a constant velocity (i.e. not accelerating)

Step 2: Identify the section with the steepest slope

- Section D of the graph has the steepest slope

Hence, the largest acceleration is shown in **section D**

Part (b)

Step 1: Recall that the gradient of a velocity-time graph gives the acceleration

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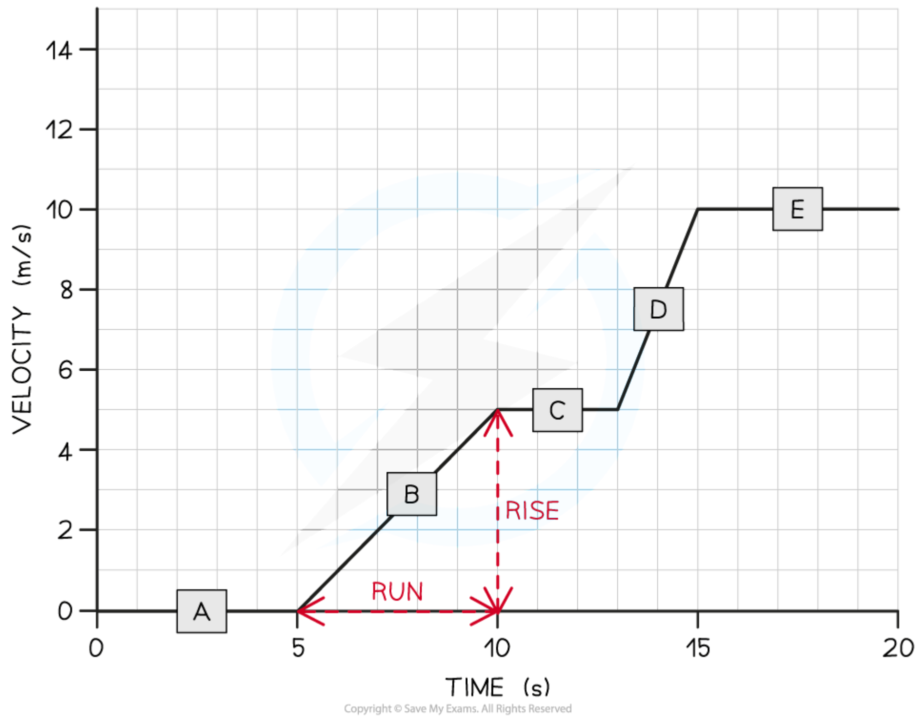
- Calculating the gradient of a slope on a velocity-time graph gives the acceleration for that time period

Step 2: Draw a large gradient triangle at the appropriate section of the graph

- A gradient triangle is drawn for the time period between 5 and 10 seconds below:



Step 3: Calculate the size of the gradient and state this as the acceleration

- The acceleration is given by the gradient, which can be calculated using:

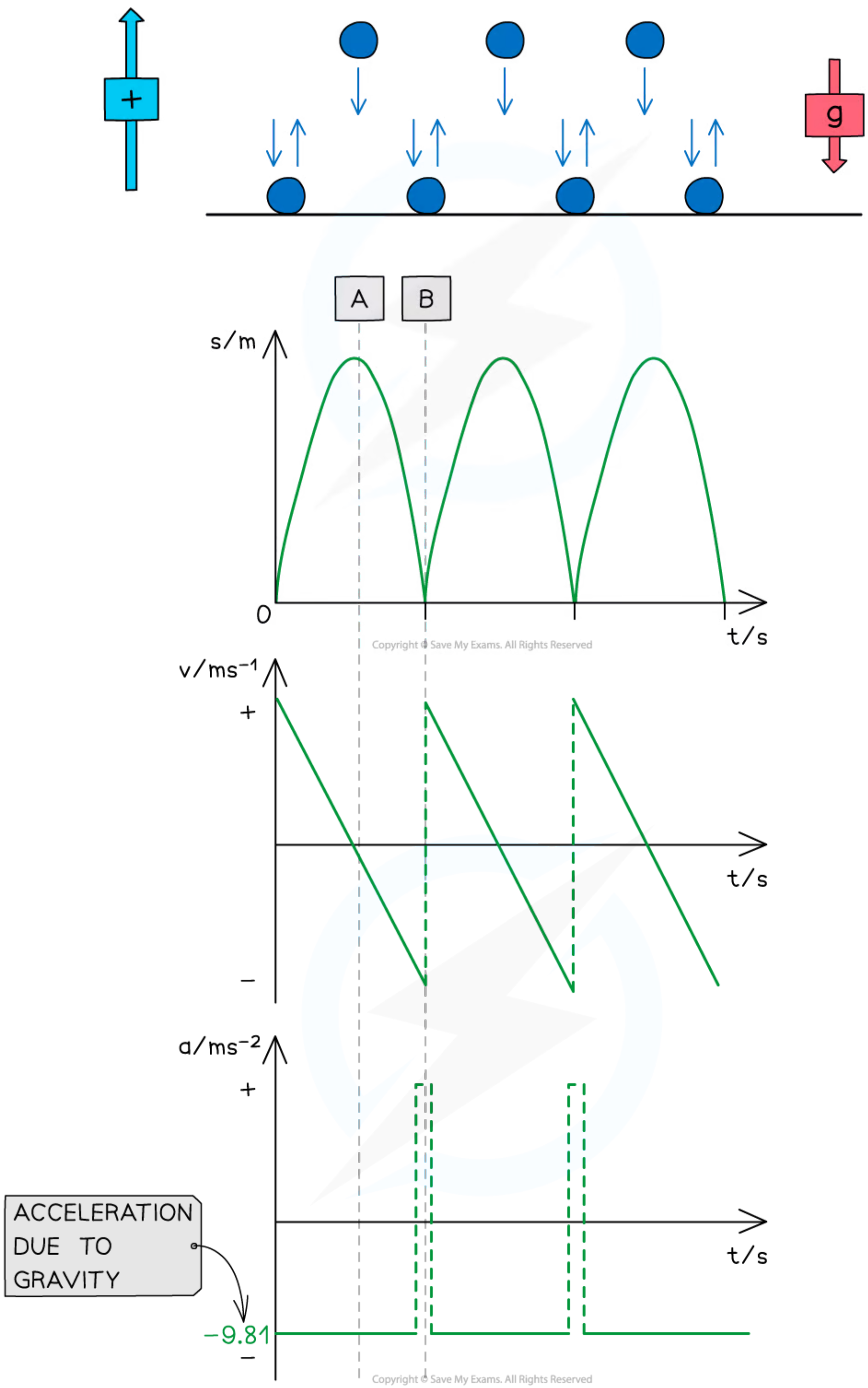
$$\text{acceleration} = \text{gradient} = 5 \div 5 = 1 \text{ m/s}^2$$

- Therefore, Tora accelerated at 1 m/s^2 between 5 and 10 seconds

Motion of a Bouncing Ball

- For a bouncing ball, the acceleration due to gravity is **always** in the same direction (in a uniform gravitational field such as the Earth's surface)
  - This is assuming there are **no** other forces on the ball, such as **air resistance**
- Since the ball changes its direction when it reaches its highest and lowest point, the direction of the velocity will change at these points
- The vector nature of velocity means the ball will sometimes have a:
  - **Positive velocity** if it is traveling in the positive direction
  - **Negative velocity** if it is traveling in the negative direction
- An example could be a ball bouncing from the ground back upwards and back down again
  - The positive direction is taken as upwards
  - This will be either stated in the question or can be chosen, as long as the direction is consistent throughout

- Ignoring the effect of air resistance, the ball will reach the same height every time before bouncing from the ground again
- When the ball is traveling upwards, it has a positive velocity which slowly decreases (decelerates) until it reaches its highest point



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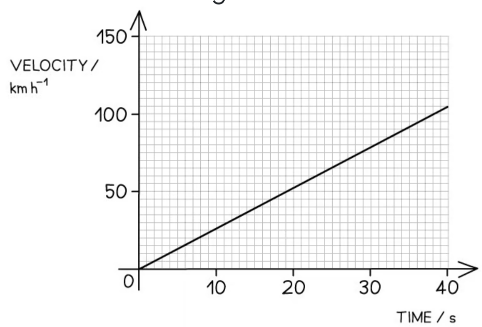
- At point **A** (the highest point):
  - The ball is at its **maximum displacement**
  - The ball momentarily has **zero velocity**
  - The **velocity** changes from **positive** to **negative** as the ball changes direction
  - The **acceleration**, *g*, is still **constant** and directed vertically downwards
- At point **B** (the lowest point):
  - The ball is at its **minimum displacement** (on the ground)
  - Its **velocity** changes instantaneously from **negative** to **positive**, but its **speed** (magnitude) **remains the same**
  - The **change** in direction causes a **momentary acceleration** (since acceleration = change in velocity / time)

Worked Example

The velocity-time graph of a vehicle travelling with uniform acceleration is shown in the diagram below.



Calculate the displacement of the vehicle at 40 s.

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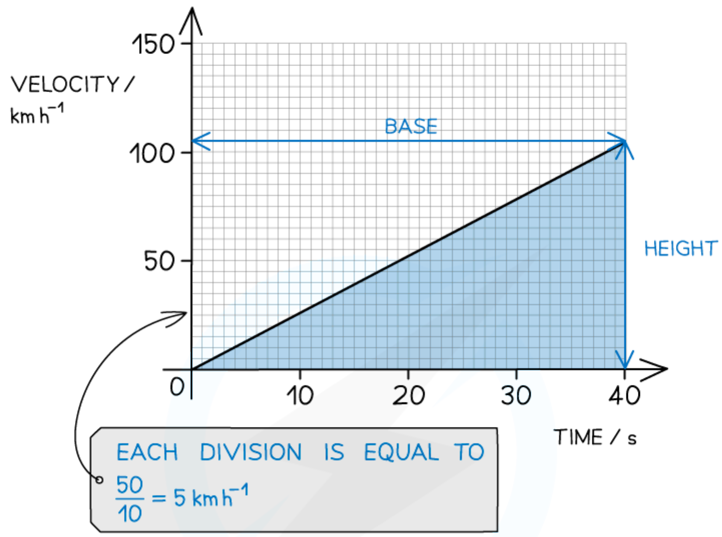
THE DISPLACEMENT IS EQUAL TO THE AREA UNDER A VELOCITY–TIME GRAPH



CONVERT km h^{-1} TO km s^{-1}

BASE = TIME = 40s

HEIGHT = VELOCITY = 105 km h^{-1}

$\frac{105}{60 \times 60} = 0.0292$ km s^{-1}

WORK OUT THE DISPLACEMENT

AREA OF A TRIANGLE = $\frac{1}{2}$ × BASE × HEIGHT

DISPLACEMENT = VELOCITY × TIME = $\frac{1}{2} \times 40 \times 0.0292$ = 0.6 km OR 600 m

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Equations of Motion for Uniform Acceleration

Deriving Kinematic Equations of Motion

- The kinematic equations of motion are a set of four equations that can describe any object moving with **constant** acceleration
- They relate the five variables:
  - s = **displacement**
  - u = **initial velocity**
  - v = **final velocity**
  - a = **acceleration**
  - t = **time interval**
- Knowing where these equations come from and how they are derived helps understand them:

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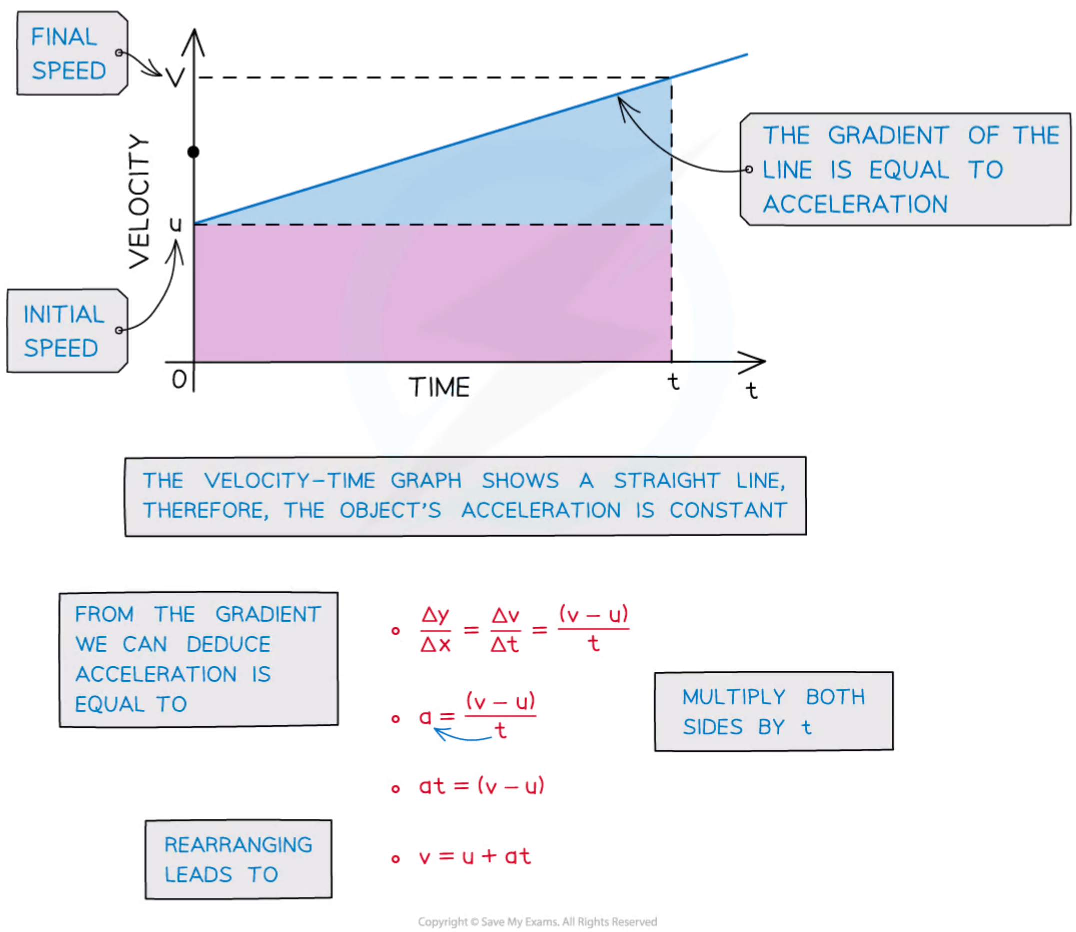
Deriving v = u + at



A graph showing how the velocity of an object varies with time

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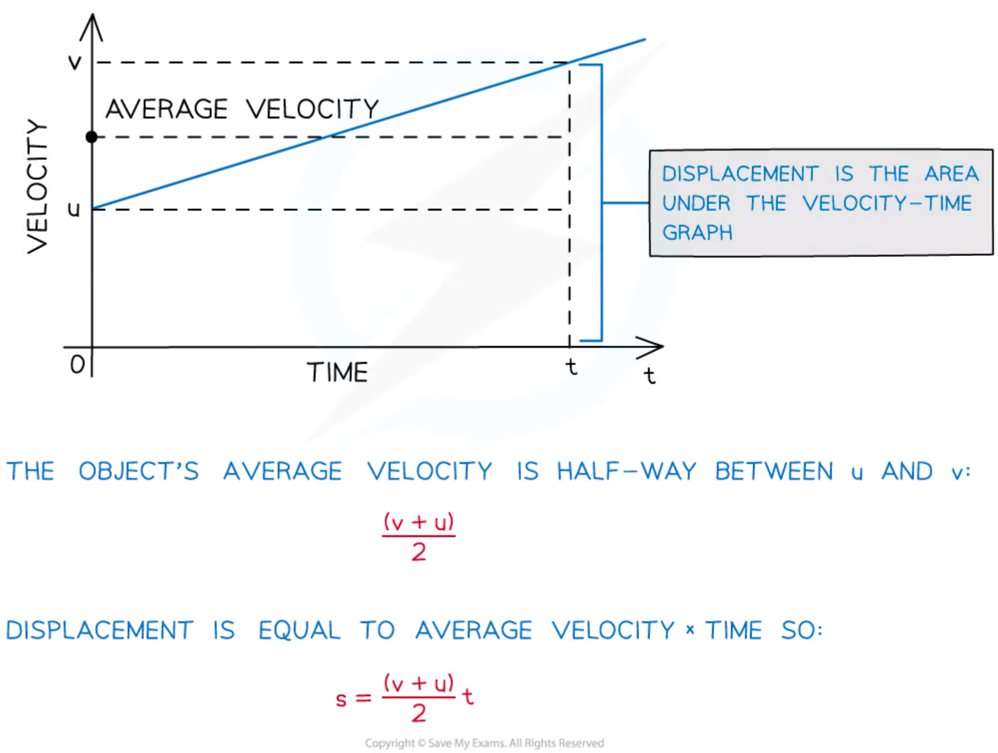
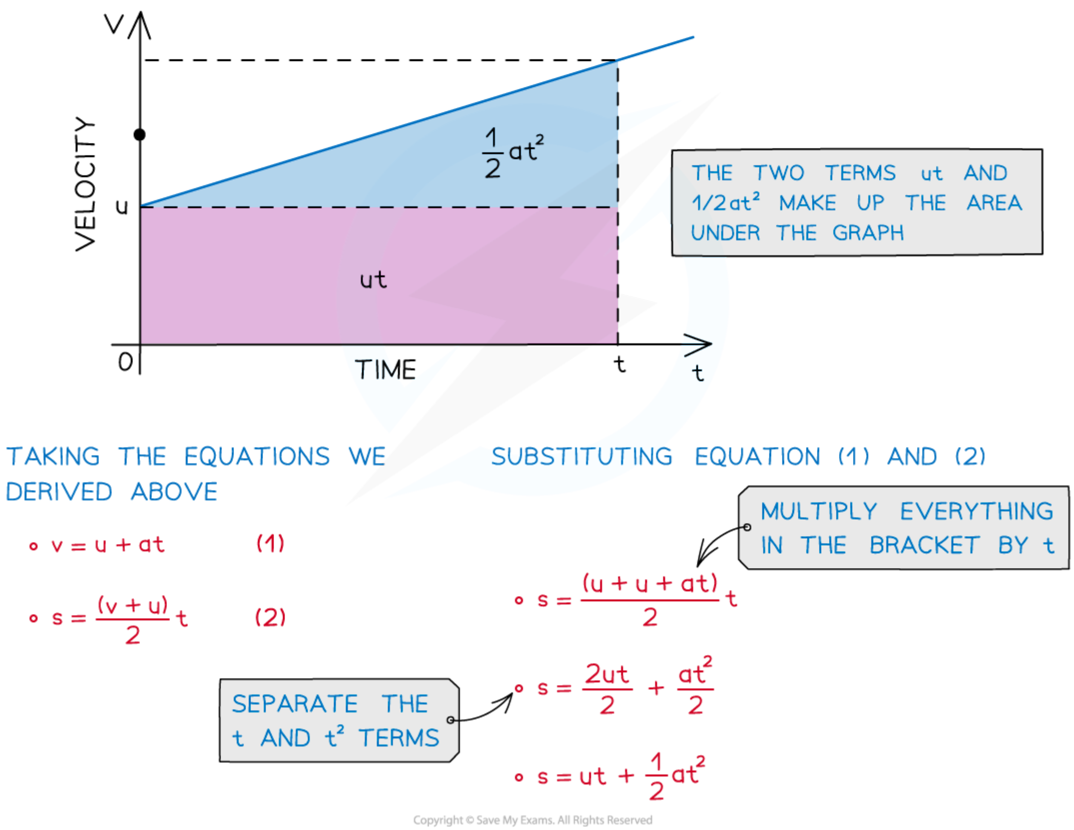
Deriving $s = \frac{(u + v)}{2}t$



The average velocity is halfway between u and v

Deriving $s = ut + \frac{1}{2}at^2$



The two terms ut and ½at² make up the area under the graph

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Deriving $v^2 = u^2 + 2as$

TAKING THE EQUATIONS WE DERIVED ABOVE

- $v = u + at \rightarrow t = \frac{v - u}{a}$ (1)
- $s = \frac{(v + u)}{2}t$ (2)

SUBSTITUTING (1) INTO (2)

- $s = \frac{(v + u)}{2} \times \frac{(v - u)}{a}$
- $s = \frac{v^2 - u^2}{2a}$ ← MULTIPLY BOTH SIDES BY 2a
- $v^2 = u^2 + 2as$

This final equation can be derived from two of the others

SUMMARY

$$v = u + at \qquad s = ut + \frac{1}{2}at^2$$

$$s = \frac{(v + u)}{2}t \qquad v^2 = u^2 + 2as$$

Summary of the four equations of uniformly accelerated motion

Key Takeaways

- These are all given in the IB DP Physics data booklet
- The key terms to look out for are:

'Starts from rest',

- This means $\boldsymbol{u} = 0$ and $\boldsymbol{t} = 0$
- This can also be assumed if the initial velocity is not mentioned

'Falling due to gravity'

- This means $\boldsymbol{a} = g = 9.81$ m s^{-2}
  - It doesn't matter which way is positive or negative for the scenario, as long as it is consistent for all the vector quantities

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- For example, if downwards is negative then for a ball travelling upwards, **s** must be positive and ***a*** must be negative

'Constant acceleration in a straight line'

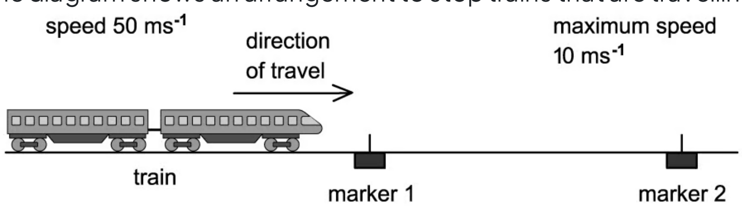
- This is a key indication SUVAT equations are intended to be used
  - For example, an object **falling** in a **uniform** gravitational field **without** air resistance

How to use the SUVAT equations

- **Step 1:** Write out the variables that are given in the question, both known and unknown, and use the context of the question to deduce any quantities that aren't explicitly given
  - e.g. for vertical motion $a = \pm 9.81$ m s^{-2}, an object which starts or finishes at rest will have $u = 0$ or $v = 0$
- **Step 2:** Choose the equation which contains the quantities you have listed
  - e.g. the equation that links *s*, *u*, *a* and *t* is $s = ut + ½at^2$
- **Step 3:** Convert any units to SI units and then insert the quantities into the equation and rearrange algebraically to determine the answer

Worked Example

The diagram shows an arrangement to stop trains that are travelling too fast.



At marker 1, the driver must apply the brakes so that the train decelerates uniformly in order to pass marker 2 at no more than 10 m s^{-1}. The train carries a detector that notes the times when the train passes each marker and will apply an emergency brake if the time between passing marker 1 and marker 2 is less than 20 s. Trains coming from the left travel at a speed of 50 m s^{-1}. Determine how far marker 1 should be placed from marker 2.

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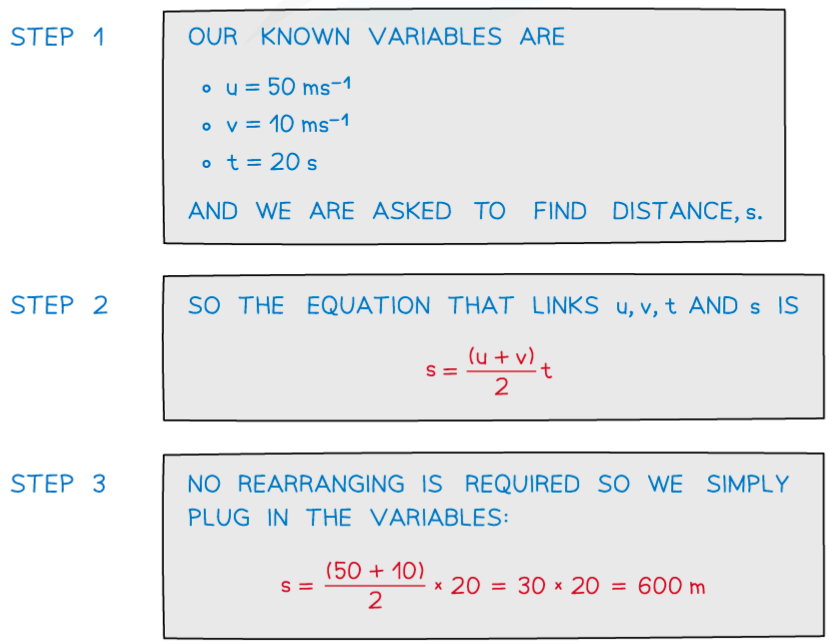

Worked Example

A cyclist is travelling directly east through a village, which is completely flat, at a velocity of 6 m s^{-1} east. They then start to constantly accelerate at 2 m s^{-2} for 4 seconds.

a) Calculate the distance that the cyclist covers in the 4 second acceleration period.

b) Calculate the cyclist's final velocity after the 4-second interval of acceleration.

Later on in their journey, this cyclist (cyclist **A**) is now cycling through a different village, still heading east at a constant velocity of 18 m s^{-1}. Cyclist **A** passes a friend (cyclist **B**) who begins accelerating from rest at a constant acceleration of 1.5 m s^{-2} in the same direction as cyclist **A** at the moment they pass.

c) Calculate how long it takes for cyclist **B** to catch up to cyclist **A**.

Part (a)

Step 1: List the known quantities

- Initial velocity, u = 6 m s^{-1} East
- Acceleration, a = 2 m s^{-2} East
- Time, t = 4 s
- Displacement, s = ? (this needs to be calculated)

Step 2: Identify and write down the equation to use

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- Since the question states **constant acceleration**, SUVAT equations can be used
- In this problem, the equation that links *s*, *u*, *a*, and *t* is

$$s = (u \times t) + (½ \times a \times t^2)$$

Step 3: Substitute known quantities into the equation and simplify where possible

$$s = (6 \times 4) + (0.5 \times 2 \times 4^2)$$

- This can be simplified to:

$$s = 24 + 16 = 40 \text{ m}$$

Part (b)

Step 1: List the known quantities

- Initial velocity, u = 6 m s^{-1} East
- Acceleration, a = 2 m s^{-2} East
- Time, t = 4 s
- final velocity, v = ? (this needs to be calculated)

Step 2: Identify and write down the equation to use

- Since the question states constant acceleration - SUVAT equation(s) - can be used
- In this problem, the equation that links *v*, *u*, *a*, and *t* is $v = u + (a \times t)$

Step 3: Substitute known quantities into the equation and simplify where possible

$$v = 6 + (2 \times 4)$$

- This can be simplified to:

$$v = 14 \text{ m s}^{-1}$$

Part (c)

Step 1: List the known quantities for cyclist A

- Initial velocity, u = 18 m s^{-1} East
- Acceleration, a = 0 m s^{-2} East
- Final velocity, v = 18 m s^{-1} East
- Time, t = ?
- Displacement, s = ?

Step 2: List the known quantities for cyclist B

- Initial velocity, u = 0 m s^{-1} East
- Acceleration, a = 1.5 m s^{-2} East
- Final velocity, v = ?
- Time, t = ?
- Displacement, s = ?

Step 3: Express the situation for cyclist A and B in terms of displacement, *s*

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- Cyclist **A** can have their situation expressed by:

$$s_A = (u \times t) + (½ \times a \times t^2)$$

$$s_A = (18 \times t) + (½ \times 0 \times t^2) = (18 \times t)$$

- Cyclist **B** can have their situation expressed by:

$$s_B = (u \times t) + (½ \times a \times t^2)$$

$$s_B = (0 \times t) + (½ \times 1.5 \times t^2) = (0.75 \times t^2)$$

Step 4: Equate the two equations and solve for *t*

- The two equations describe the displacement of each cyclist respectively
- When equating them, this will find the time when the cyclists are at the same location

$$s_A = s_B = (18 \times t) = (0.75 \times t^2)$$

$$0 = (0.75 \times t^2) - (18 \times t)$$

$$0 = (0.75 \times t - 18) \times t$$

- Therefore, solving for *t*, it can be two possible answers:

$$t = \mathbf{0\ s} \text{ or } 18 / 0.75 = \mathbf{24\ s}$$

Step 4: State the final answer

- Since the question is seeking the time when the two cyclists meet after first passing each other, the final answer is **24 s**

Exam Tip

This is one of the most important sections of this topic - usually, there will be one, or more, questions in the exam about solving problems with SUVAT equationsThe best way to master this section is to practice as many questions as possible!

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Acceleration of Free Fall Experiment

- The acceleration of free fall, g, is defined as:

  The acceleration of any object in response to the gravitational attraction between the Earth and the object

- Any object released on the Earth will accelerate **downwards** to the centre of the Earth as long as there are no external forces acting on it
  - On Earth, the acceleration of free fall is equal to **g = 9.81 m s^{-2}**

Determining *g* in the Laboratory

Aims of the Experiment

- The overall aim of the experiment is to calculate the value of the acceleration due to gravity, *g*
- This is done by measuring the time it takes for a ball-bearing to fall a certain distance
  - The acceleration can then be calculated using an equation of motion

Variables

- Independent variable = height, *h*
- Dependent variable = time, *t*
- Control variables:
  - Same steel ball-bearing
  - Same electromagnet
  - Distance between ball-bearing and top of the glass tube

Equipment List

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Equipment	Purpose
Metre ruler	To measure the distance between the light gates
Steel ball−bearing	To measure the distance and time taken to drop. The ball must be made from a magnetic material (iron, steel etc.)
Electromagnet	To drop the ball−bearing through the glass tube at specific height every time
Two light gates	To determine the time taken to drop a certain distance
Timer	To measure the time taken for the ball to drop between the light−gates. The timer must be activated to start when the ball passes the first light gate and stop when it passes the second
Tall clamp stand (retort stand)	To hold the glass tube and electromagnet in line with each other
Glass tube	To guide the ball−bearing vertically downwards
Cushion	To stop the ball−bearing from being damaged or damaging the surface when landing

- Resolution of measuring equipment:
  - Metre ruler = 1 mm
  - Timer = 0.01 s

Method

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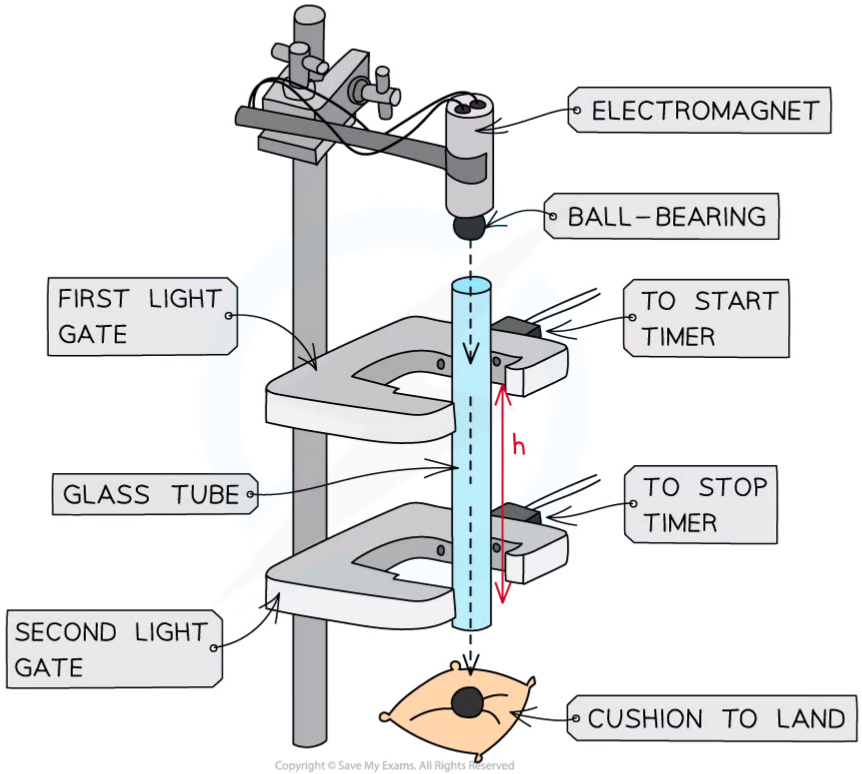


Apparatus setup to measure the distance and time for the ball bearing to drop

This method is an example of the procedure for varying the height the ball-bearing falls and determining the time taken – this is just one possible relationship that can be tested

1. Set up the apparatus by attaching the electromagnet to the top of a tall clamp stand. Do not switch on the current till everything is set up
2. Place the glass tube directly underneath the electromagnet, leaving space for the ball-bearing. Make sure it faces directly downwards and not at an angle
3. Attach both light gates around the glass tube at a starting distance of around 10 cm
4. Measure this distance between the two light gates as the height, *h* with a metre ruler
5. Place the cushion directly underneath the end of the glass tube to catch the ball-bearing when it falls through
6. Switch the current on the electromagnet and place the ball-bearing directly underneath so it is attracted to it
7. Turn the current to the electromagnet off. The ball should drop
8. When the ball drops through the first light gate, the timer starts
9. When the ball drops through the second light gate, the timer stops
10. Read the time on the timer and record this as time, *t*
11. Increase *h* (eg. by 5 cm) and repeat the experiment. At least 5 – 10 values for *h* should be used
12. Repeat this method at least 3 times for each value of *h* and calculate an average *t* for each

- An example of a table with some possible heights would look like this:

Example Table of Results

HEIGHT h/m	TIME t_1/s	TIME t_2/s	TIME t_3/s	AVERAGE TIME t/s
0.10				
0.15				
0.20				
0.25				
0.30				
0.35				

Analysis of Results

- The acceleration is found by using one of the SUVAT equations
- The known quantities are
  - Displacement $s = h$
  - Time taken = t
  - Initial velocity $u = u$
  - Acceleration $a = g$

- The following SUVAT equation can be rearranged:

$$s = ut + \frac{1}{2}at^2$$

$$2s = 2ut + at^2$$

$$\frac{2s}{t} = 2u + at$$

- Substituting in the values and rearranging to fit the straight line equation gives:

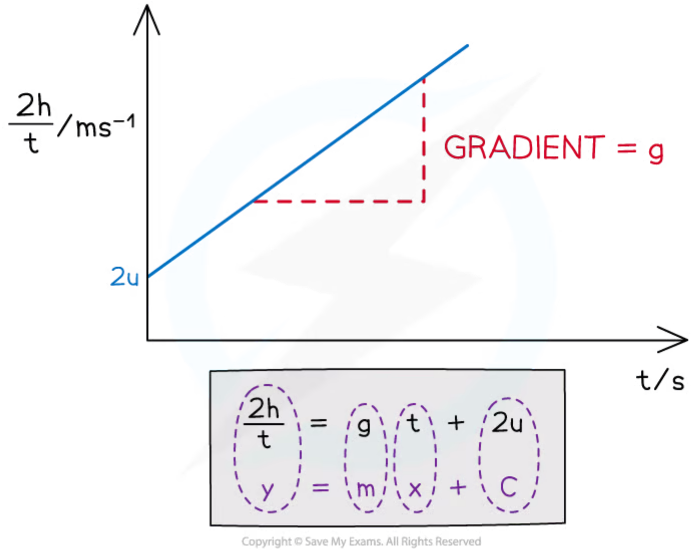
$$\frac{2h}{t} = gt + 2u$$

- Comparing this to the equation of a straight line: y = mx + c
  - $y = 2h/t$ (m s^{-1})
  - $x = t$
  - Gradient, $m = a = g$ (m s^{-2})
  - y-intercept = $2u$

1. Plot a graph of the $2h/t$ against t
2. Draw a line of best fit
3. Calculate the gradient - this is the acceleration due to gravity g

4. Assess the uncertainties in the measurements of *h* and *t*. Carry out any calculations needed to determine the uncertainty in *g* due to these

YOUR NOTES ⬇



The graph of 2h/t against t produces a straight-line graph where the acceleration is the gradient

Evaluating the Experiment

Systematic Errors:

- Residue magnetism after the electromagnet is switched off may cause *t* to be recorded as longer than it should be

Random Errors:

- Large uncertainty in *h* from using a metre rule with a precision of 1 mm
- Parallax error from reading *h*
- The ball may not fall accurately down the centre of each light gate
- Random errors are reduced through repeating the experiment for each value of *h* at least 3–5 times and finding an average time, *t*

Safety Considerations

- The electromagnetic requires current
  - Care must be taken to not have any water near it
  - To reduce the risk of electrocution, only switch on the current to the electromagnet once everything is set up
- A cushion or a soft surface must be used to catch the ball-bearing so it doesn't roll off / damage the surface
- The tall clamp stand needs to be attached to a surface with a G clamp so it stays rigid

YOUR NOTES
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Worked Example

A student investigates the relationship between the height that a ball-bearing is dropped between two light gates and the time taken for it to drop.

Height h/m	Time t_1/s	Time t_2/s	Time t_3/s	Average time t/s
0.10	0.14	0.15	0.14	
0.20	0.18	0.20	0.21	
0.30	0.25	0.25	0.26	
0.40	0.29	0.28	0.30	
0.50	0.32	0.31	0.32	
0.60	0.34	0.35	0.34	

Calculate the value of g from the table.

Step 1: Complete the table

- Calculate the average time for each height
- Add an extra column 2h / t

Height h/m	Time t_1/s	Time t_2/s	Time t_3/s	Average time t/s	$\frac{2h}{t}$ / ms^{-1}
0.10	0.14	0.15	0.14	0.14	1.43
0.20	0.18	0.20	0.21	0.20	2.00
0.30	0.25	0.25	0.26	0.25	2.40
0.40	0.29	0.28	0.30	0.29	2.76
0.50	0.32	0.31	0.32	0.32	3.13
0.60	0.34	0.35	0.34	0.34	3.53

Step 2: Draw graph of 2h/t against time t

YOUR NOTES
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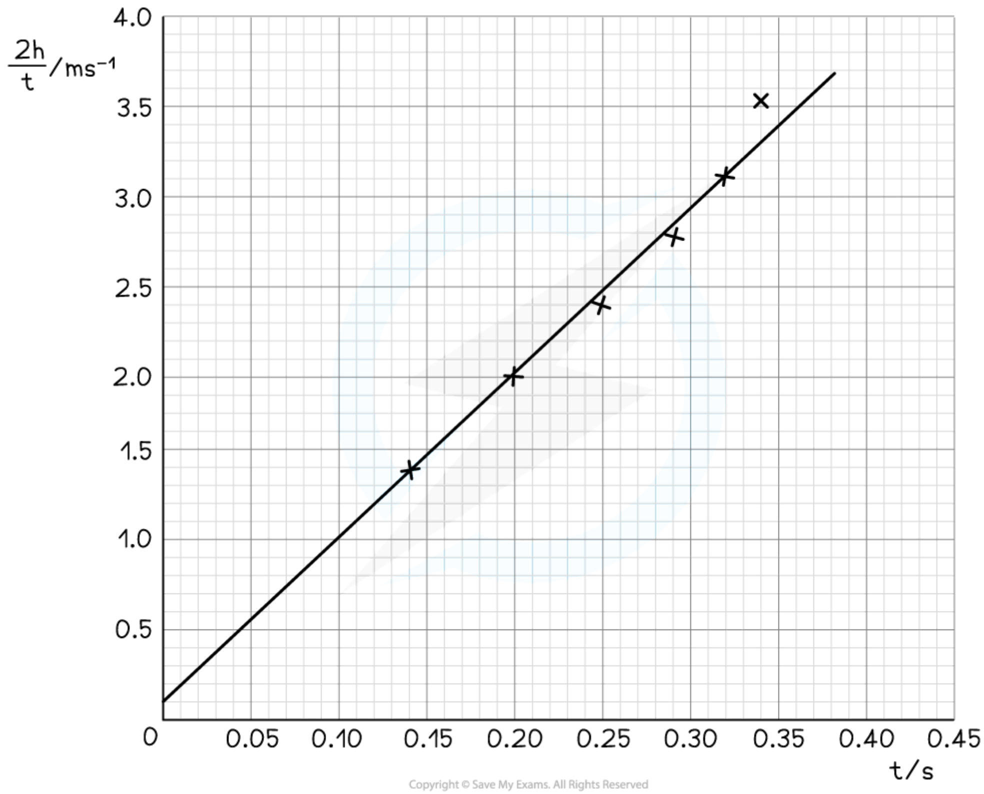
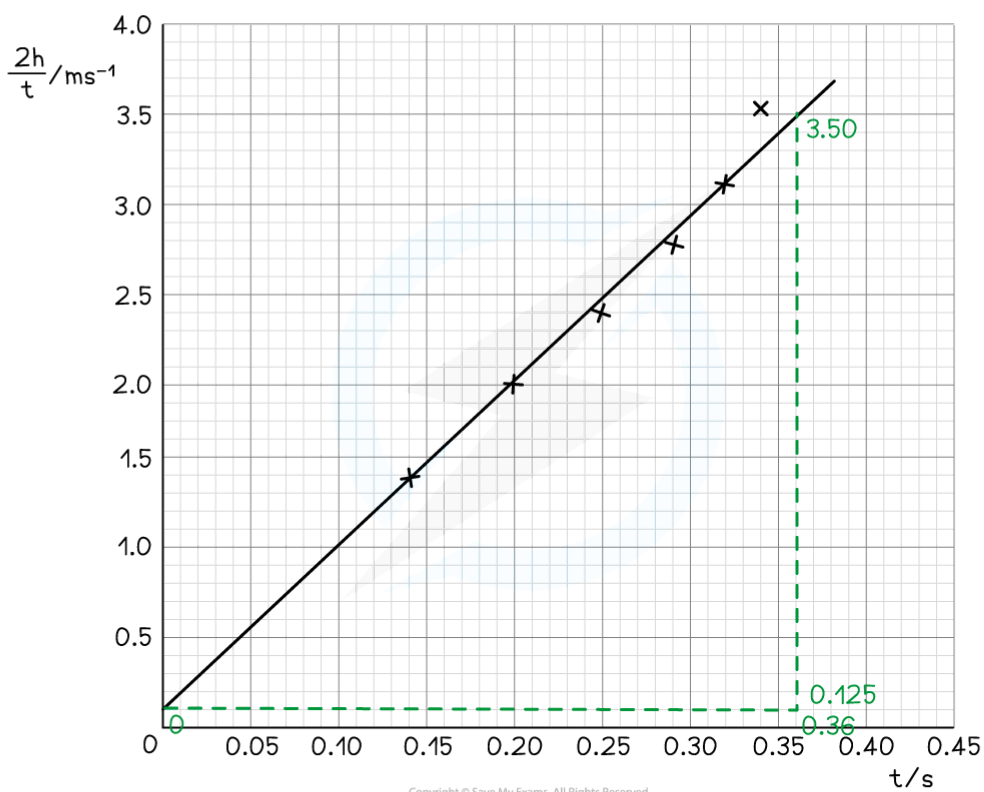


- Make sure the axes are properly labeled and the line of best fit is drawn with a ruler

Step 3: Calculate the gradient of the graph



YOUR NOTES
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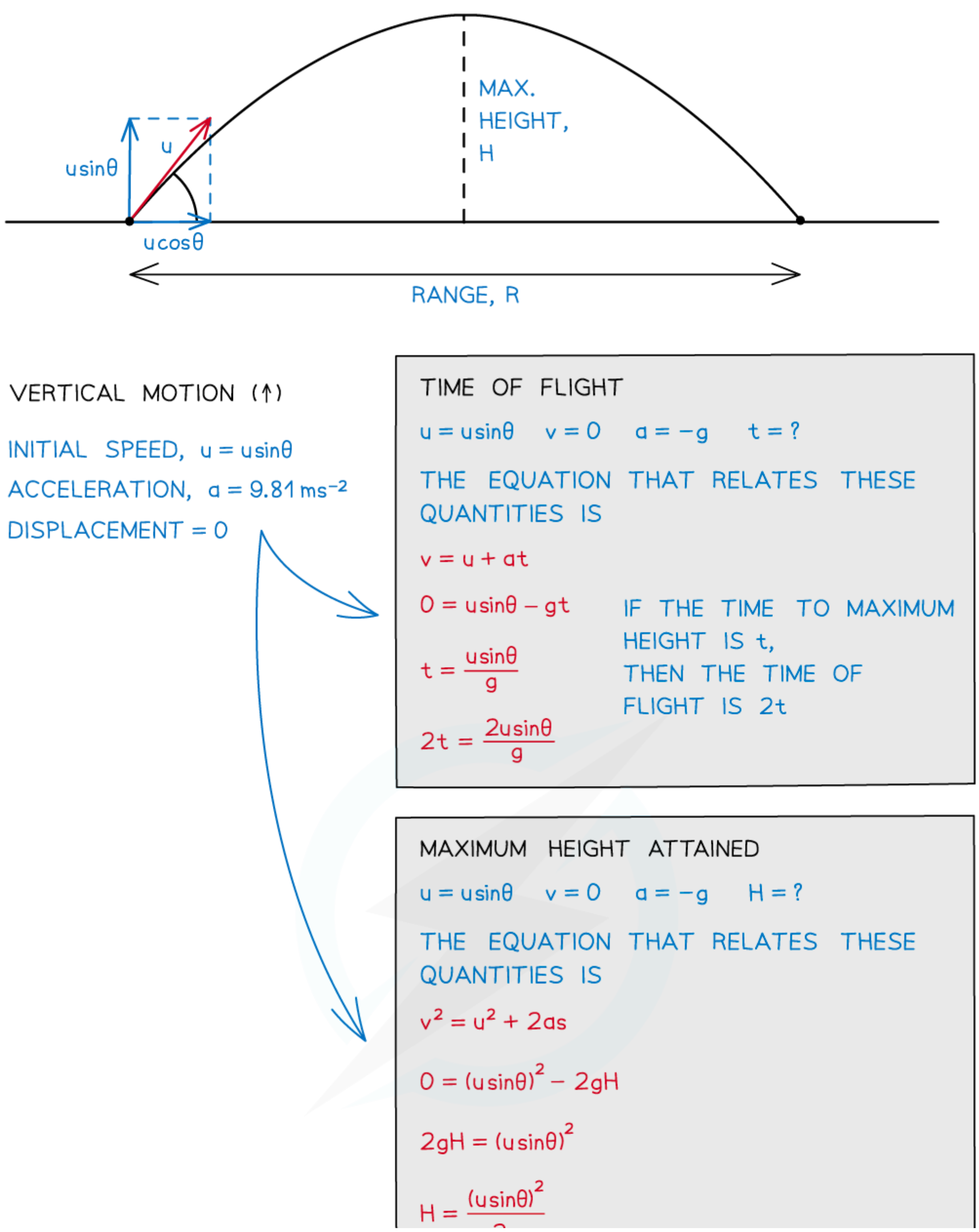
- The gradient is calculated by:

$$g = \frac{3.50 - 0.125}{0.36 - 0} = 9.375 = \mathbf{9.38\ m\ s^{-2}}$$

YOUR NOTES ↓

Projectile Motion

- The trajectory of an object undergoing projectile motion consists of a **vertical** component and a **horizontal** component
  - These need to be evaluated separately
- Some key terms to know, and how to calculate them, are:
  - **Time of flight:** how long the projectile is in the air
  - **Maximum height attained:** the height at which the projectile is momentarily at rest
  - **Range:** the horizontal distance traveled by the projectile



YOUR NOTES
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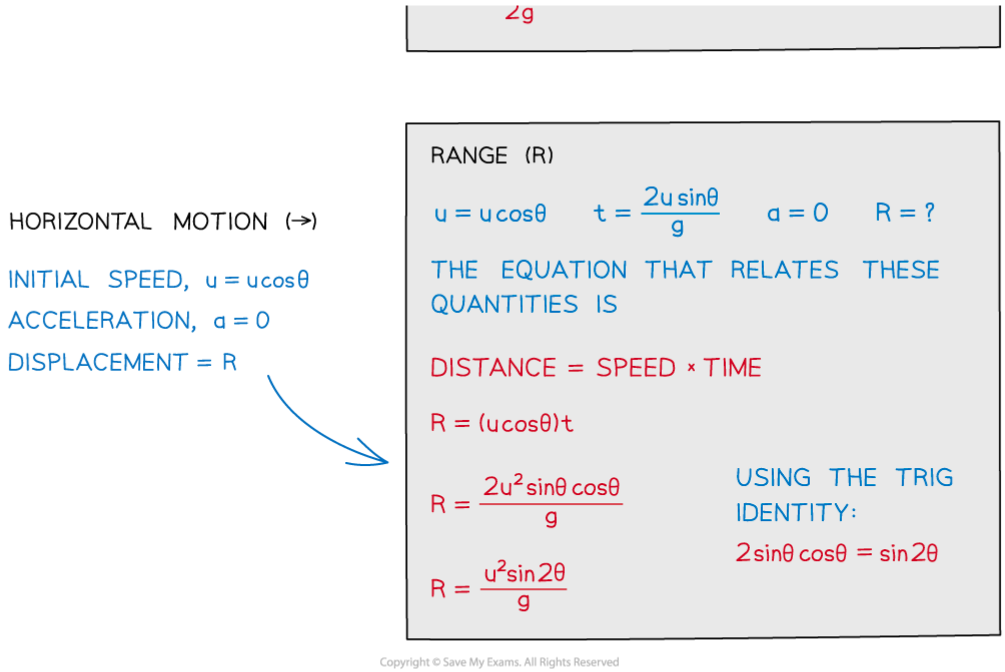


How to find the time of flight, maximum height and range

Problems involving projectile motion

- There are two main considerations for solving problems involving two-dimensional motion of a projectile
    - Constant velocity in the horizontal direction
    - Constant acceleration in a perpendicular direction
- The only force acting on the projectile, after it has been released, is **gravity**
- There are three possible scenarios for projectile motion:
    - **Vertical** projection
    - **Horizontal** projection
    - **Projection** at an **angle**

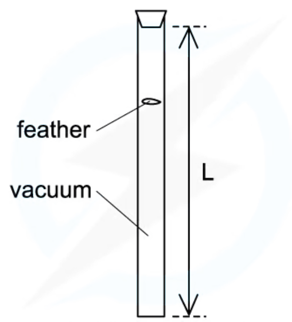
YOUR NOTES ↓

Worked Example

To calculate vertical projection (free fall)

A science museum designed an experiment to show the fall of a feather in a vertical glass vacuum tube.

The time of fall from rest is 0.5 s.



What is the length of the tube, L?

IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION. FIRST WE MUST LIST THE KNOWN VARIABLES.

$a = 9.81\ ms^{-2}$ $\quad u = 0$ $\quad t = 0.5\ s$ $\quad L = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$L = \frac{1}{2}gt^2$$

$$L = \frac{1}{2} \times 9.81 \times 0.5^2 = 1.2\ m$$

YOUR NOTES
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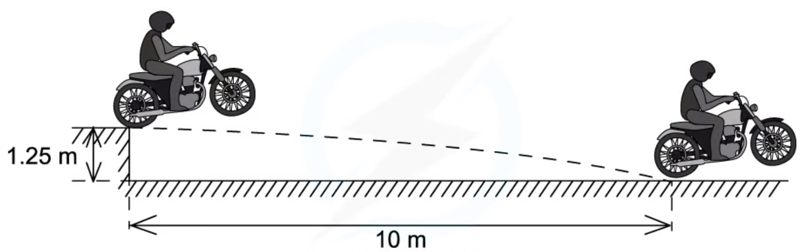
Worked Example

To calculate horizontal projection

A motorcycle stunt-rider moving horizontally takes off from a point 1.25 m above the ground, landing 10 m away as shown.

What was the speed at take-off? (ignoring air resistance)



IN THIS PROBLEM, WE NEED TO CONSIDER BOTH VERTICAL AND HORIZONTAL MOTION. LET'S CONSIDER THE VERTICAL MOTION FIRST. THE KNOWN VARIABLES ARE

$s = 1.25$ m $\quad a = 9.81$ ms^{-2} $\quad u = 0 \quad t = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2s}{g}}$$

$$t = \sqrt{\frac{2 \times 1.25}{9.81}} = 0.5\text{s}$$

NEXT LET'S CONSIDER THE HORIZONTAL MOTION. THE KNOWN VARIABLES ARE

$s = 10$ m $\quad a = 0 \quad t = 0.5$ s $\quad u = ?$

SINCE THE ACCELERATION IS ZERO, WE CAN USE

$$\text{VELOCITY} = \frac{\text{DISPLACEMENT}}{\text{TIME}}$$

$$v = \frac{10}{0.5} = 20 \text{ ms}^{-1}$$

YOUR NOTES
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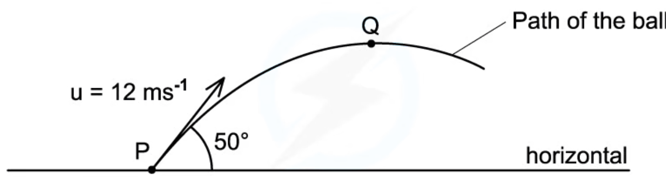
Worked Example

To calculate projection at an angle

A ball is thrown from a point P with an initial velocity u of 12 m s^{-1} at 50° to the horizontal.

What is the value of the maximum height at Q? (ignoring air resistance)



IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION UP TO THE POINT Q. FIRST WE MUST LIST THE KNOWN VARIABLES

$u = 12\sin(50)$ $a = -9.81\ ms^{-2}$ $v = 0$ $H = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$v^2 = u^2 + 2as$$

$$2as = v^2 - u^2$$

$$s = \frac{(v^2 - u^2)}{2a}$$

$$H = \frac{0 - (12\sin 50)^2}{2 \times (-9.81)}$$

$$H = \frac{(12\sin 50)^2}{19.62} = 4.3\ m$$

Exam Tip

Make sure you don't make these common mistakes:

- Forgetting that deceleration is negative as the object rises
- Confusing the direction of sin θ and cos θ
- Not converting units (mm, cm, km etc.) to metres

Further, it is worth noting that projectile motion is typically symmetrical when air resistance is ignored allowing for use of the peak to find the time of total flight or total horizontal distance by doubling the amount to get from the start point to the peak.

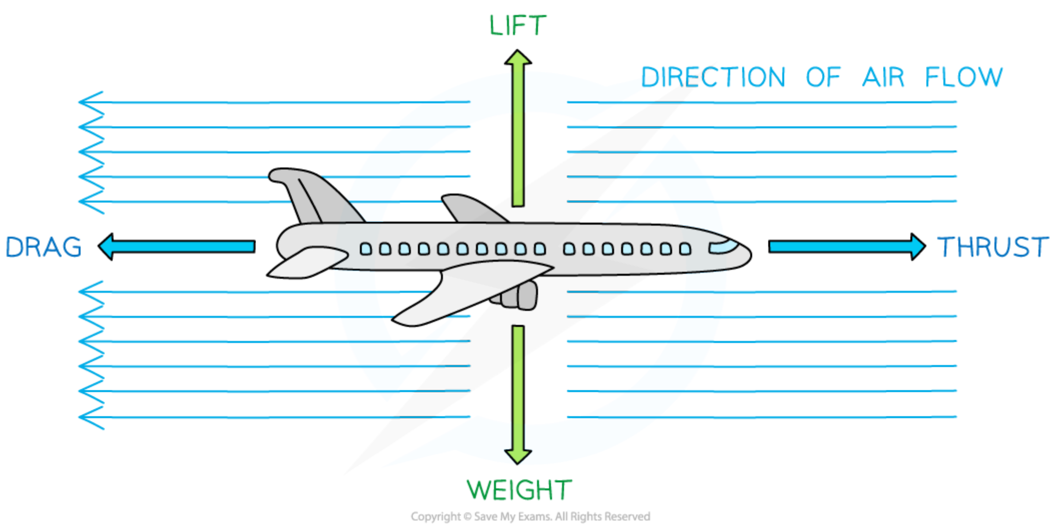
2.1.8 Terminal Speed

YOUR NOTES

Fluid Resistance & Terminal Speed

Fluid Resistance

- When an object moves through a fluid (a gas or a liquid) then there are resistive forces for that movement.
  - These forces are known as drag forces
- Examples of drag forces are **friction** and **air resistance**
- Drags forces:
  - Are always in the **opposite** direction to the motion of the object
  - Never speed an object up or start them moving
  - Slow down an object or keep them moving at a constant speed
  - Convert kinetic energy into heat and sound
- Lift is an upwards force on an object moving through a fluid. It is perpendicular to the fluid flow
  - For example, as an aeroplane moves through the air, it pushes down on the air to change its direction
  - This causes an equal and opposite reaction upwards on the wings (lift) due to Newton's third law



Drag forces are always in the opposite direction to the thrust (direction of motion). Lift is always in the opposite direction to the weight

- A key component of drag forces is it increases with the **speed** of the object
- This is shown in the diagram below:

YOUR NOTES
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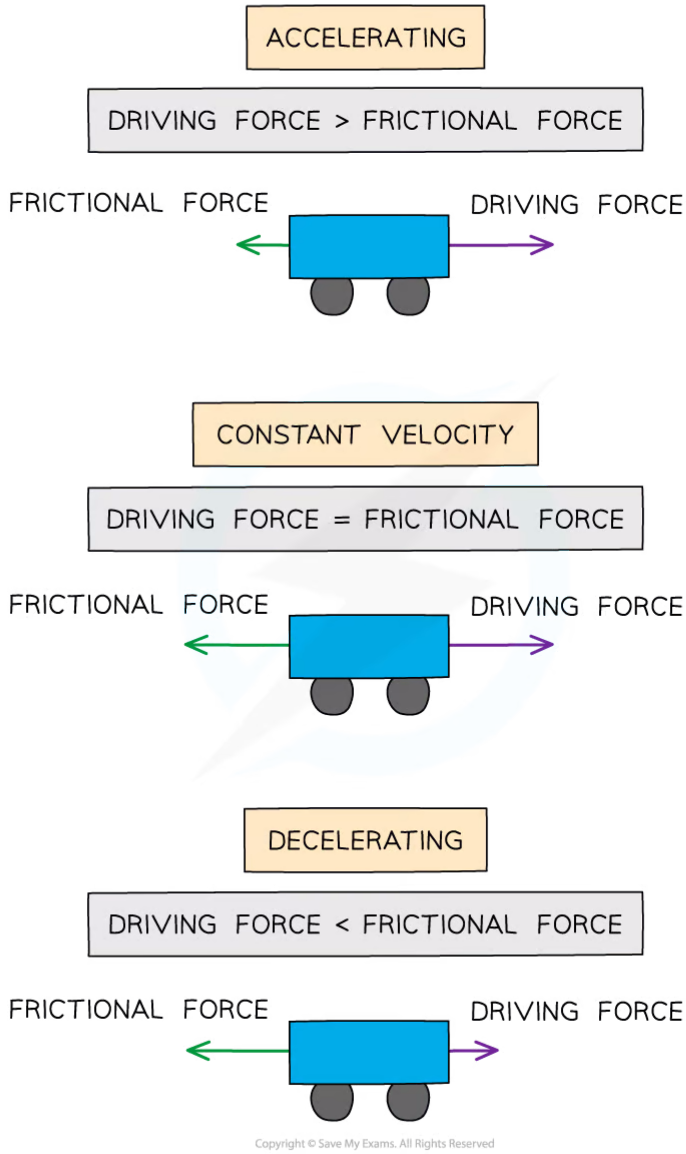

Frictional forces on a car increase with speed

Air Resistance & Projectile Motion

- Air resistance decreases the **horizontal** component of the velocity of a projectile
  - This means both its range and maximum height is decreased compared to no air resistance

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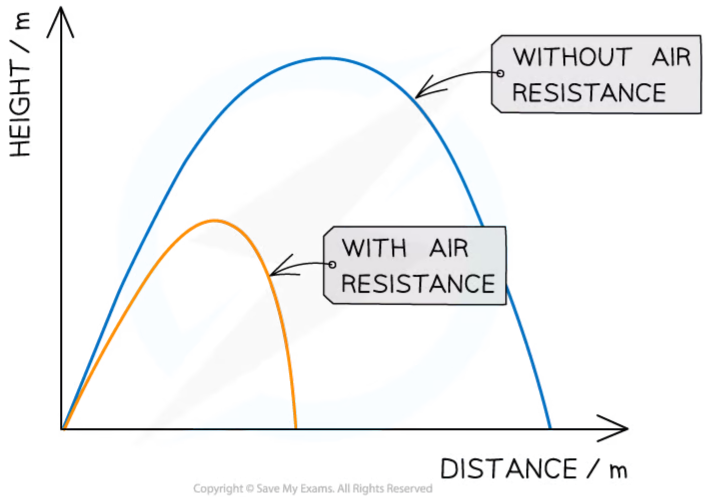

A projectile with air resistance travels a smaller distance and has a lower maximum height than one without air resistance

- The angle and speed of release of a projectile is varied to produce either a longer flight path or cover a larger distance, depending on the situation
    - For sports such as the long jump or javelin, an optimum angle against air resistance is used to produce the greatest distance
    - For gymnastics or a ski jumper, the initial vertical velocity is made as large as possible to reach a greater height and longer flight path

Terminal Velocity

- For a body in free fall, the only force acting is its weight and its acceleration *g* is only due to gravity.
- The drag force increases as the body accelerates
    - This increase in velocity means the drag force also increases
- Due to Newton's Second Law, this means the resultant force and therefore acceleration decreases (recall ***F = ma***)
- When the drag force is equal to the gravitational pull on the body, the body will no longer accelerate and will fall at a constant velocity
    - This velocity is called the **terminal velocity**
- Terminal velocity can occur for objects falling through a gas or a liquid

YOUR NOTES
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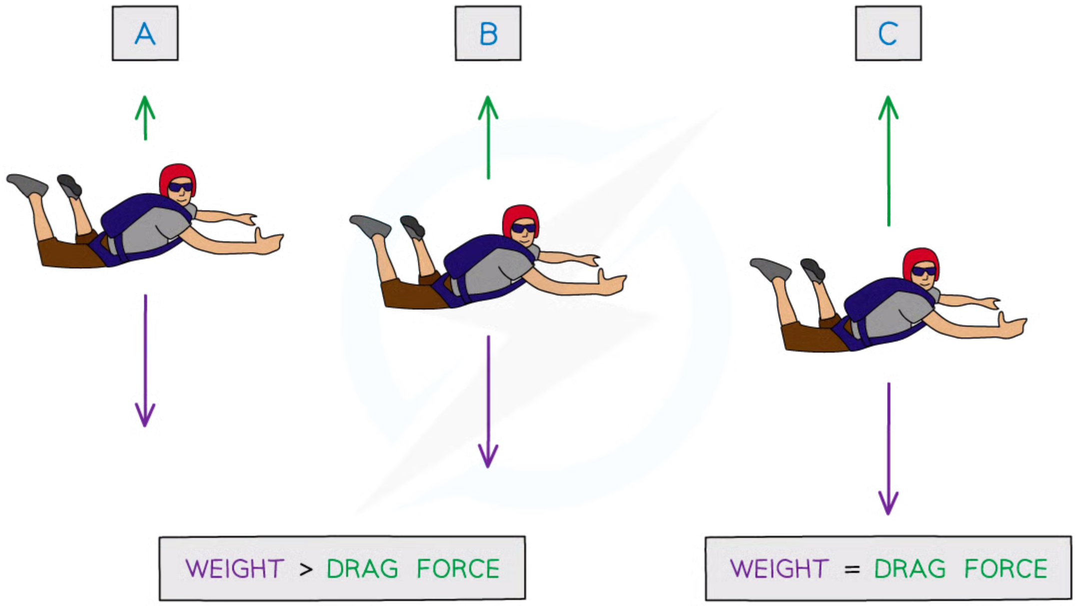


THE SKYDIVER IS IN FREEFALL.

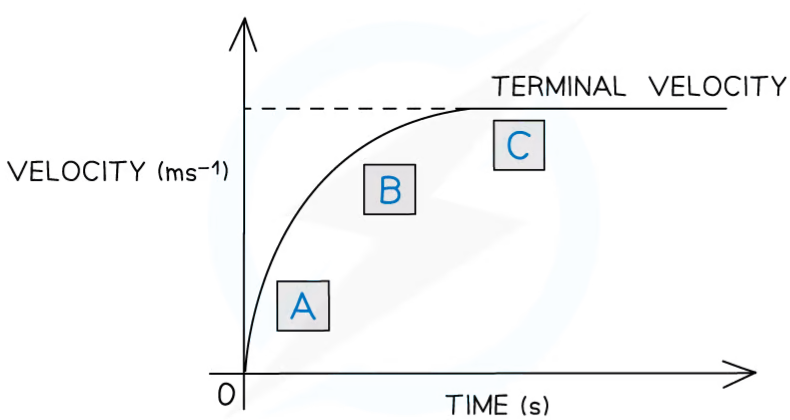
THEIR VELOCITY INCREASES DUE TO THE DOWNWARD FORCE OF THEIR WEIGHT.

THE INCREASE IN VELOCITY MEANS AIR RESISTANCE ALSO INCREASES AND ACCELERATION DECREASES.

EVENTUALLY THE SKYDIVER REACHES A VELOCITY WHERE THEIR WEIGHT EQUALS THE FORCE OF AIR RESISTANCE.

THEIR ACCELERATION IS 0.

THIS IS THE TERMINAL VELOCITY.



YOUR NOTES
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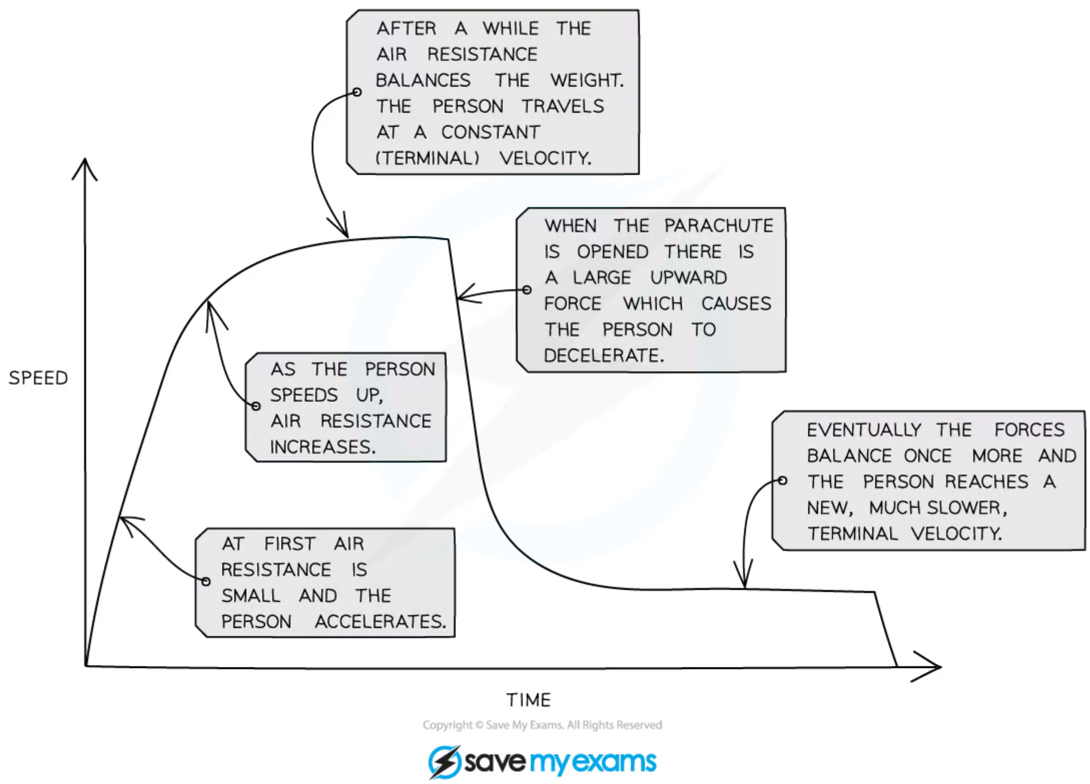
A skydiver in freefall reaching terminal velocity

- The graph shows how the velocity of the skydiver varies with time
- Since the acceleration is equal to the gradient of a velocity-time graph, the acceleration decreases and eventually becomes zero when terminal velocity is reached
- After the skydiver deploys their parachute, they decelerate to a **lower terminal velocity** to reduce the impact on landing

- This is demonstrated by the graph below:



A graph showing the changes in speed of the skydiver throughout their entire journey in freefall

YOUR NOTES
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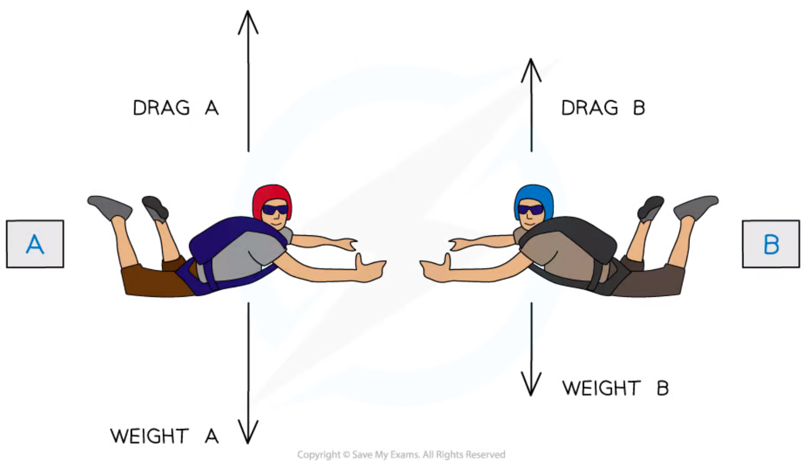
Worked Example

Skydivers jump out of a plane at intervals of a few seconds.

Skydivers **A** and **B** want to join up as they fall.



If **A** is heavier than **B**, who should jump first?

- Skydiver **B** should jump first since he will take longer to reach terminal velocity
- This is because skydiver **A** has a higher mass, and hence, weight
- More weight means higher acceleration and hence higher speed, therefore, **A** will reach terminal velocity faster than **B**

Exam Tip

A common misconception is that skydivers move upwards when their parachutes are deployed - however, this is not the case, they are in fact **decelerating** to a lower terminal velocityIf a question considers air resistance to be '**negligible**' this means in that question, air resistance is taken to be so small it will not make a difference to the motion of the body. You can take this to mean there are no drag forces acting on the body.

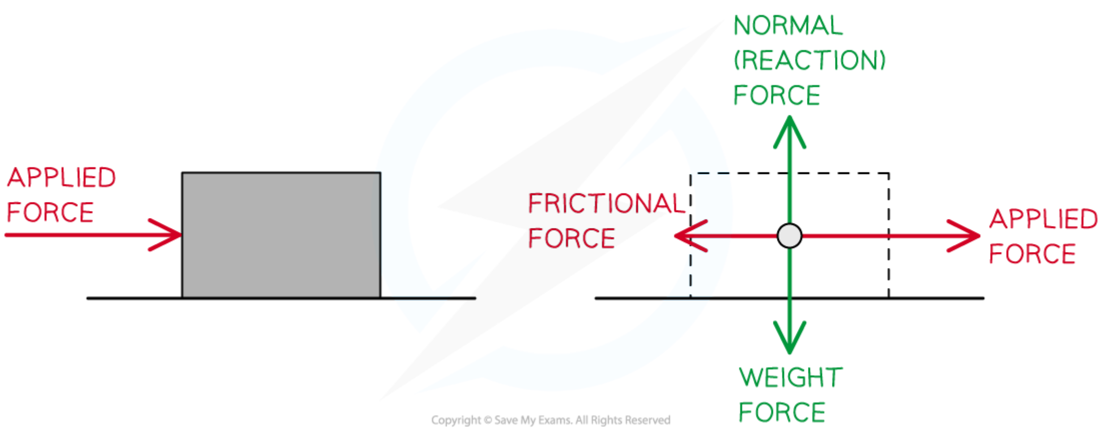
YOUR NOTES

2.2 Forces

2.2.1 Free-Body Diagrams

Vectors

- In physics, during force interactions, it is common to represent situations as simply as possible without losing information
  - When considering force interactions objects can be represented as point particles
  - These point particles should be placed at the center of mass of the object
- Force vectors that act upon that objects should be drawn with their tail on that point particle
  - The length of the force vector corresponds to its strength
  - The longer the vector, the greater the force magnitude
- The below example shows the forces acting on an object when pushed to the right over a rough surface



Point particle representation of the forces acting on a moving object

- The below example shows an object sitting on a slope in equilibrium

YOUR NOTES
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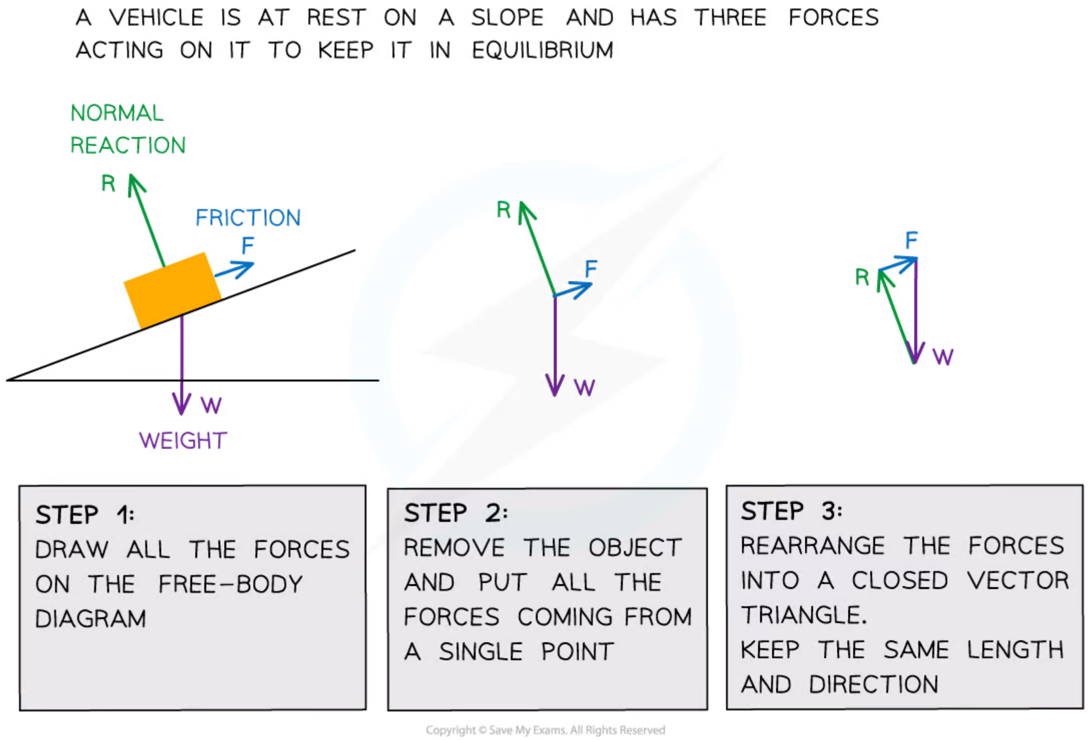
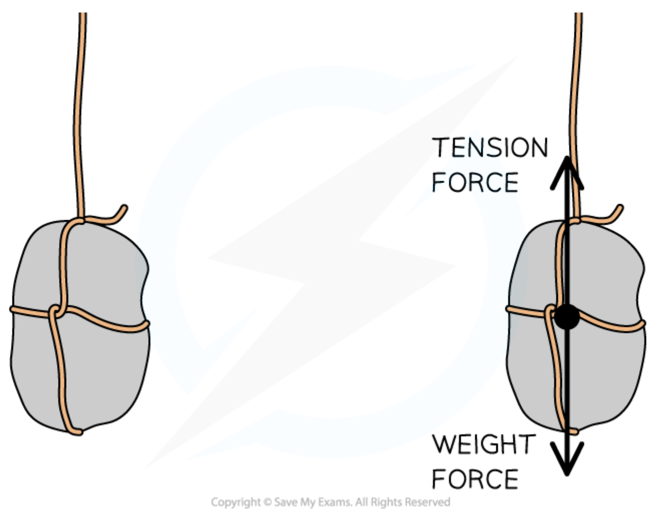


Three forces on an object in equilibrium form a closed vector triangle

- The below example shows the forces acting on an object suspended from a stationary rope



Free-body diagram of an object suspended from a stationary rope

Exam Tip

When labeling force vectors, it is important to use conventional and appropriate naming or symbols such as:

- ***w*** or Weight force or ***mg***
- ***N*** *or* ***R*** for normal reaction force (depending on your local context either of these could be acceptable)

Using unexpected notation can lead to losing marks so try to be consistent with expected conventions.

YOUR NOTES
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Free-Body Diagrams

YOUR NOTES
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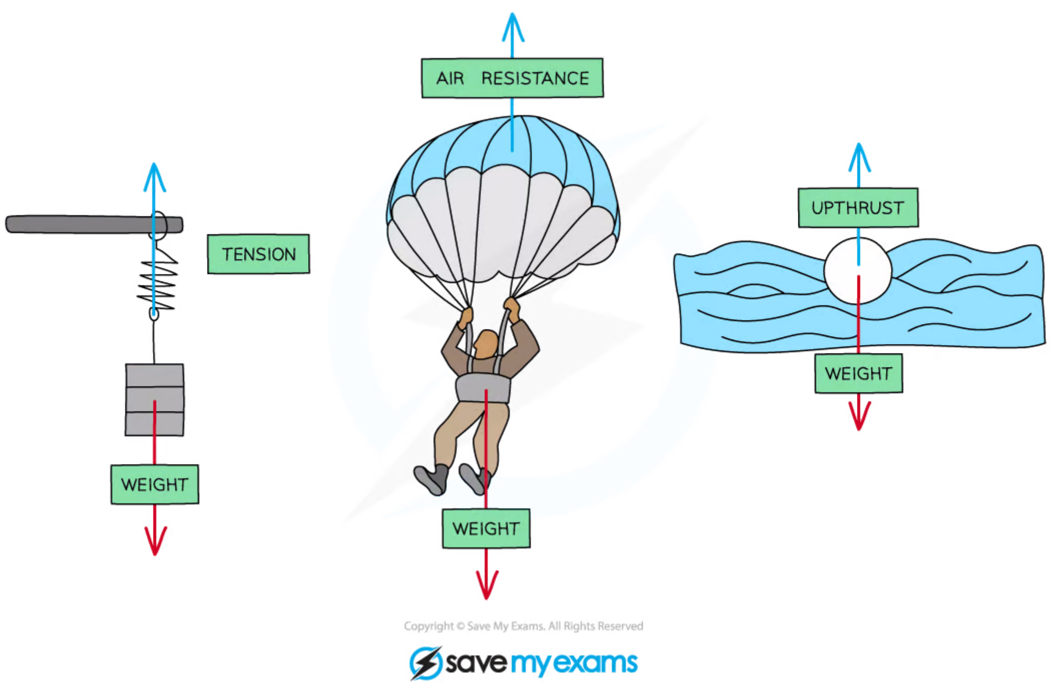
- Free body diagrams are useful for modeling the forces that are acting on an object
- Each force is represented as a **vector** arrow, where each arrow:
  - Is scaled to the **magnitude** of the force it represents
  - Points in the **direction** that the force acts
  - Is **labelled** with the name of the force it represents or an appropriate symbol

- Free body diagrams can be used:
  - To identify which forces act in which plane
  - To resolve the net force in a particular direction

- The rules for drawing a free-body diagram are the following:
  - **Rule 1:** Draw a point in the centre of mass of the body
  - **Rule 2:** Draw the body free from contact with any other object
  - **Rule 3:** Draw the forces acting on that body using vectors with correct direction and proportional length



Free body diagrams can be used to show the various forces acting on objects

- You must be able to apply the following forces with their symbols to free-body diagrams:
  - Weight (W)
  - Tension (T)
  - Normal Reaction Force (N)
  - Upthrust (U)
  - Frictional Forces (F_r)

YOUR NOTES
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Worked Example

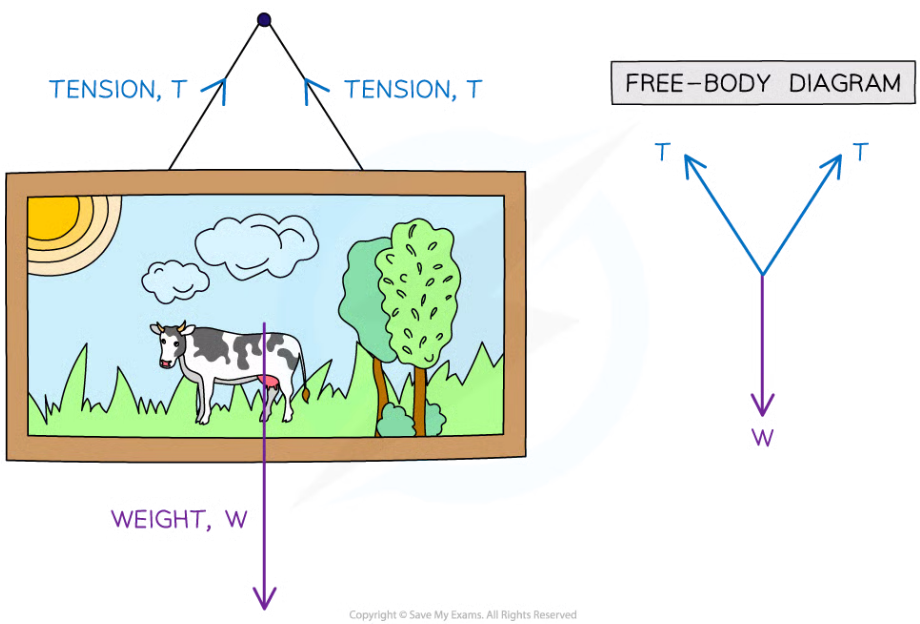
Draw free-body diagrams for the following scenarios:

a) A picture frame hanging from a nail

b) A box sliding down a slope

Part (a)

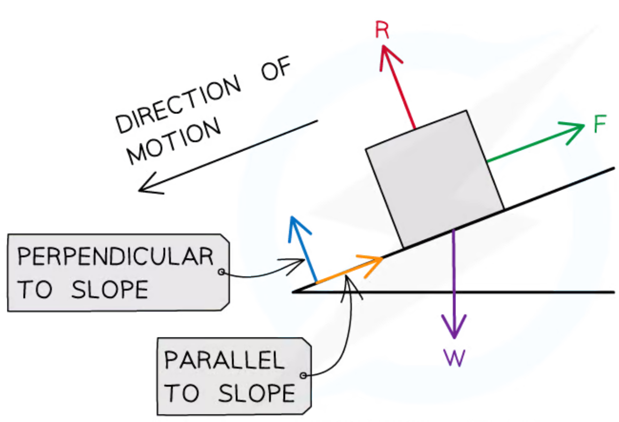
A picture frame hanging from a nail:



- The size of the arrows should be such that the 3 forces would make a closed triangle as they are **balanced**

Part (b)

A box sliding down a slope:

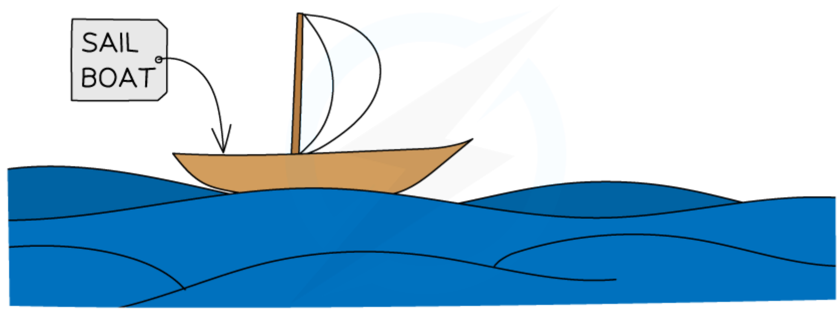


- There are three forces acting on the box:
  - The **normal contact force**, *R*, acts perpendicular to the slope
  - **Friction**, *F*, acts parallel to the slope and in the opposite direction to the direction of motion
  - **Weight**, *W*, acts down towards the Earth

Worked Example

Draw a free-body diagram of a toy sailboat with weight 30 N floating in water that is being pulled to the right by an applied force of 35 N with a total resistive force of 5 N.

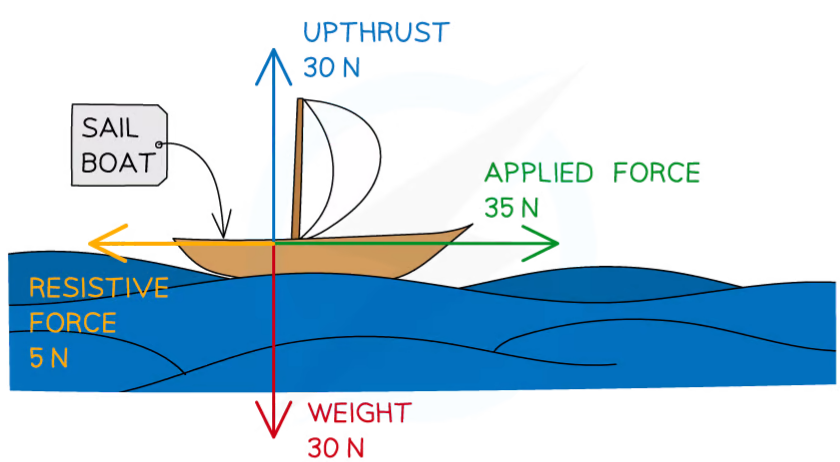
Step 1: Draw the object in a simplified diagram



Step 2: Identify all of the forces acting upon the object in the question, including any forces that may be implied

- **Weight**: 30 N down
- **Upthrust** from the water (since the object is floating): 30 N up
- **Applied force**: 35 N to the right
- **Resistive force:** 5 N to the left

Step 3: Draw in all of the force vectors (arrows), making sure the arrows start at the object and are directed away



- An approximation can be made as to the final resultant force due to all of the forces
  - Decide whether the resultant force is approximately up or down
  - Decide whether the resultant force is approximately left or right
  - For example, the resultant force is directed **up** and to the **right**

YOUR NOTES
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2.2.2 Newton's First Law

YOUR NOTES
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Newton's First Law

- Newton's First Law states:

  A body will remain at rest or move with constant velocity unless acted on by a resultant force

- If the net force acting on an object is zero it is said to be in **translational equilibrium**
- If the forces on a body are balanced (the resultant force is 0), the body must be either:
  - At rest
  - Moving at a constant velocity

- Since force is a vector, it is easier to split the forces into **horizontal** and **vertical** forces
- If the forces are balanced:
  - The forces to the left = the forces to the right
  - The forces up = the forces down

- The resultant force is the single force obtained by combining **all** the forces on the body

Worked Example

If there are no external forces acting on the car, other than friction, and it is moving at a constant velocity, what is the value of the frictional force *F*?

SINCE THE CAR IS MOVING AT CONSTANT VELOCITY, THERE IS NO RESULTANT FORCE.

THIS MEANS THE DRIVING AND FRICTIONAL FORCES ARE BALANCED.

F IS ALSO EQUAL TO 6 kN

YOUR NOTES
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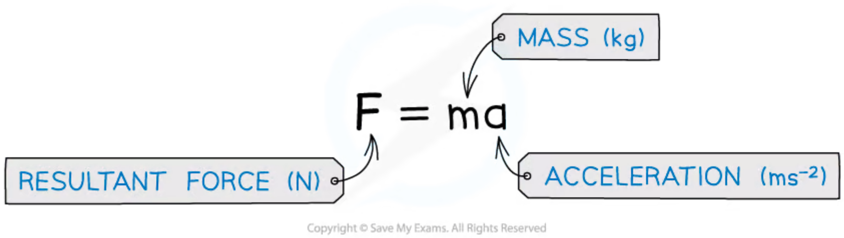
Newton's Second Law

- Newton's Second Law states:

> **The resultant force is equal to the rate of change in momentum. The change in momentum is in the same direction as the resultant force**

- This can also be written as:



- This relationship means that objects will **accelerate** if there is a **resultant force** acting upon them
- This is derived from the definition of momentum as follows:

$$\text{Momentum } p = mv$$

$$\text{Rate of change in momentum} = \frac{\Delta p}{\Delta t} = m\frac{\Delta v}{\Delta t}$$

$$\text{Force } F = m\frac{\Delta v}{\Delta t}$$

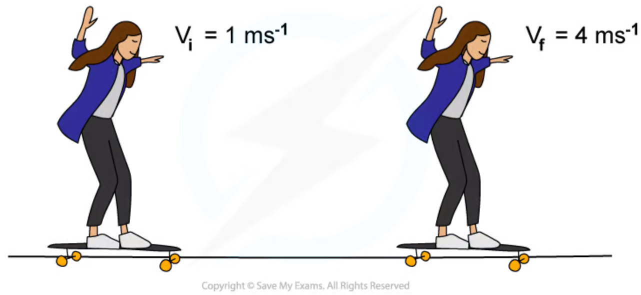
$$\text{Since } a = \frac{\Delta v}{\Delta t},\ \mathbf{F = ma}$$

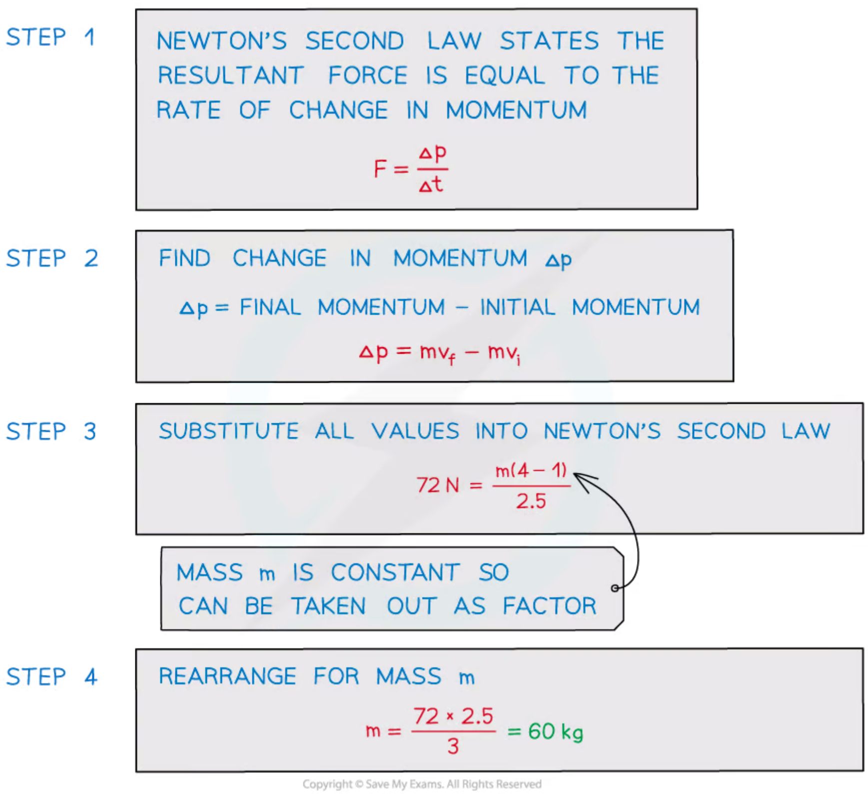
- An **unbalanced** force on a body **means** it experiences a **resultant force**
  - If the resultant force is along the direction of motion, it will speed up (**accelerate**) or slow down (**decelerate**) the body
  - If the resultant force is at an angle, it will change the direction of the body

YOUR NOTES

Worked Example

A girl is riding her skateboard down the road and increases her speed from 1 m s^{-1} to 4 m s^{-1} in 2.5 s. If the force driving her forward is 72 N, calculate the combined mass of the girl and the skateboard.





Resultant Force

- Since force is a vector, every force on a body has a magnitude and direction
  - The resultant force is, therefore, the **vector sum** of all the forces acting on the body
- The direction is given by either the positive or negative direction as shown in the examples below

YOUR NOTES
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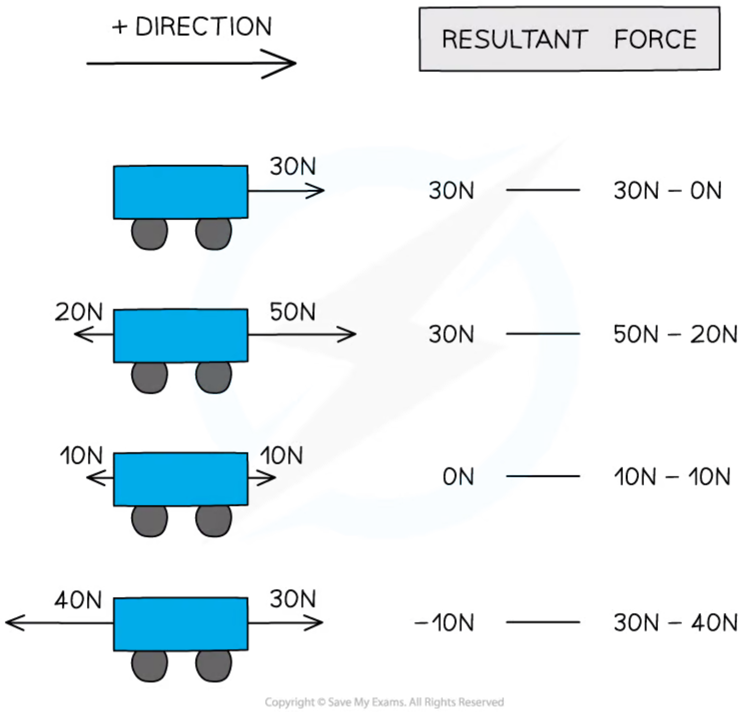


Resultant forces on a body can be positive or negative depending on their direction

- The resultant force could also be at an **angle**, in which case, the **magnitude** and **direction** of the resultant force can be determined using either:
  - **Calculation** (usually simple geometry, such as Pythagoras Theorem or the sine and cosine rules)
  - **Scale drawing**

Acceleration

- Since acceleration is a **vector**, it can be either positive or negative depending on the direction of the resultant force
  - If the resultant force is in the **same** direction as the motion of an object, the acceleration is **positive**
  - If the resultant force is in the **opposite** direction to the motion of an object, the acceleration is **negative**

- An object may continue in the same direction however with a resultant force in the opposite direction to its motion, it will slow down and eventually come to a stop
- If drag forces are ignored, or severely reduced, the acceleration is **independent** of the **mass** of an object
  - This has been shown in experiments by astronauts who have dropped a feather and a hammer on the Moon from the same height
  - Both the hammer and feather drop to the Moon's surface **at the same time**

YOUR NOTES
⬇

Worked Example

Three forces, 4 N, 8 N, and 24 N act on an object with a mass of 5 kg. Which acceleration is not possible with any combination of these three forces?

A. 1 m s^{-2}

B. 4 m s^{-2}

C. 7 m s^{-2}

D. 10 m s^{-2}

Step 1: List the values given

- Three possible forces at any angle of choice: 4 N, 8 N, and 24 N
- Mass of object = 5 kg

Step 2: Consider the relevant equation

- Newton's second law:

$$F = m \times a$$

Step 3: Rearrange to make acceleration the focus

$$a = F \div m$$

Step 4: Investigate the minimum possible acceleration

- It is best to consider the edges of this problem before dealing with more difficult combinations that is why it is prudent to check the minimum and maximum acceleration
- The minimum would occur when the forces were acting against each other
- The minimum possible would be when only the 4 N force was acting on the body
- Now check the acceleration:

$$4 \text{ N} \div 5 \text{ kg} = 0.8 \text{ m s}^{-2}$$

Step 4: Investigate the maximum possible acceleration

- The maximum would occur when all three forces are acting in the same direction
- Therefore adding to:

$$4 + 8 + 24 = 36 \text{ N}$$

- Now check the acceleration:

$$36 \text{ N} \div 5 \text{ kg} = 7.2 \text{ m s}^{-2}$$

Step 5: Consider this range and the options

- Since option **D** is higher than 7.2 m s^{-2}; it is not possible that these three forces can produce 10 m s^{-2} acceleration for this mass

YOUR NOTES

- **Option D** is the **correct** answer

Worked Example

A rocket produces an upward thrust of 15 MN and has a weight of 8 MN.

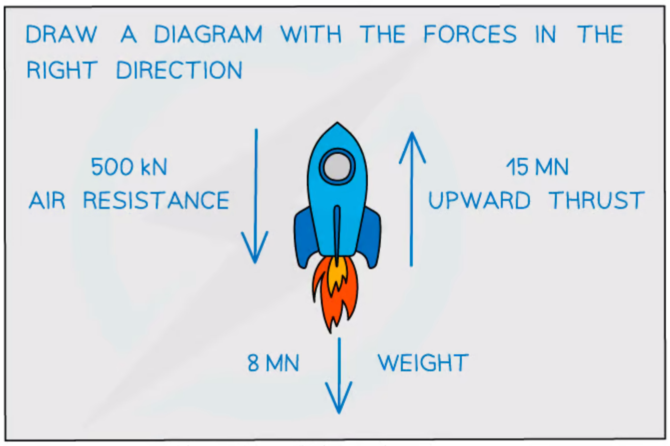
A. When in flight, the force due to air resistance is 500 kN.

Determine what is the resultant force on the rocket.

B. The mass of the rocket is 0.8×10^5 kg.

Calculate the magnitude and direction of the acceleration of the rocket.

A. STEP 1



YOUR NOTES

STEP 2 — CALCULATE THE RESULTANT FORCE ON THE ROCKET

$$F = 15\text{ MN} - (500\text{ kN} + 8\text{ MN})$$

UPWARD FORCES (15 MN) — DOWNWARD FORCES (500 kN + 8 MN)

UNIT CONVERSIONS: $1\text{ kN} = 1 \times 10^3\text{ N}$ $\quad 1\text{ MN} = 1 \times 10^6\text{ N}$

STEP 3 — CONVERT ALL VALUES TO THE SAME UNITS (NEWTONS)

$$F = 15 \times 10^6\text{ N} - (500 \times 10^3\text{ N} + 8 \times 10^6\text{ N})$$

$$F = 6.5 \times 10^6\text{ N}$$

$$F = 6.5\text{ MN} \quad \text{UPWARDS}$$

UPWARDS: IN THE POSITIVE DIRECTION

B. STEP 1 — NEWTONS SECOND LAW

$$F = ma$$

STEP 2 — REARRANGE FOR ACCELERATION a

$$a = \frac{F}{m}$$

STEP 3 — SUBSTITUTE IN VALUES FOR F AND m

$$a = \frac{6.5 \times 10^6\text{ N}}{0.8 \times 10^5\text{ kg}} = 81\text{ ms}^{-2} \quad \text{UPWARDS}$$

UPWARDS: ACCELERATION IS ALWAYS IN THE SAME DIRECTION AS THE RESULTANT FORCE

Exam Tip

The direction you consider positive is your choice, as long as the signs of the numbers (positive or negative) are consistent throughout the question. It is a general rule to consider the direction the object is initially travelling in as positive. Therefore all vectors in the direction of motion will be positive and opposing vectors, such as drag forces, will be negative.

2.2.4 Newton's Third Law

YOUR NOTES
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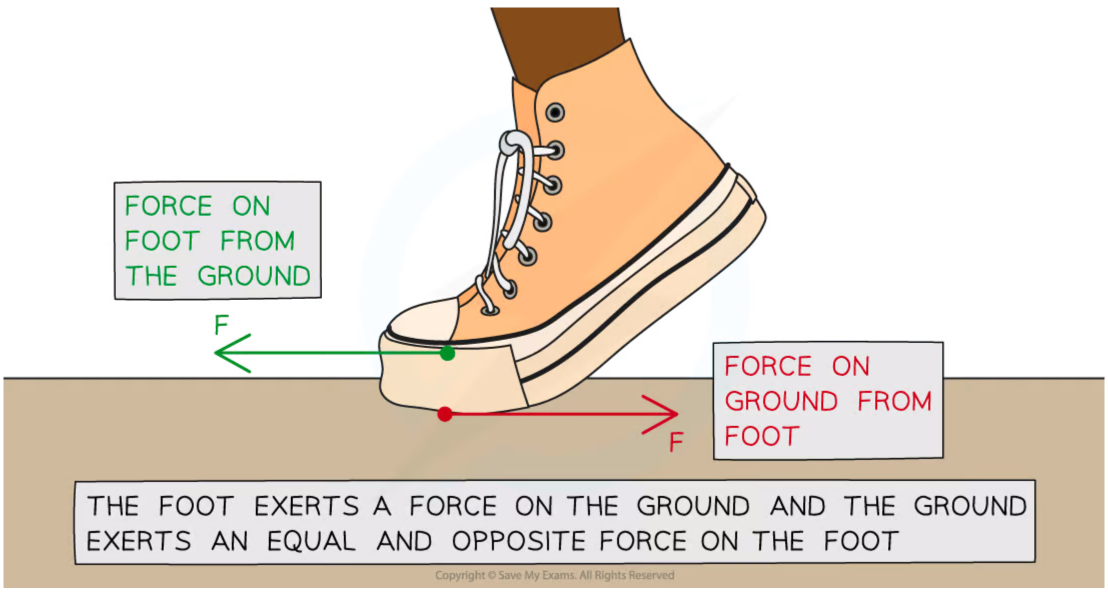
Newton's Third Law

- Newton's **third law of motion** states:

  Whenever two bodies interact, the forces they exert on each other are equal and opposite

- Newton's third law explains the following important principles about forces:
  - All forces arise in **pairs** – if object A exerts a force on object B, then object B exerts an **equal** and **opposite** force on object A
  - Force pairs are of the **same type** – for example, if object A exerts a **gravitational force** on object B, then object B exerts an equal and opposite **gravitational force** on object A

- Newton's third law explains the forces that enable someone to walk
- The image below shows an example of a pair of equal and opposite forces acting on two objects (the ground and a foot):



Newton's Third Law: The foot pushes the ground backwards, and the ground pushes the foot forwards

- One force is from the foot that pushes the ground backwards
- The other is an **equal and opposite** force from the ground that pushes the foot forwards

YOUR NOTES
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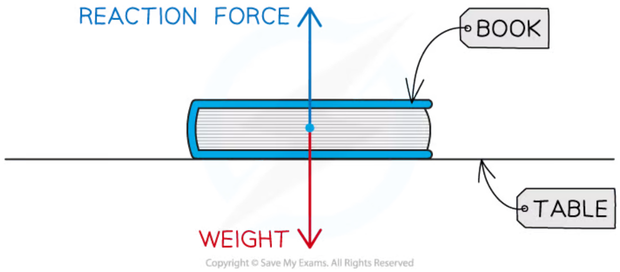
Worked Example

A physics textbook is at rest on a dining room table. Eugene draws a free body force diagram for the book and labels the forces acting on it.



Eugene says the diagram is an example of Newton's third law of motion. William disagrees with Eugene and says the diagram is an example of Newton's first law of motion. By referring to the free-body force diagram, state and explain who is correct.

Step 1: State Newton's first law of motion

- Objects will remain at rest, or move with a constant velocity unless acted on by a resultant force

Step 2: State Newton's third law of motion

- Whenever two bodies interact, the forces they exert on each other are equal and opposite

Step 3: Check if the diagram satisfies the two conditions for identifying Newton's third law

- In each case, Newton's third law identifies pairs of equal and opposite forces, of the same type, acting on two different objects
- The diagram only involves **one object**
- Furthermore, the forces acting on the object are different types of force - one is a **contact force** (from the table) and the other is a **gravitational force** on the book (from the Earth) - its **weight**
- The image below shows how to apply Newton's third law correctly in this case, considering the pairs of forces acting:

YOUR NOTES
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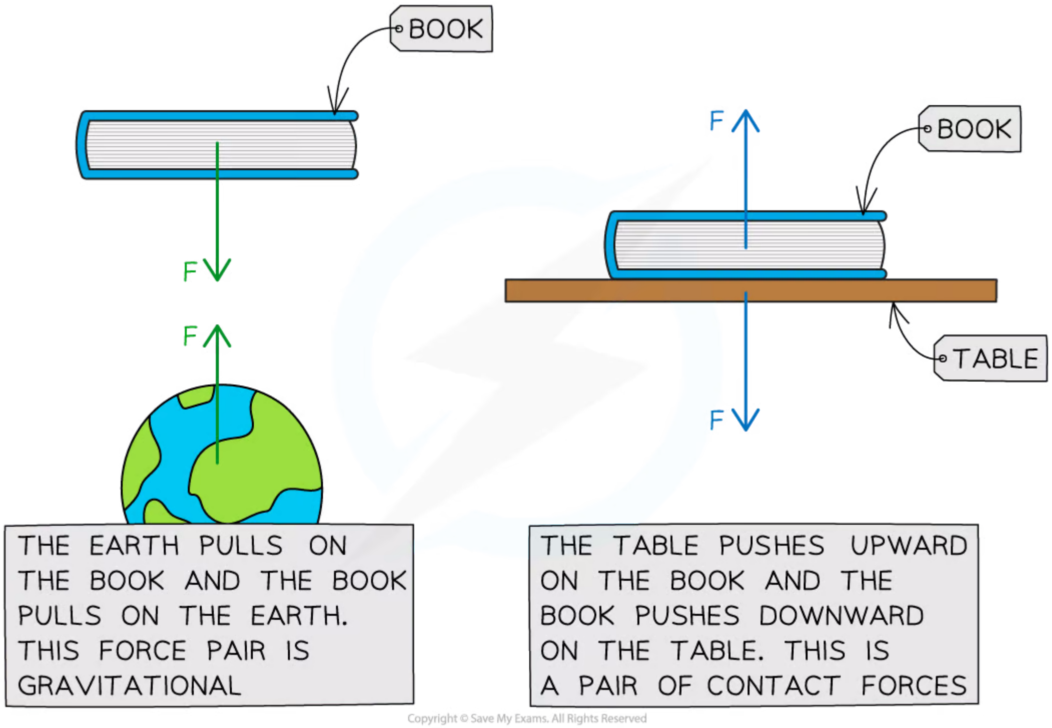

Step 4: Conclude which person is correct

- In this case, **William is correct**
- The free-body force diagram in the question is an example of **Newton's first law**
- The book is **at rest** because the two forces acting on it are **balanced** - i.e. there is no **resultant force**

YOUR NOTES
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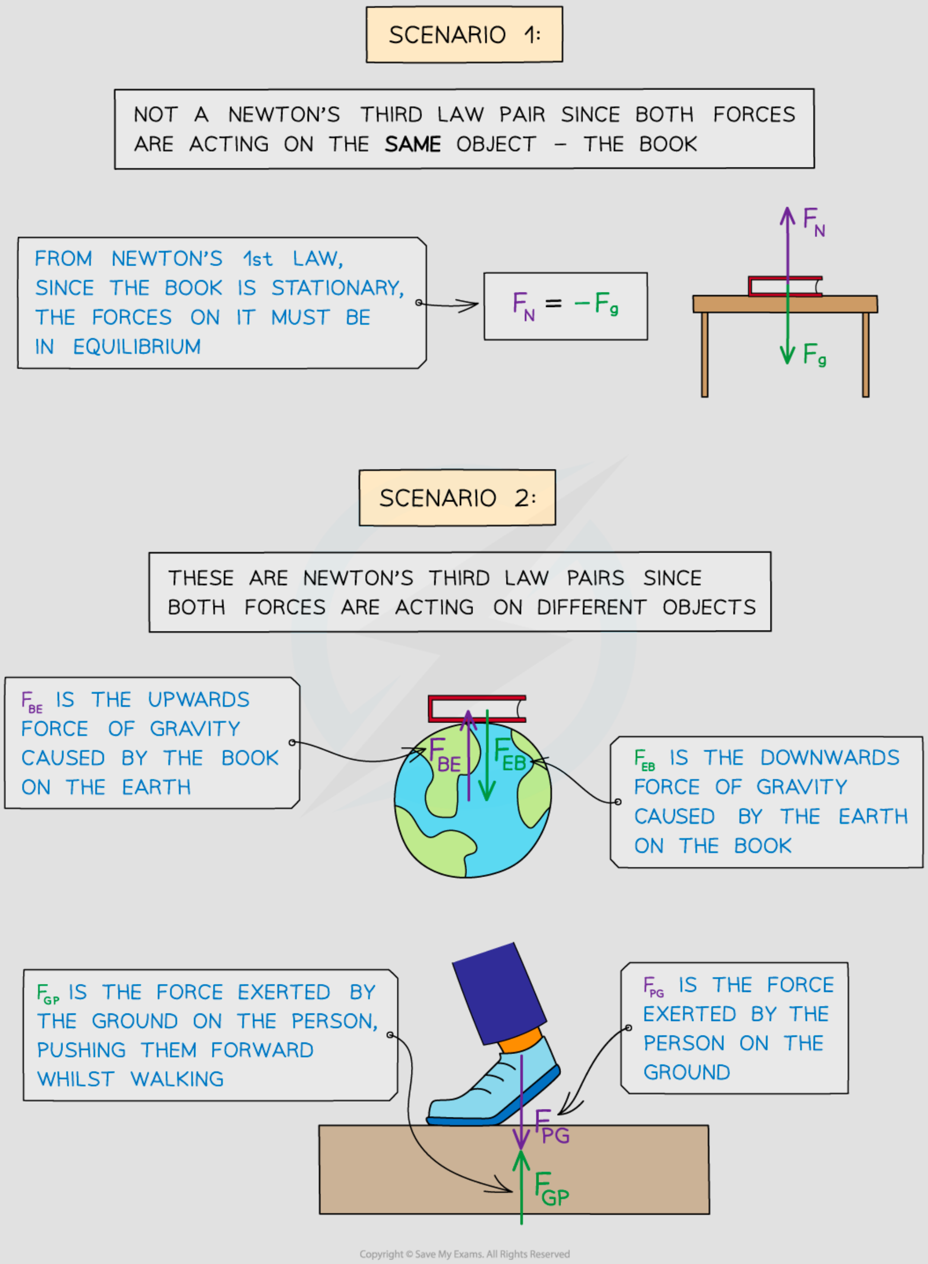
Exam Tip

Remember that pairs of equal and opposite forces in Newton's third law act on **two different objects**. It's a really common mistake to confuse Newton's third law with Newton's first law, so applying this check will help you distinguish between them. Newton's first law involves forces acting on a **single** object. These differences are shown in Scenario 1 (Newton's first law) vs. Scenario 2 (Newton's third law)



YOUR NOTES
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Applying Newton's Laws of Motion

- Newton's laws of motion are strong tools to understand the motion of objects with forces acting upon them
- Below are two worked examples demonstrating different situations involving the application of Newton's laws

Worked Example

Two blocks of mass 1 kg and 4 kg respectively are attached by a tight massless rope between them. The 1 kg block sits on the left and the 4 kg block sits on the right. The 1 kg mass has a 100 Newton force applied to it pulling it to the left. What is the acceleration of both blocks and the tension in the rope as they move across a frictionless surface?

A The acceleration is 15 m s^{-2} to the left and the tension is 20 N

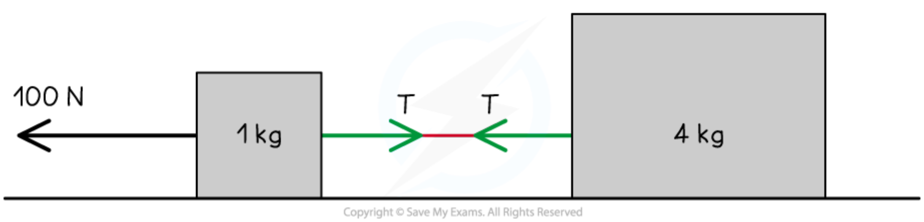
B The acceleration is 20 m s^{-2} to the left and the tension is 40 N

C The acceleration is 15 m s^{-2} to the left and the tension is 60 N

D The acceleration is 20 m s^{-2} to the left and the tension is 80 N

Step 1: Consider the whole of the system

- Together the 1 kg and 4 kg blocks are both being pulled along by the 100 N force (since the rope is tight)
- This is a frictionless flat surface, therefore the only forces in the system are the pulling force and the tension force(s) in the rope
- Therefore the acceleration can be found using Newton's second law



Step 2: Find the acceleration

- Using Newton's second law:

$$F = m \times a$$

- Rearrange for ***a***

$$a = F \div m$$

$$a = 100 \div 5 = 20 \text{ m s}^{-2} \text{ to the left}$$

YOUR NOTES
⬇

Step 3: Examine the 4 kg mass only

- Since the system is moving with an acceleration of 20 m s^{-2} to the left, the force on only the 4 kg mass can be found

$$F = m \times a$$

$$F = 4 \times 20 = 80 \text{ N to the left}$$

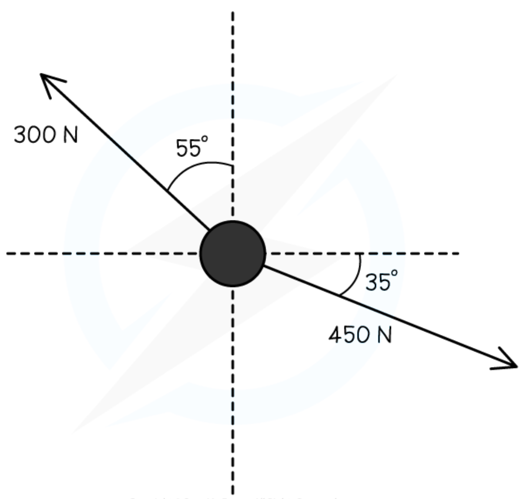
- The only force acting on the 4 kg mass is the rope and therefore tension force
- This means there is 20 N pulling force on the 1 kg block only

Step 4: State your answer

- Since acceleration, a = 20 m s^{-2} to the left and the tension force = 80 N to the left
- Therefore, the answer is **option D**

Worked Example

A stationary object is subject to a 300 N force towards the left and at 55 degrees leftwards with respect to the vertical and a 450 N force to the right and 35° downwards with respect to the horizontal.Calculate what is the magnitude and direction of the third force that would make this object remain stationary (to the nearest N).



Step 1: Recall Newton's first law

- Newton's First Law states:

  A body will remain at rest or move with constant velocity unless acted on by a resultant force

- Therefore, for this object to remain stationary, the resultant force must have a magnitude of 0 Newtons

Step 2: Resolve the 300 N force into its horizontal and vertical components

YOUR NOTES
⬇

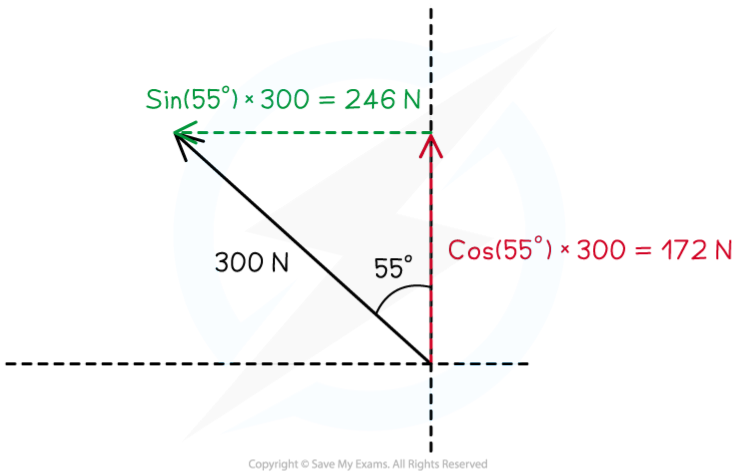
- The horizontal component can be resolved from:

$$\sin(55°) \times 300 = 246 \text{ N}$$

- This is directed to the **left**
- The vertical component can be resolved from:

$$\cos(55°) \times 300 = 172 \text{ N}$$

- This is in an **upwards** direction



Step 3: Resolve the 450 N force into its horizontal and vertical components

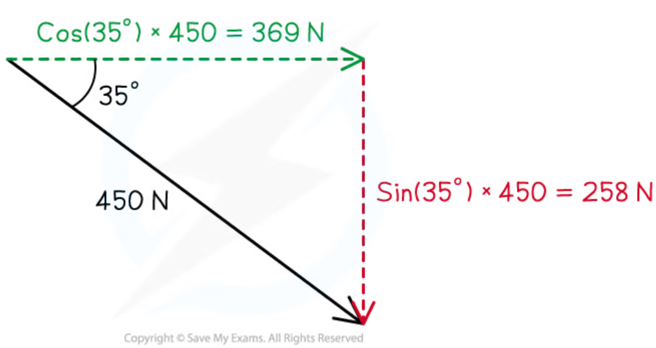
- The horizontal component can be resolved from:

$$\cos(35°) \times 450 = 369 \text{ N}$$

- This is directed to the **right**
- The vertical component can be resolved from:

$$\sin(35°) \times 450 = 258 \text{ N}$$

- This is in a **downwards** direction



Step 4: Combine the horizontal components

- The two forces provide 369 N to the right and 246 N to the left

YOUR NOTES
⬇

- Therefore, since these are opposing directions:

369 – 246 = 123 N to the right

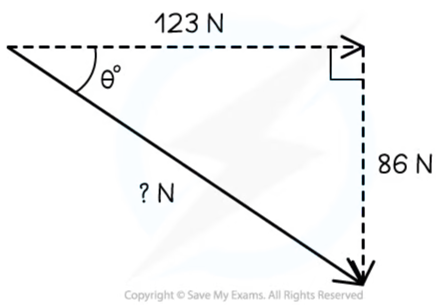
Step 5: Combine the vertical components

- The two forces provide 258 N downwards and 172 N upwards
- Therefore, since these are opposing directions:

258 – 172 = 86 N downwards

Step 6: Make a right angle triangle using these two force vectors

- There should be a longer size 123 N magnitude vector arrow to the right and then a 86 N magnitude vector arrow downwards
- These can be connected from start to finish by a third vector which is the hypotenuse of this right-angled triangle



Step 7: Use the two vectors magnitudes to find the angle from the horizon

- Since the vectors are to the right and downwards in this right-angle triangle, neither is the hypotenuse
- Therefore the angle from the horizontal downwards can be found by using *tan:*

$$\tan(\theta°) = \text{Opp.} \div \text{Adj.} = 86 \div 123$$

$$\theta° = \tan^{-1}(86 \div 123) = \tan^{-1}(0.699)$$

$$\theta° \approx 35°$$

Step 8: Use Pythagoras theorem or trigonometry to find the magnitude of the resultant force

- Method 1: Using Pythagoras theorem
  - $c^2 = b^2 + a^2$ where b and a are the vector magnitudes for the horizontal and vertical components found so far and c is the hypotenuse magnitude

$$c^2 = 86^2 + 123^2 = 7396 + 15129 = 22525$$

$$c = \sqrt{22\,525} \approx 150 \text{ N}$$

- Method 2: Using trigonometry
  - Using the horizontal component and the angle found in step 7, cos can be used

$$\cos(35°) = \text{Adj.} \div \text{Hyp.} = 123 \div \text{Hyp.}$$

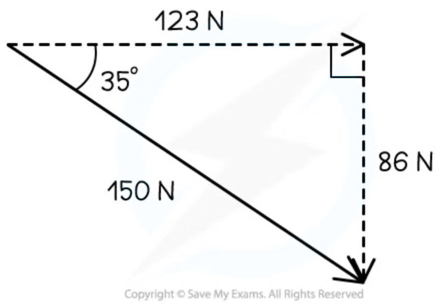
YOUR NOTES
⬇

- Therefore:

$$123 \div \cos(35°) = \text{Hyp.} \approx 150 \text{ N}$$

Step 9: State the final answer

- The third force which would cause this object to remain stationary is **150 N right** and **35° downwards** from the horizontal



Exam Tip

You are expected to know Newton's three laws of motion from memory and how they apply to physical situations. So be sure to practice and use them without having to review them before carrying out a problem.

2.2.6 Friction

YOUR NOTES
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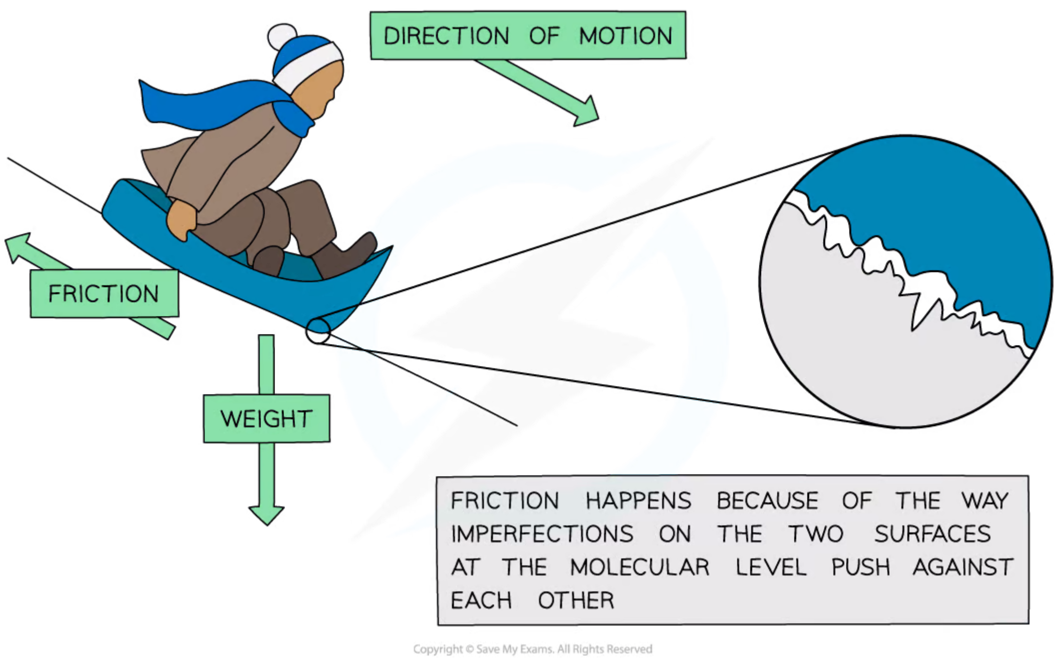
Friction

- Friction is a force that works in **opposition** to the motion of an object
- It occurs between two **solid bodies** that are **in contact** with one another
    - The opposition of **friction slows** down the motion of the object
- When friction is present, energy is transferred in the form of **heat**
    - This raises the **temperature** (thermal energy) of the object and its surroundings
    - The work done against the frictional forces causes this rise in the temperature
- **Imperfections** at the interface between the object and the surface bump into and rub up against each other
    - Not only does this slow the object down but also causes an increase in **thermal energy**



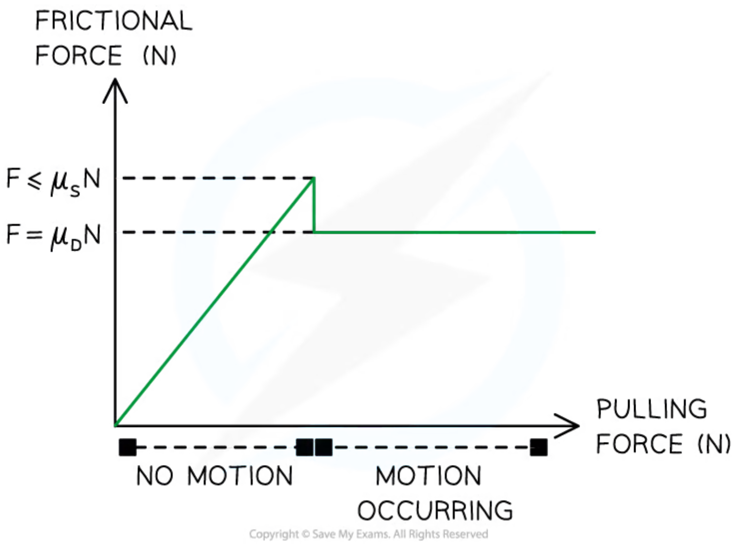
The interface between the ground and the sled is bumpy which is the source of the frictional force

Static & Dynamic Friction

- There are two kinds of friction to consider for IB DP Physics
    - **Static friction** occurs when two solid objects are in contact and **no movement** is occurring between the two objects
    - **Dynamic friction** occurs once one of the objects is **moving** past the other, such as in the sled example above
- Both of these forms of friction depend on the **normal reaction force** of the object sitting upon the other

YOUR NOTES
⬇

- Static friction will **match** any pushing force that acts against until it can no longer hold the two objects stationary
  - Static friction **increases** in magnitude **until movement begins** and dynamic friction occurs
- For any given situation, **static friction** should reach a maximum value that is **larger** than that of **dynamic friction**
  - For a constant pushing force, **dynamic** friction will be a **constant**
- This is because there are more forces at work keeping an object stationary than there are forces working to resist an object once it is in motion



The relationship between frictional forces and motion

- The equation for static friction is given by:

$$F \leq \mu_S \times R$$

- Where:
  - ***F*** = static frictional force (N)
  - μ_S = coefficient of static friction
  - ***R*** = normal reaction force (N)
- The **coefficient** of static **friction** is a number between 0 and 1 but does not include those numbers
  - It is a ratio of the force of static friction and the normal force
  - The **larger** the coefficient of static friction, the **harder** it is to move those two objects past one another
- The equation for dynamic friction is given by:

$$F = \mu_D \times R$$

- Where:
  - ***F*** = dynamic frictional force (N)
  - μ_D = coefficient of dynamic friction

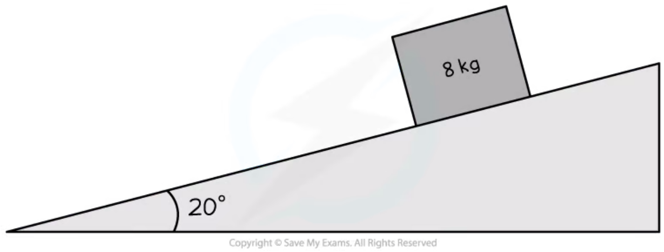
YOUR NOTES
↓

- R = normal reaction force (N)

- The coefficient of dynamic friction has similar properties to that of static friction
- Yet, **dynamic friction** has a **definite force** value for a given situation
  - Whereas the force of **static friction** has an **increasing force** value

Worked Example

An 8.0 kg block sits on an incline of 20 degrees from the horizontal. It is stationary and does have a frictional force acting upon it.



Determine the minimum possible value of the coefficient of static friction.

Step 1: List the known quantities

- Mass of the block, m = 8.0 kg
- Angle between the slope and the horizontal, θ = 20°

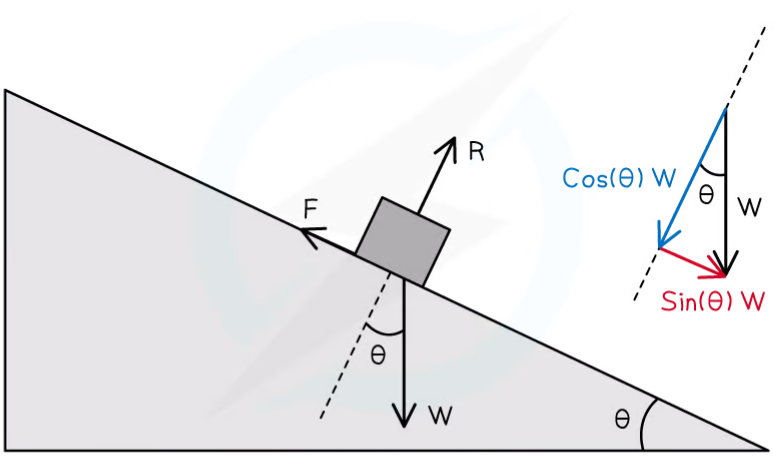
Step 2: Determine the weight of the block

- The weight will act directly downwards and comes from the interaction of mass and acceleration due to gravity

$$W = m \times g$$

$$8.0 \times 9.8 = 78.4 \text{ N downwards}$$

Step 3: Break the weight down into components based on the slope angle



- The component that is parallel to the slope and provides a force moving the block down the slope can be found from:

$$\sin(20°) \times W = F$$

$$F = \sin(20°) \times 78.4 = 26.8 \text{ N}$$

- The component that is perpendicular to the slope and the same magnitude as the normal reaction force can be found from:

$$\cos(20°) \times W = F$$

$$F = \cos(20°) \times 78.4 = 73.7 \text{ N}$$

Step 4: Use the equation of static friction to find the minimum value of the coefficient of static friction

- The equation for static friction is:

$$F \leq \mu_S \times R$$

- In this case, the ***F*** is the 26.8 N pushing the block down the slope
- The ***R*** is the normal reaction force which has the same magnitude as the perpendicular component of the weight force which is 73.7 N
- Therefore the value can be added and μ_S solved for:

$$26.8 \leq \mu_S \times 73.7$$

- Rearrange for μ_S

$$26.8 \div 73.7 \leq \mu_S$$

$$0.36 \leq \mu_S$$

Step 5: State the final answer

- The coefficient for static friction must be at least **0.36 or greater** for this situation

YOUR NOTES
⬇

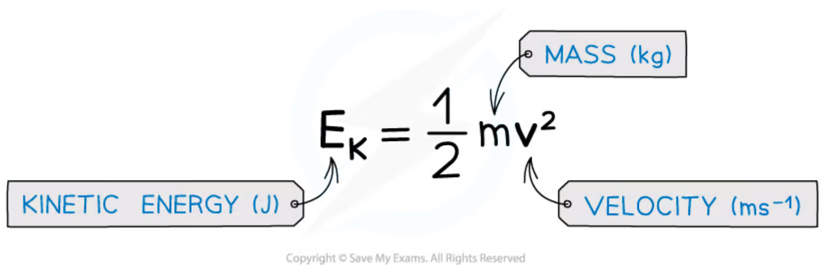
YOUR NOTES

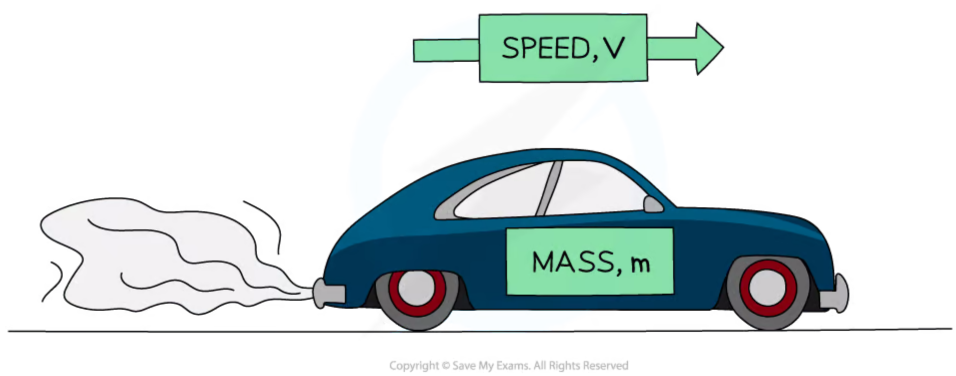
2.3 Work, Energy & Power

2.3.1 Kinetic Energy

Kinetic Energy

- Kinetic energy (E_k) is the energy an object has due to its **motion** (or velocity)
  - The faster an object is moving, the greater its kinetic energy
- When an object is falling, it is **gaining** kinetic energy since it is gaining speed
  - This energy transferred from the gravitational potential energy it is losing
  - An object will maintain this kinetic energy unless its speed changes
- Kinetic energy can be calculated using the following equation:

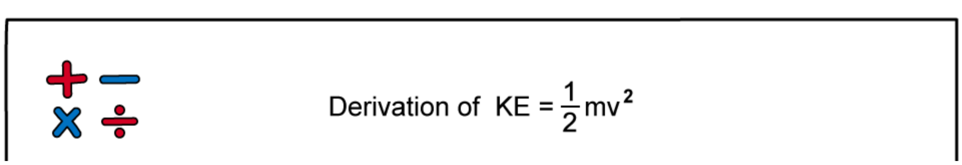




Kinetic energy (KE): The energy an object has when it is moving

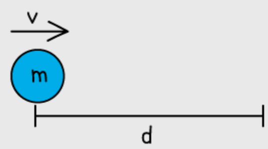
Derivation of Kinetic Energy Equation

- A force can make an object accelerate; work is done by the force and energy is transferred to the object
- Using this concept of work done and an equation of motion, the extra work done due to an object's speed can be derived
- The derivation for this equation is shown below:



YOUR NOTES ↓

CONSIDER A MASS m AT REST WHICH ACCELERATES TO A SPEED v OVER A DISTANCE d



WORK DONE IN ACCELERATING THE MASS

$$W = F \times d$$

AND $F = ma$ FROM NEWTON'S SECOND LAW

RECALL THE SUVAT EQUATION

$$v^2 = u^2 + 2as$$

IF $u = 0$ AND $s = d$

$$v^2 = 2ad$$

REARRANGING FOR a

$$a = \frac{v^2}{2d}$$

SUBSTITUTE BACK INTO $F = ma$

$$F = ma = \frac{mv^2}{2d}$$

SUBSTITUTE THIS FORCE F INTO THE WORK DONE EQUATION

$$W = \frac{mv^2}{2d} \times d = \frac{1}{2}mv^2$$

THE MASS IS NOW ABLE TO DO EXTRA WORK $= \frac{1}{2}mv^2$ DUE TO ITS SPEED

IT HAS KINETIC ENERGY $= \frac{1}{2}mv^2$

YOUR NOTES
↓

Worked Example

A body travelling with a speed of 12 m s^{-1} has kinetic energy 1650 J. If the speed of the body is increased to 45 m s^{-1}, estimate what is its new kinetic energy.

STEP 1

EQUATION FOR KINETIC ENERGY

$$KE = \frac{1}{2}mv^2$$

STEP 2

MASS WILL NOT CHANGE, SO CAN BE CALCULATED FROM ITS INITIAL KINETIC ENERGY

REARRANGE FOR MASS m

$$m = \frac{2 \times KE}{v^2} = \frac{2 \times 1650}{12^2} = 23 \text{ kg}$$

STEP 3

SUBSTITUTE INTO KINETIC ENERGY EQUATION

USING VALUE OF MASS AND NEW VALUE OF VELOCITY

$$KE = \frac{1}{2} \times 23 \times 45^2 = 23000 \text{ J} \quad (2 \text{ s.f})$$

Exam Tip

When using the kinetic energy equation, note that only the speed is squared, not the mass or the ½. If a question asks about the 'loss of kinetic energy', remember **not** to include a negative sign since energy is a **scalar** quantity.

YOUR NOTES
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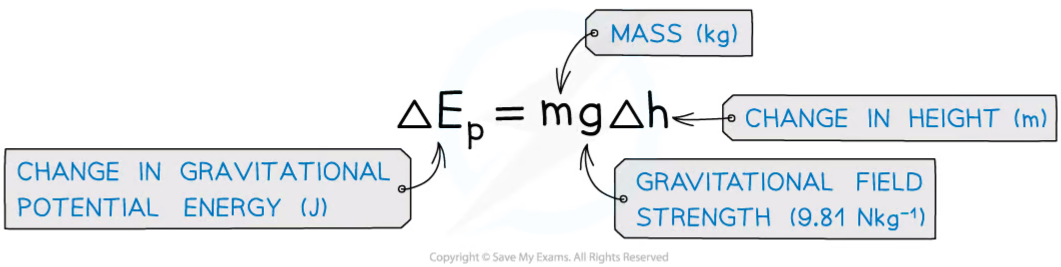
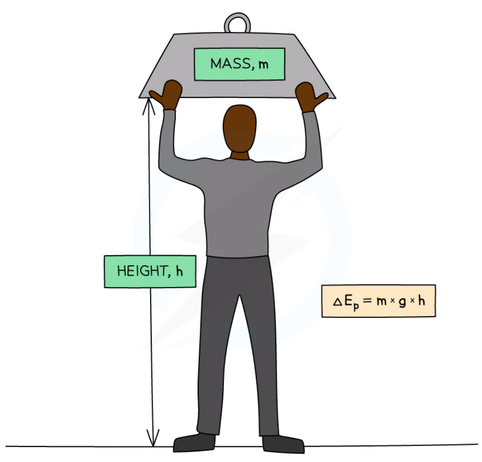
Gravitational Potential Energy

- Gravitational potential energy (E_p) is energy stored in a mass due to its position in a gravitational field
  - If a mass is **lifted** up, it will **gain** E_p (converted **from** other forms of energy)
  - If a mass **falls**, it will **lose** E_p (and be converted **to** other forms of energy)

- The equation for gravitational potential energy for energy changes in a **uniform gravitational field** is:





Gravitational potential energy (GPE): The energy an object has when lifted up

- The potential energy on the Earth's surface at ground level is taken to be equal to 0
- This equation is only relevant for energy changes in a **uniform gravitational field** (such as near the Earth's surface)

Derivation of GPE Equation

- When a heavy object is lifted, work is done since the object is provided with an upward force against the downward force of gravity
  - Therefore **energy is transferred to the object**

- This equation can therefore be derived from the work done

YOUR NOTES
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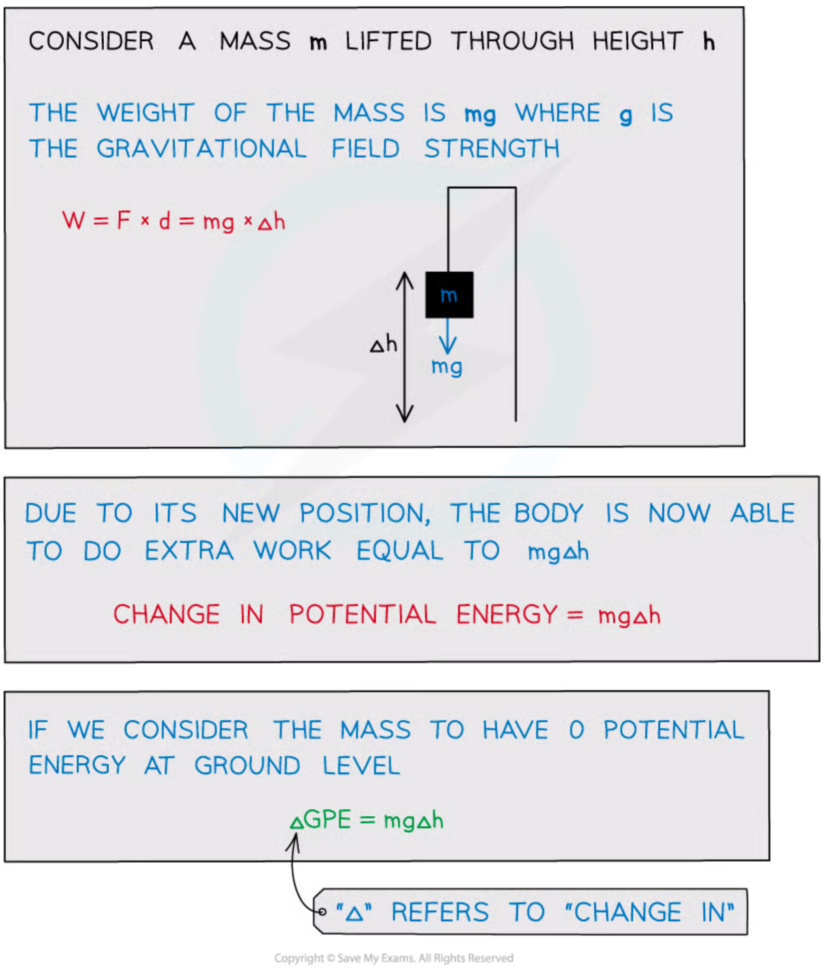
Derivation of GPE = mgh



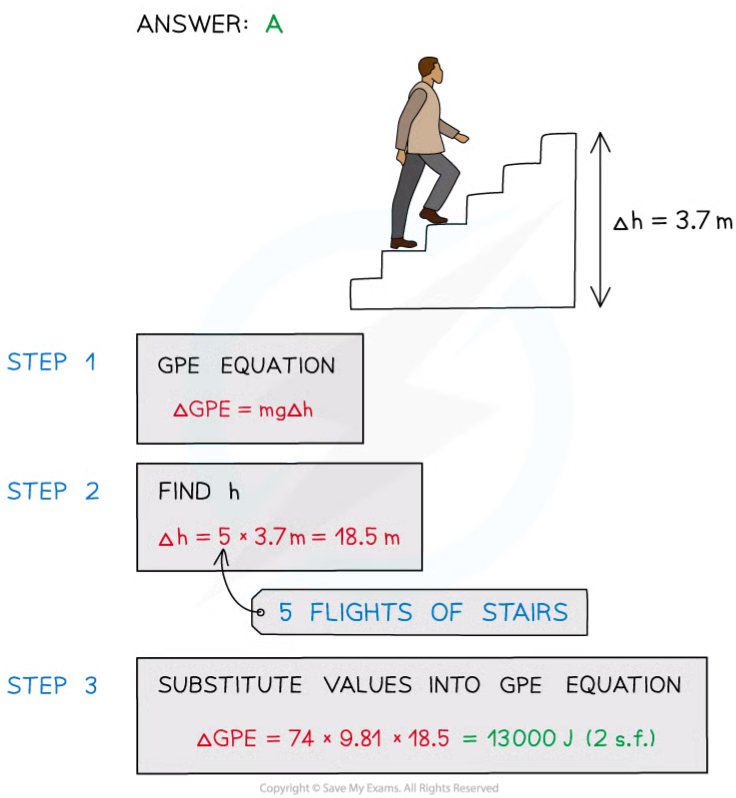
Worked Example

To get to his apartment a man has to climb five flights of stairs.

The height of each flight is 3.7 m and the man has a mass of 74 kg.

What is the approximate gain in the man's gravitational potential energy during the climb?

A. 13 000 J **B.** 2700 J **C.** 1500 J **D.** 12 500 J



GPE vs Height

- The two graphs below show how GPE changes with height for a ball being thrown up in the air and when falling down (**ignoring air resistance**)

YOUR NOTES
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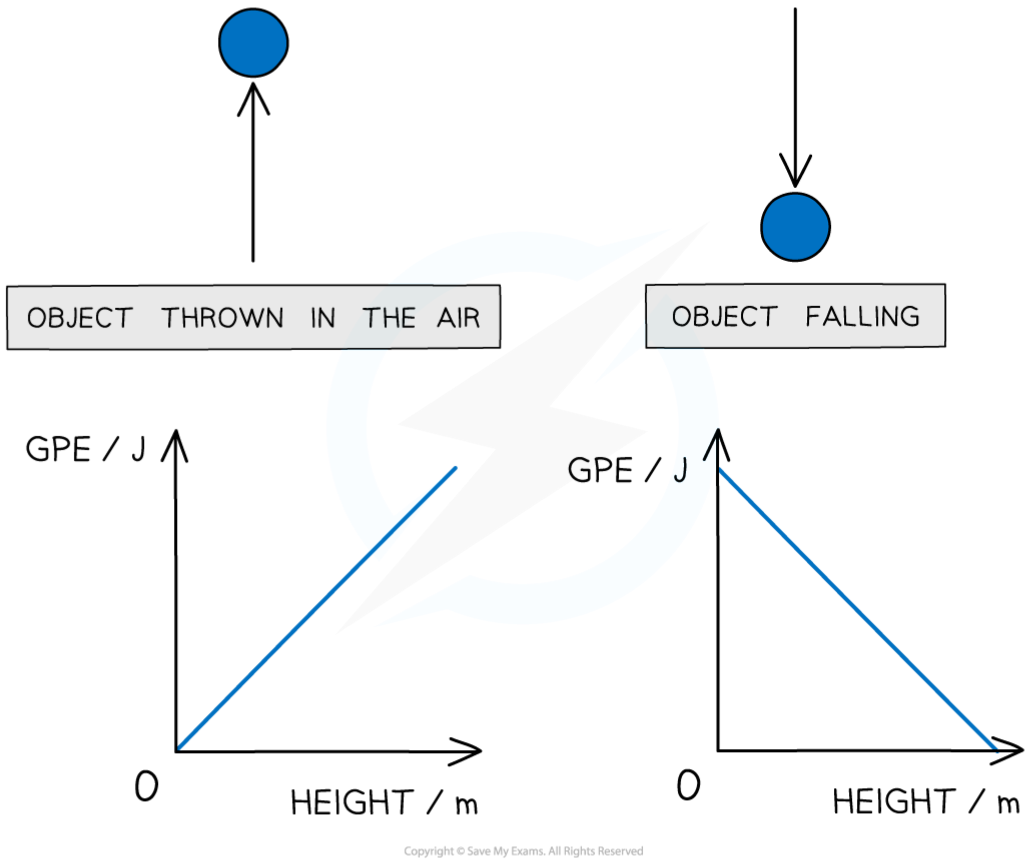

Graphs showing the linear relationship between GPE and height

- Since the graphs are straight lines, GPE and height are said to have a **linear** relationship
    - These graphs would be identical for GPE against time instead of height

Relationship between GPE & KE

- There are many scenarios that involve the transfer of kinetic energy into gravitational potential, or vice versa
- Some examples are:
    - A swinging pendulum
    - Objects in freefall
    - Sports that involve falling, such as skiing and skydiving

- Using the principle of conservation of energy, and taking any drag forces as negligible:

Loss in potential energy = Gain in kinetic energy

YOUR NOTES
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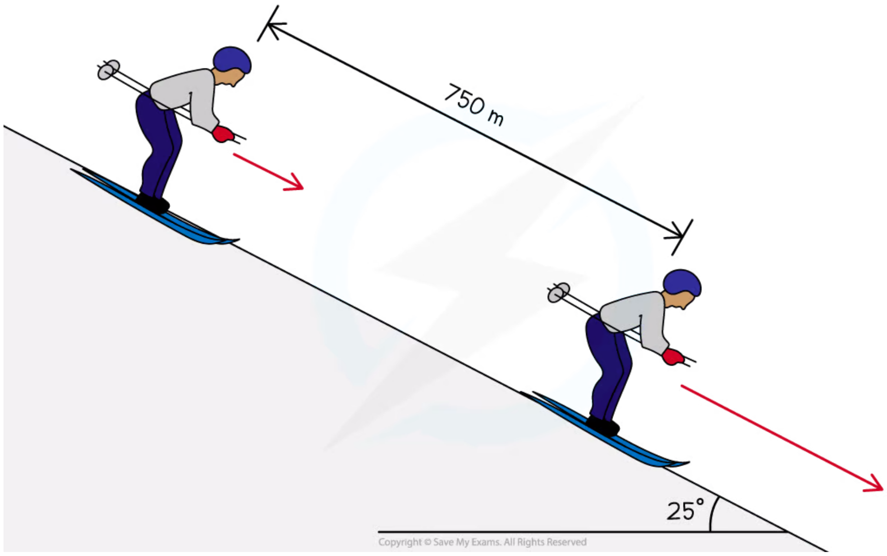
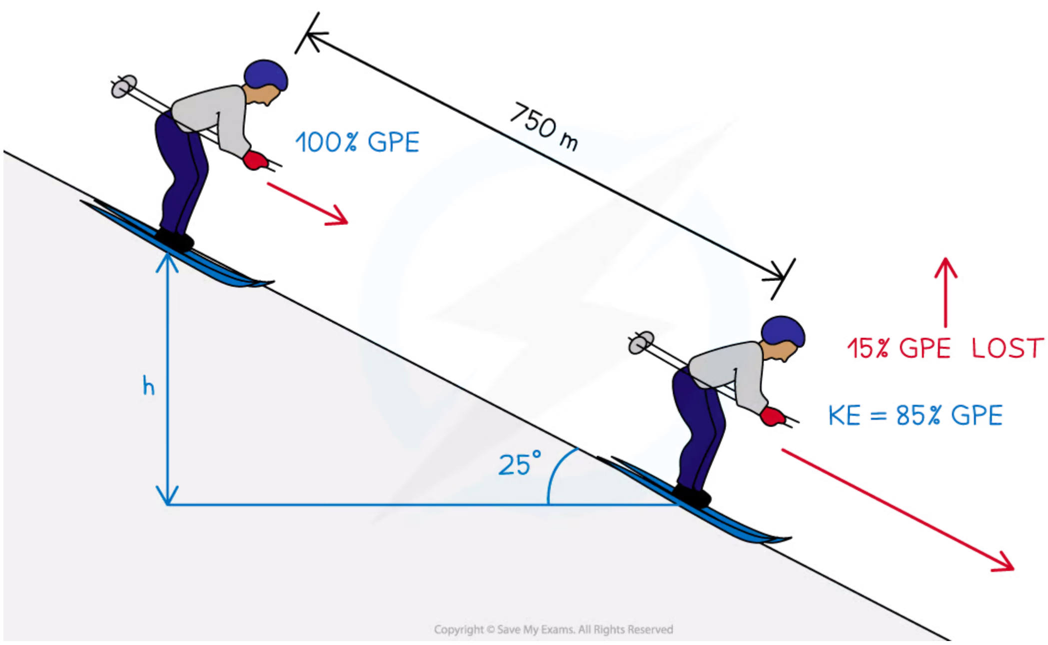
Worked Example

The diagram below shows a skier on a slope descending 750 m at an angle of 25° to the horizontal.



Calculate the final speed of the skier, assuming that he starts from rest and 15% of his initial gravitational potential energy is **not** transferred to kinetic energy.

Step 1: Write down the known quantities



- Vertical height, h = 750 sin 25°
- $E_k = 0.85\ E_p$

Step 2: Equate the equations for E_k and E_p

$E_k = 0.85\ E_p$

$\frac{1}{2} mv^2 = 0.85 \times mgh$

Step 3: Rearrange for final speed, *v*

$$\frac{1}{2} \cancel{m}v^2 = 0.85 \times \cancel{m}gh$$

$$v^2 = 0.85 \times 2gh$$

$$v = \sqrt{0.85 \times 2gh}$$

Step 4: Calculate the final speed, *v*

$$v = \sqrt{0.85 \times 2 \times 9.81 \times 750 \sin 25°} = 72.7$$

Final speed, $v = 73$ m s^{-1}

YOUR NOTES
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YOUR NOTES
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Elastic Potential Energy

- Elastic potential energy is defined as

  The energy stored within a material (e.g. in a spring) when it is stretched or compressed

- It can be found from the **area under the force-extension graph** for a material deformed within its limit of proportionality
  - A material **within its limit of proportionality** obeys Hooke's law
- Therefore, for a material obeying Hooke's Law, elastic potential energy can be calculated using:

HOOKE'S LAW: $F = kx$

$$EPE = \frac{1}{2}Fx = \frac{1}{2}(kx)x$$

$$\text{ELASTIC POTENTIAL ENERGY} = \frac{1}{2}kx^2$$

- Where:
  - k = force constant of the spring ($N\ m^{-1}$)
  - x = extension (m)
- In your data booklet the extension x is written as Δx
  - This just means the change in x
- It is very dangerous if a wire under large stress suddenly breaks
- This is because the elastic potential energy of the strained wire is converted into kinetic energy

EPE = KE

$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2$$

$$v \propto x$$

- This equation shows that the greater the extension of a wire, x, the greater the speed, v, it will have on breaking

YOUR NOTES
⬇

Worked Example

A car's shock absorbers make a ride more comfortable by using a spring that absorbs energy when the car goes over a bump. One of these springs, with a force constant of 50 kN m^{-1} is fixed next to a wheel and compressed a distance of 10 cm.Calculate the energy stored by the compressed spring.

Step 1: List the known values

- Force constant, $k = 50$ kN $m^{-1} = 50 \times 10^3$ N m^{-1}
- Compression, $x = 10$ cm $= 10 \times 10^{-2}$ m

Step 2: Write the relevant equation

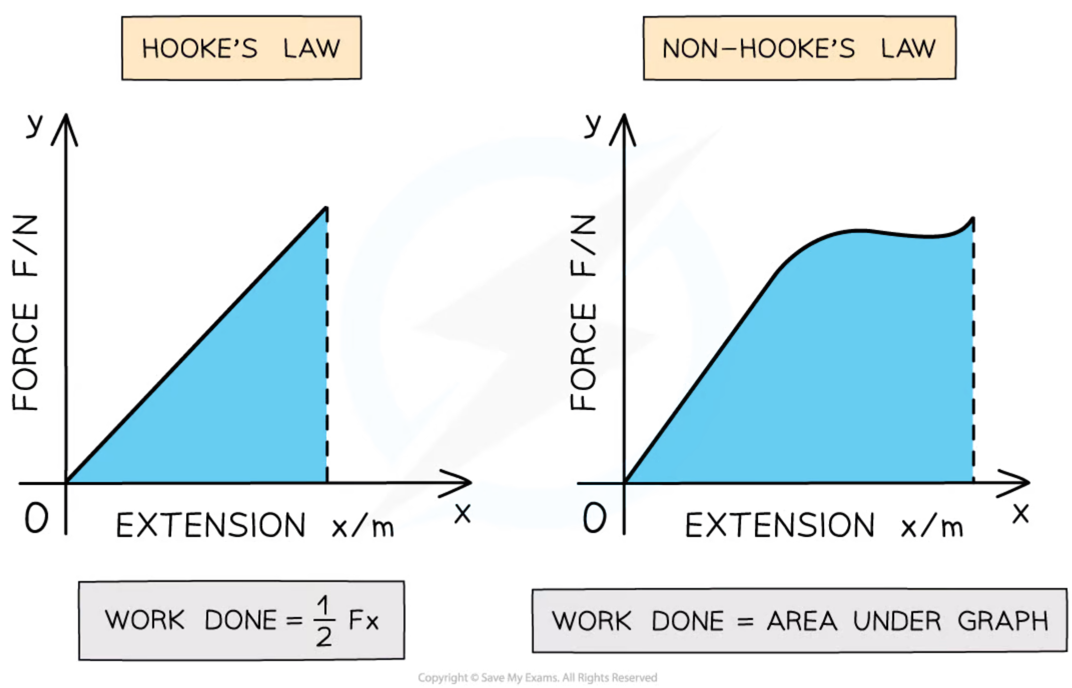
$$EPE = ½kx^2$$

Step 3: Substitute in the values

$$EPE = ½ \times (50 \times 10^3) \times (10 \times 10^{-2})^2 = \mathbf{250\ J}$$

Force-Extension graphs

- Work has to be done to stretch a material
- Before a material reaches its elastic limit (whilst it obeys Hooke's Law), all the work is done is stored as **elastic potential energy (EPE)**
- The work done, or the elastic potential energy is the **area under the force-extension graph**



Work done is the area under the force-extension graph

- This is true for whether the material obeys Hooke's law or not

YOUR NOTES

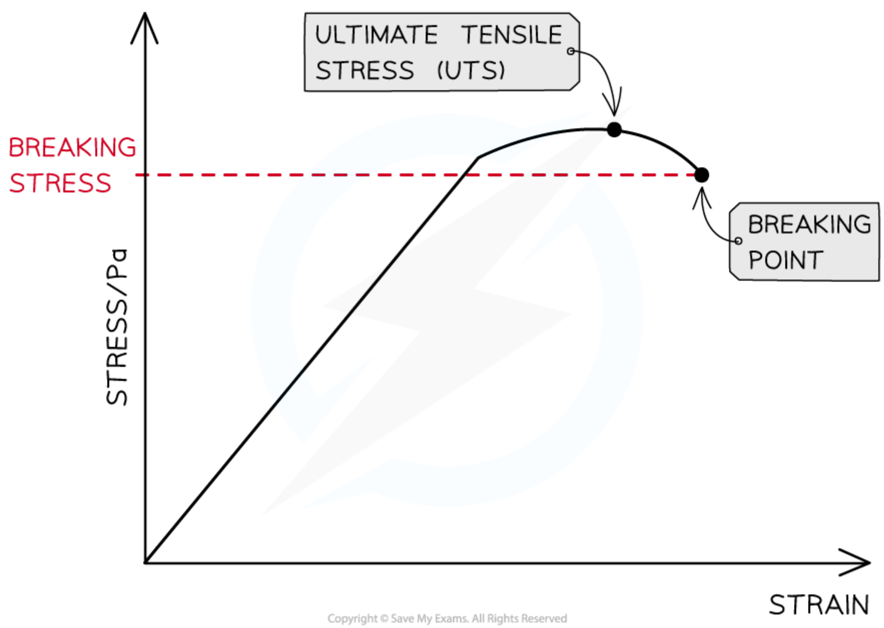
- For the region where the material **obeys** Hooke's law, the work done is the area of a **right-angled triangle** under the graph
- For the region where the material **doesn't obey** Hooke's law, the area is the **full region** under the graph. To calculate this area, split the graph into separate segments and add up the individual areas of each

Breaking Stress

- As greater force is applied on a material, the **stress** on it increases
- The breaking stress is the **maximum stress a material can stand before it fractures (breaks)**
  - A material with high breaking stress is considered **ductile**, which means it can extend more before breaking because of plastic deformation
  - A common example of this is copper, as well as being a good electrical conductor, copper is ductile so it is a suitable material for making wires

- The ultimate tensile stress (UTS) is sometimes also marked on a stress-strain graph
  - This is the **maximum stress that the material can withstand**

- The UTS and breaking stress can depend on the condition of the material such as its temperature
  - This is very important for engineers when considering materials for a particular structure
  - The material might need to stand extreme temperatures and loads which are taken into consideration

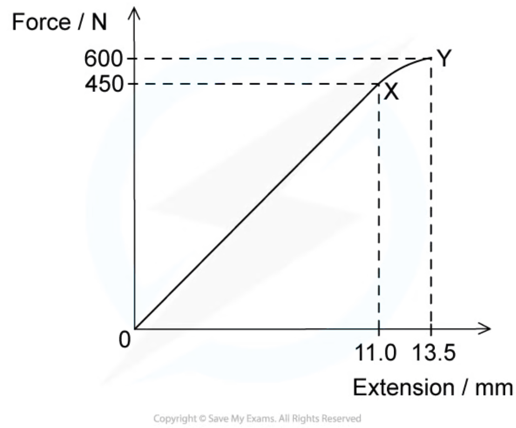


The ultimate breaking stress is the point on the stress-strain graph of maximum stress. The breaking point is where the material fractures

YOUR NOTES
⬇

Worked Example

The graph shows the behaviour of a sample of a metal when it is stretched until it starts to undergo plastic deformation.



Calculate the total work done in stretching the sample from zero to an extension of 13.5 mm. Simplify the calculation by treating the curve XY as a straight line.

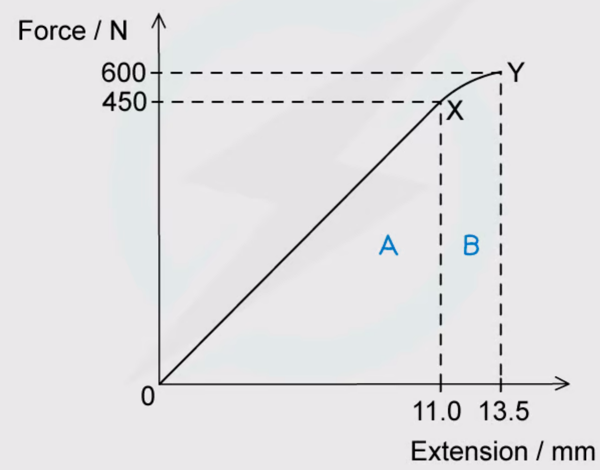
YOUR NOTES

STEP 1

WORK DONE = AREA UNDER THE FORCE–EXTENSION GRAPH

STEP 2

SPLIT GRAPH INTO THE TWO AREAS



STEP 3

CALCULATE AREA A

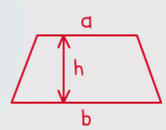
AREA OF A RIGHT ANGLED TRIANGLE = $\frac{1}{2} \times \text{BASE} \times \text{HEIGHT}$

AREA = $\frac{1}{2} \times 11 \times 10^{-3} \times 450 = 2.475$ J

STEP 4

CALCULATE AREA B

AREA OF TRAPEZIUM = $\left(\frac{a + b}{2}\right) \times h$



AREA = $\left(\frac{450 + 600}{2}\right) \times 2.5 \times 10^{-3} = 1.313$ J

STEP 5

TOTAL AREA = 2.475 + 1.313 = 3.79 J (3 s.f.)

YOUR NOTES
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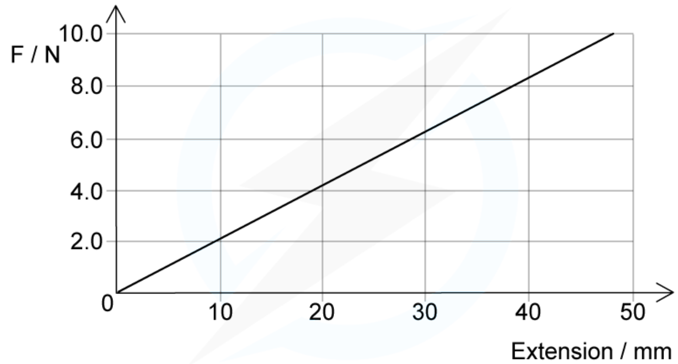
Worked Example

A spring is extended with varying forces; the graph below shows the results.



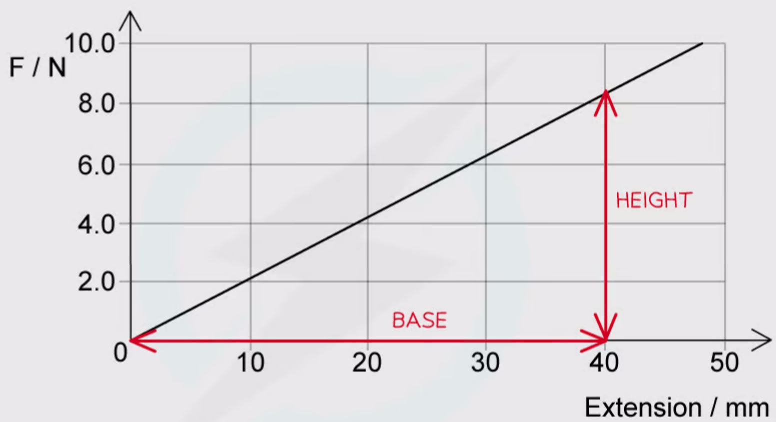
Calculate the energy stored in the spring when the extension is 40 mm.

STEP 1

ENERGY STORED = AREA UNDER THE GRAPH

STEP 2

CALCULATE AREA UNDER GRAPH FOR EXTENSION OF 40mm



AREA OF TRIANGLE = $\frac{1}{2}$ × BASE × HEIGHT

AREA = $\frac{1}{2}$ × 40 × 10^{-3}m × 8.1 N = 0.16 J

STEP 3

ENERGY STORED = 0.16 J

YOUR NOTES
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Work Done

- Work is defined as

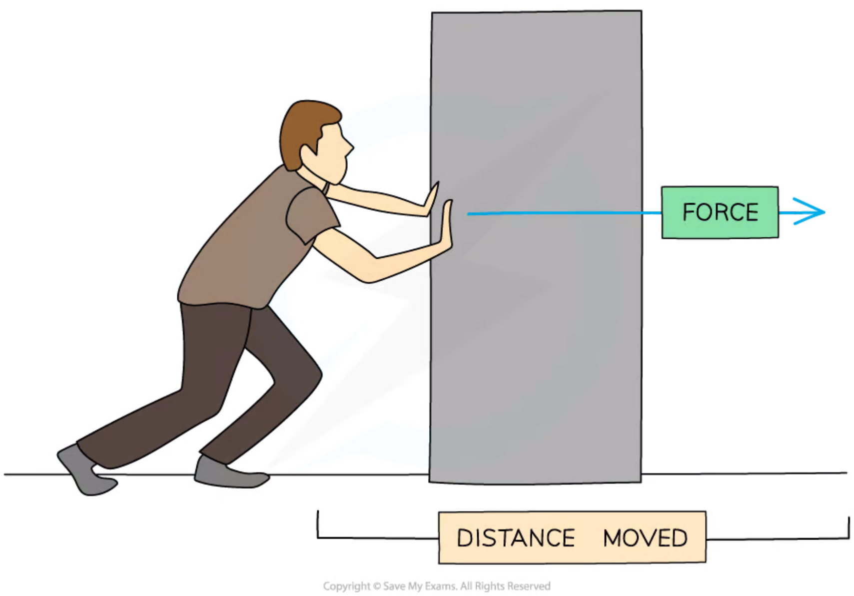
  The amount of energy transferred when an external force causes an object to move over a certain distance

- If the force is parallel to the direction of the object's displacement, the work done can be calculated using the equation:

$$W = Fs$$

- Where:
  - W = work done (J)
  - F = average force applied (N)
  - s = displacement (m)

- In the diagram below, the man's pushing force on the block is doing work as it is transferring energy to the block



Work is done when a force is used to move an object over a distance

- When pushing a block, **work is done against friction** to give the box kinetic energy to move
- The kinetic energy is transferred to other forms of energy such as heat and sound
  - Usually, if a force acts **in** the direction that an object is moving then the object will **gain energy**
  - If the force acts in the **opposite** direction to the movement then the object will **lose energy**

- When plotting a graph of average force applied against displacement, the **area** under the graph is equal to the **work done**

YOUR NOTES
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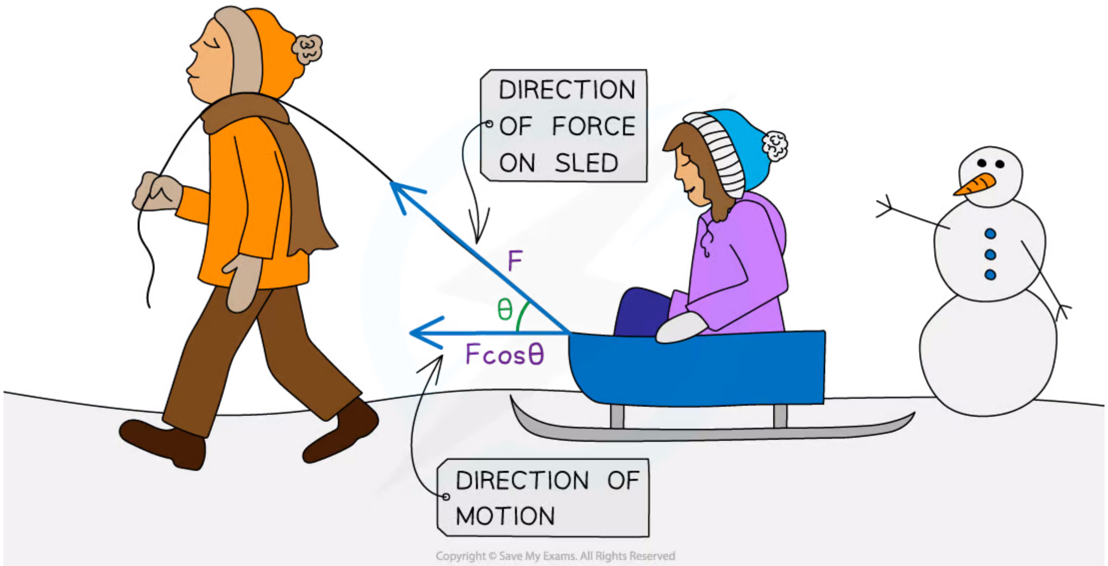
- Sometimes the direction of motion of an object is **not parallel** to the direction of the force
- If the force is at an **angle θ** to the object's displacement, the work done is calculated by:

$$W = Fs \cos \theta$$

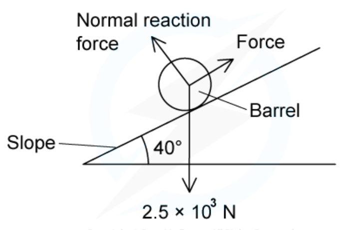
- Where **θ** is the angle, in degrees, between the direction of the force and the motion
  - When **θ** is 0 (the force is in the direction of motion) then **cos θ = 1** and ***W = Fs***
- This may not always be cos θ, since this is just for horizontal motion
- For vertical motion, it would be sin θ
  - Always consider the horizontal and vertical components of the force
  - The component needed is the one that is **parallel to the displacement**



When the force is at an angle, only the component of the force in the direction of motion is considered for the work done

Worked Example

The diagram shows a barrel of weight 2.5×10^3 N on a frictionless slope inclined at 40° to the horizontal.



A force is applied to the barrel to move it up the slope at a constant speed. The force is parallel to the slope. What is the work done in moving the barrel a distance of 6.0 m up the slope? **A.** 7.2×10^3 J **B.** 2.5×10^4 J **C.** 1.1×10^4 J **D.** 9.6×10^3 J

YOUR NOTES
↓

ANSWER: D

STEP 1

WORK DONE EQUATION

$W = F \times d$

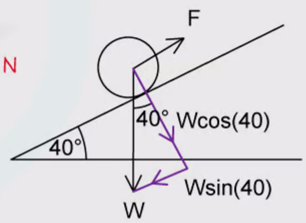
STEP 2

CALCULATE THE FORCE IN THE DIRECTION OF TRAVEL

THE FORCE NEEDED TO PUSH THE BARREL NEEDS TO OVERCOME THE COMPONENT OF THE BARREL'S WEIGHT. SINCE THE FORCE IS PARALLEL TO THE SLOPE, THE COMPONENT OF THE WEIGHT WE NEED IS THE ONE PARALLEL TO THE SLOPE.

$F = W\sin(40) = 2.5 \times 10^3 \times \sin(40) = 1607$ N

THIS IS THE FORCE IN THE SAME DIRECTION AS THE DISPLACEMENT



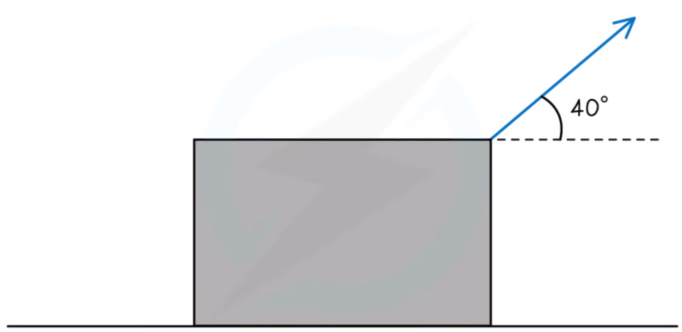
STEP 3

SUBSTITUTE F AND d INTO THE WORK DONE EQUATION

$W = 1607 \text{ N} \times 6.0 \text{ m} = 9.6 \times 10^3$ J

Worked Example

An 80 kg person pulls a 15 kg box along using a rope which is at 40° from the horizontal as shown below. The person is pulling with a force of 40 N and moves the box 20 m horizontally from its starting position against a constant friction force of 5.0 N. Calculate the work that has been done on the box in the direction of its motion.



Step 1: List the known quantities

- The angle between the rope and the horizontal, $\theta = 40°$
- The pulling force (along rope) = 40 N
- Horizontal distance moved by box, s = 20 m
- Frictional force = 5.0 N

YOUR NOTES
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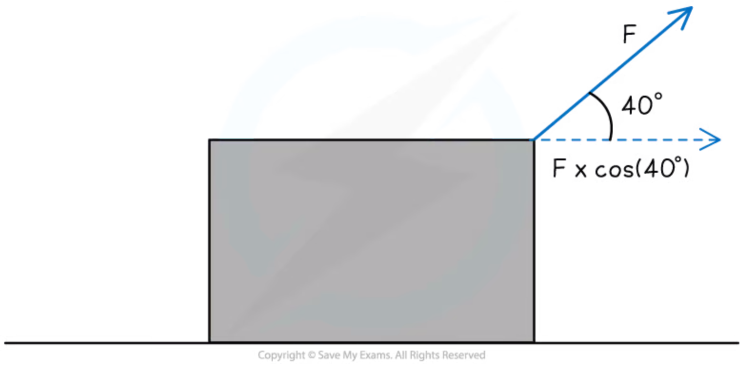
Step 2: Resolve the pulling force in the rope into its horizontal component

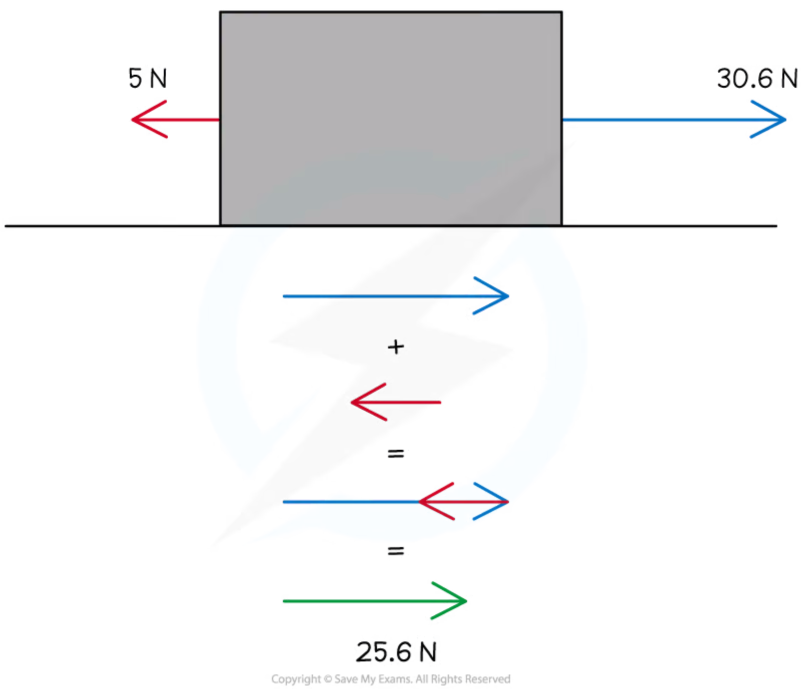


- The horizontal component of the pulling force is the only part of the pulling force aligned with the direction of work
- Hence, that is the component that is needed to continue solving this problem
- The horizontal component can be resolved from:

$$\cos(40°) \times 40 = 30.6 \text{ N to the right}$$

Step 3: Find the net force for the motion

- The net force can be found from the interaction between the horizontal component of the pulling force and the friction force:



$$30.6 - 5.0 = 25.6 \text{ N to the right}$$

Step 4: Use the equation for work

- Use the equation for work given the net force and distance moved in the horizontal

$$W = Fs \cos \theta$$

- The **cos** θ has already been accounted for so that the net force could be found when combining with friction
- Therefore:

$$W = F \times s$$

$$W = 25.6 \times 20 = \mathbf{512\ J}$$

Step 5: State the final answer

- The work done on this box in the horizontal direction is:

$$W = \mathbf{512\ J}$$

Exam Tip

Sometimes exam questions will include more values than you need to use in the solution - this is purposefully done to confuse you. For example, in the worked example above, the question supplies the mass of the person and the box, however, these quantities are not needed for the calculation.

YOUR NOTES

2.3.5 Power

YOUR NOTES
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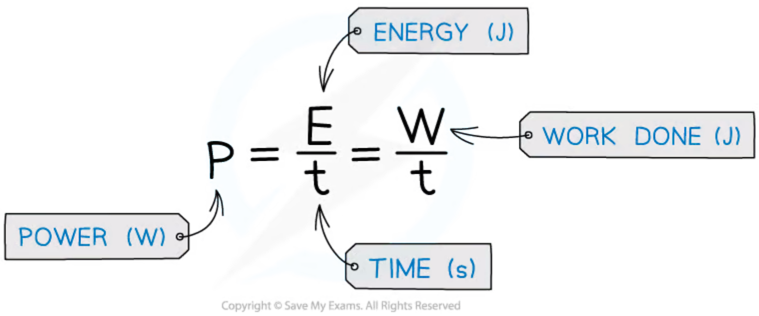
Power

- The power of a mechanical process is the **rate at which energy is transferred**
  - Since work done is equal to the energy transferred, power can also be defined as the rate of doing work or **the work done per unit time**
- Power can be calculated using the equation:



- Power is also used in electricity, with labels on lightbulbs which indicate their power, such as 60 W or 100 W
  - These indicate the amount of energy transferred by an electrical current rather than by a force doing work

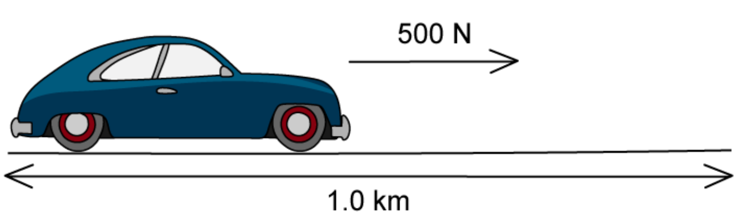
The Watt

- Power is measured in **Watts (W)**
- The Watt, *W*, is commonly used as the unit power (and radiant flux)
  - It is defined as **1 W = 1 J s^{-1}**
- The SI unit for energy is kg m^2 s^{-3}
- One watt is defined as:

A transfer of energy of 1 J in 1 s

Worked Example

A car engine exerts the following force for 1.0 km in 200 s.



Determine what is the average power developed by the engine.

YOUR NOTES
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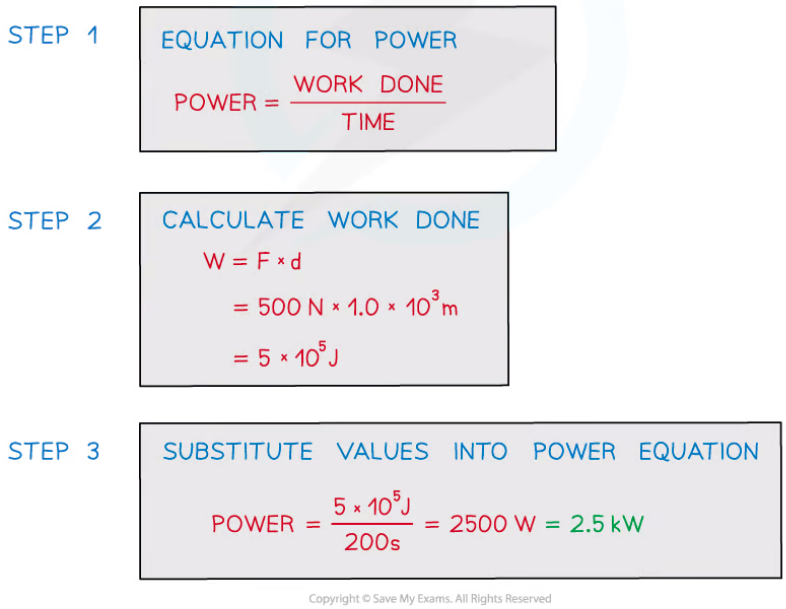
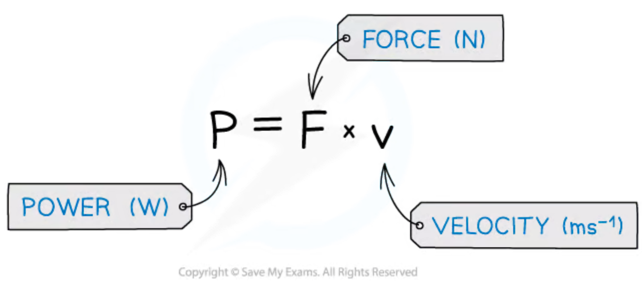


Power & Force

- Moving power is defined by the equation:



- This equation is only relevant where a **constant force** moves a body at **constant velocity**. Power is required in order to produce an acceleration
- The force must be applied in the **same** direction as the velocity

Derivation of *P* = *Fv*

- The derivation for this equation is shown below:

YOUR NOTES

Derivation of P = F × v

POWER IS THE RATE OF CHANGE OF WORK

$$\text{POWER} = \frac{W}{t}$$

WORK DONE = FORCE × DISTANCE

$$W = F \times d$$

AT CONSTANT VELOCITY, $d = v \times t$ THEREFORE

$$W = F \times v \times t$$

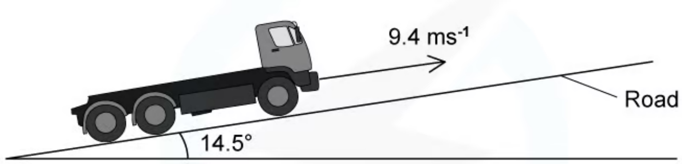
$$P = \frac{W}{t} = \frac{F \times v \times t}{t}$$

CANCELLING t

$$P = F \times v$$

Worked Example

A lorry moves up a road that is inclined at 14.5° to the horizontal.



The lorry has a mass of 3500 kg and is travelling at a constant speed of 9.4 m s^{-1}. The force due to air resistance is negligible. Calculate the useful power from the engine to move the lorry up the road.

YOUR NOTES
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STEP 1

EQUATION FOR POWER

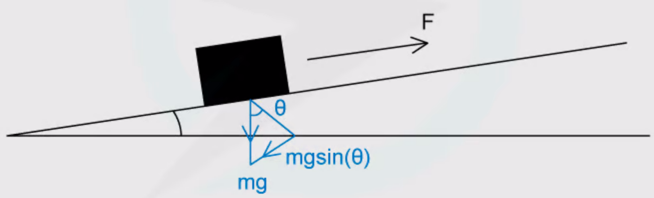
$P = F \times v$

STEP 2

CALCULATE THE FORCE

THE FORCE NEEDED TO MOVE THE LORRY UP THE ROAD IS THAT WHICH OVERCOMES THE COMPONENT OF ITS WEIGHT ACTING DOWN THE SLOPE

$F = mg\sin\theta = 3500 \times 9.81 \times \sin(14.5) = 8596.8\ \text{N}$



STEP 3

SUBSTITUTE INTO POWER EQUATION

$P = 8\,596.8 \times 9.4 = 80\,809.9\ \text{W} = 81\,000\ \text{W} = 81\ \text{kW}\ \ (2.s.f.)$

Exam Tip

The force represented in exam questions will often be a drag force. Whilst this is in the opposite direction to its velocity, remember the force needed to calculate the power is equal to (or above) this drag force to overcome it therefore you equate it to that value.

YOUR NOTES
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Principle of Conservation of Energy

- The Principle of Conservation of Energy states that:

  Energy cannot be created or destroyed, it can only be transferred from one form to another

- This means the total amount of energy in a closed system **remains constant**, although how much of **each form** there is **may change**

Types of Energy

FORM	WHAT IS IT?
KINETIC	THE ENERGY OF A MOVING OBJECT.
GRAVITATIONAL POTENTIAL	THE ENERGY SOMETHING GAINS WHEN YOU LIFT IT UP, AND WHICH IT LOSES WHEN IT FALLS.
ELASTIC	THE ENERGY OF A STRETCHED SPRING OR ELASTIC BAND.(SOMETIMES CALLED STRAIN ENERGY)
CHEMICAL	THE ENERGY CONTAINED IN A CHEMICAL SUBSTANCE.
NUCLEAR	THE ENERGY CONTAINED WITHIN THE NUCLEUS OF AN ATOM.
INTERNAL	THE ENERGY SOMETHING HAS DUE TO ITS TEMPERATURE (OR STATE). (SOMETIMES REFERRED TO AS THERMAL OR HEAT ENERGY)

YOUR NOTES
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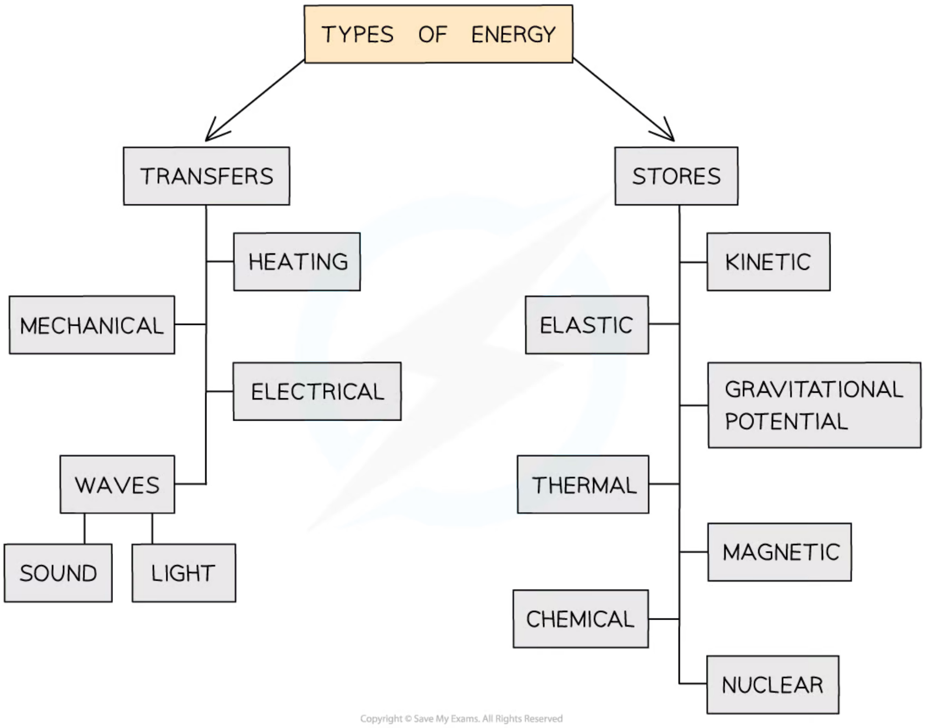

Energy types can be separated into transfers or stores

- The differences between **energy stores** and **energy transfers** can be confusing, so it is important to learn their differences and the different types of energy involved in each

Energy Stores

- **Energy stores** keep or store energy within one part of a system
- Kinetic energy store: the amount of energy in a kinetic energy store depends on the speed of the object
  - A runner with a high kinetic energy store in her muscles can run more quickly than a runner with a smaller kinetic energy store
  - A car with a high kinetic energy store within the engine can drive more quickly than a car with a smaller kinetic energy store
    - A racing car has a higher kinetic energy store than a tractor
- Gravitational potential energy store: the amount of energy stored in an object depending on its height
  - The higher an object, the more gravitational potential energy it has
    - When a diver stands on a board 5 m high he has more gravitational potential energy than when standing on a board 3 m high
- Magnetic energy store: magnets store magnetic energy until a magnetic material is present in its field
  - A piece of iron will be moved when it enters a magnetic field
  - The magnetic energy store is transferred to the kinetic energy of the iron
- Chemical energy store: energy is stored as chemical energy and a chemical reaction takes place to release and then transfer it
  - Chemical energy is stored in our body for use when we think and move

YOUR NOTES
⬇

  - Chemical energy is stored in a battery to be transferred to electrical energy
- Thermal energy store: all objects store thermal energy
  - An object that is hotter stores more thermal energy than an object that is colder
- Nuclear energy store: in a nuclear reactor, energy is stored as Uranium-235 until it is bombarded by neutrons and a huge amount of thermal energy is released

Energy Transfers

- **Energy transfers:** give or transfer energy to different parts of a system
- They act as a pathway around an energy system
  - Electrical transfer: when charge flows to produce an electric current
    - The current transfers the energy
  - Mechanical transfer: this occurs when a force is applied to move an object
    - This could be pushing a book across a desk
    - It could also be sound waves passing through a material causing the particles to move
  - Heating transfer: The internal energy of an object is determined by the temperature of the object
    - Energy is transferred from hotter to cooler areas
  - Waves transfer: When sound travels through a material it causes the particles to vibrate
    - Energy is transferred from an object that is moving/vibrating to generate the sound waves through other objects by the movement of particles
    - Light energy is transferred from the sun so we can see

Energy Dissipation

- When energy is transferred from one form to another, not all the energy will end up in the desired form (or place)
- **Dissipation** is used to describe ways in which energy is **wasted**
- Any energy not transferred to useful energy stores is wasted because it is lost to the surroundings
- These are commonly in the form of **thermal (heat), light, or sound** energy
- What counts as **wasted energy** depends on the system
- For example, in a **television**:

$$\text{electrical energy} \rightarrow \text{light energy} + \text{sound energy} + \text{thermal energy}$$

- Light and sound energy are useful energy transfers whereas thermal energy (from the heating up of wires) is wasted

YOUR NOTES
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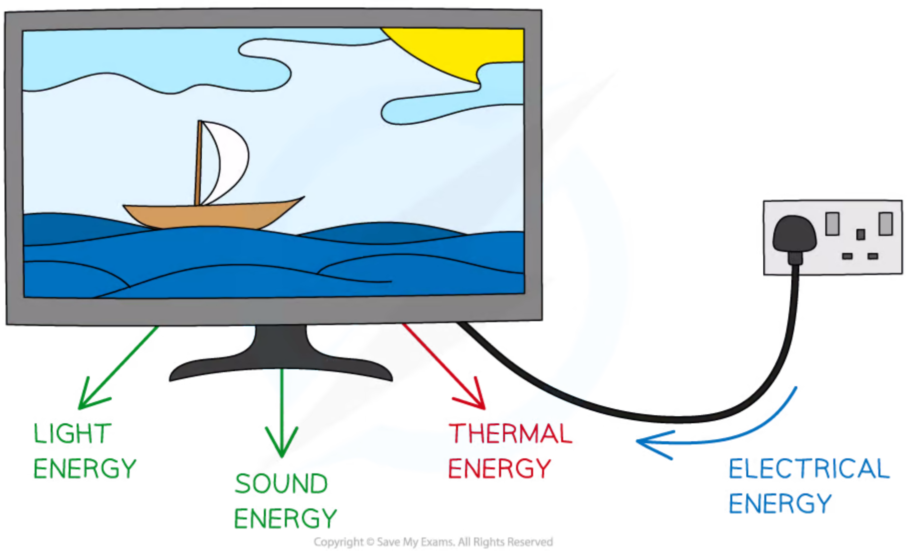


Useful and wasted energy conversions for a television

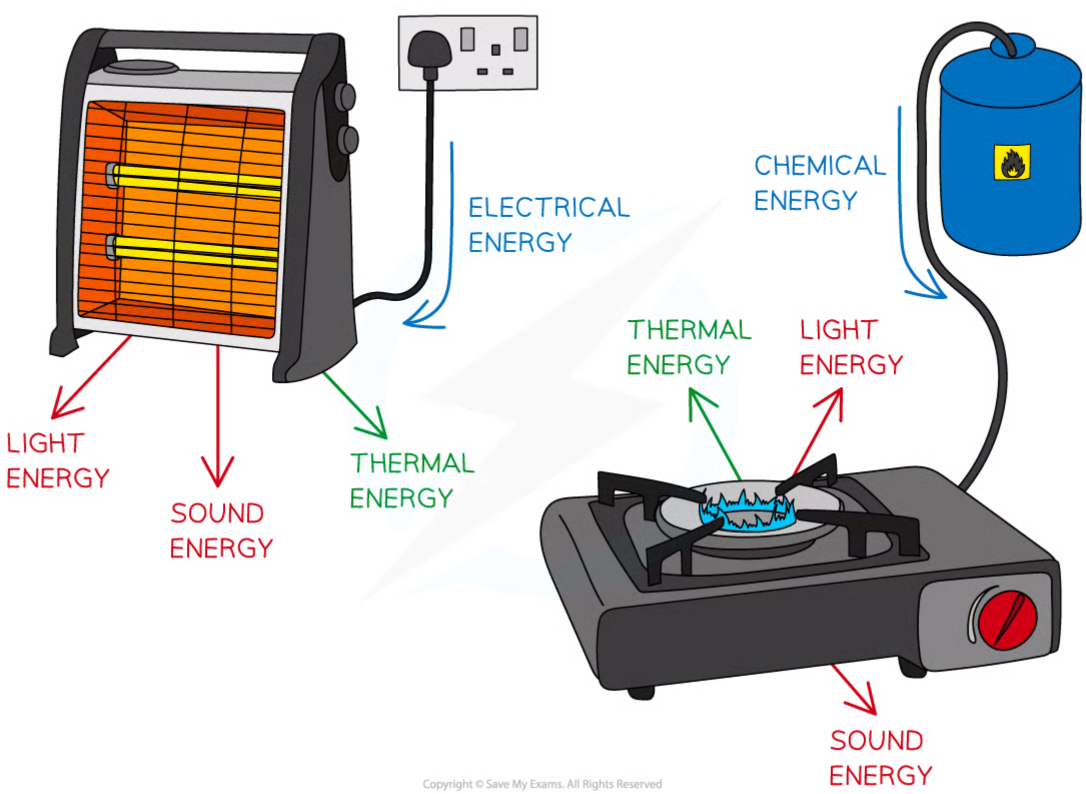
- The energy changes in an **electrical heater**:

electrical energy → thermal energy + sound energy + light energy

- In a **gas cooker**, the energy transfers are similar but the initial source of energy is different:

chemical energy → thermal energy + sound energy + light energy

- In both these cases, thermal energy is **useful**, whereas sound and light are not



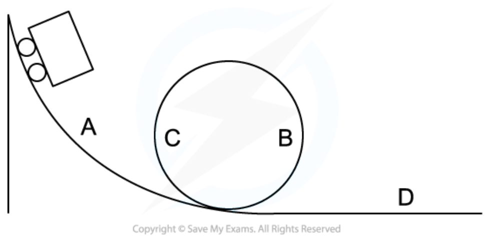
YOUR NOTES

Useful and wasted energy conversions in an electric heater and gas cooker

Worked Example

The diagram shows a rollercoaster going down a track.

The rollercoaster takes the path A → B → C → D.



Which statement is true about the energy changes that occur for the rollercoaster down this track?

A. KE - GPE - GPE - KE

B. KE - GPE - KE - GPE

C. GPE - KE - KE - GPE

D. GPE - KE - GPE - KE

ANSWER: D

- **At point A:**
  - The rollercoaster is raised above the ground, therefore it has **GPE**
  - As it travels down the track, **GPE** is converted to **KE** and the roller coaster speeds up
- **At point B:**
  - **KE** is converted to **GPE** as the rollercoaster rises up the loop
- **At point C:**
  - This **GPE** is converted back into **KE** as the rollercoaster travels back down the loop
- **At point D:**
  - The flat terrain means the rollercoaster only has **KE**

Applications of Energy Conservation

- In mechanical systems when energy is transferred between stores it is equivalent to the work done:
  - A falling object (in a vacuum, where no energy is not dissipated into the surroundings): **gravitational potential energy → kinetic energy**
  - Horizontal mass on a spring: **elastic potential energy → kinetic energy**
- We can also say energy is transferred between stores and transfers:
  - A battery connected to a bulb: **chemical energy → electrical energy → light energy** (if connected to a bulb)
  - A car: **chemical energy → mechanical energy → kinetic energy**

YOUR NOTES
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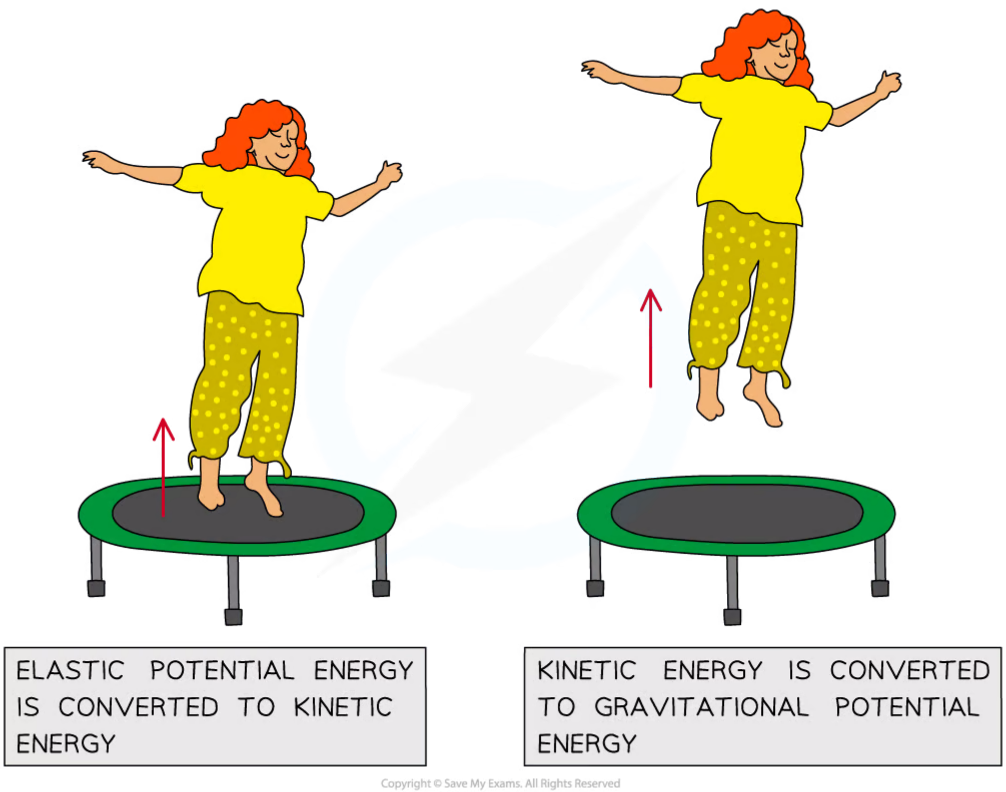


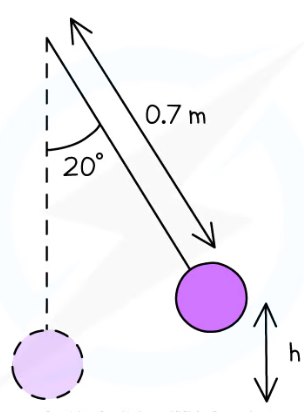
Energy transfers whilst jumping on a trampoline

- There may also be work done against resistive forces such as **friction**
- For example, if an object travels up a rough inclined surface, then

Loss in kinetic energy = Gain in gravitational potential energy + Work done against friction

Worked Example

A simple pendulum has a mass of 640 g and a length of 0.7 m. It is pulled out to an angle of 20° from the vertical. The pendulum is released. Assuming negligible air resistance, calculate the maximum speed of the pendulum bob as it passes through the vertical position.



YOUR NOTES
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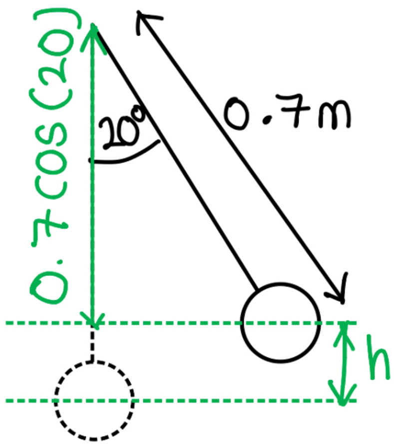
Step 1: Determine the energy transfers

When the pendulum bob is at its maximum height, it has gravitational potential energy.

As it is dropped to the vertical position, all this is converted into kinetic energy

Step 2: Calculate the height dropped



Height Δh dropped = 0.7 – 0.7cos(20)

Step 3: Calculate the loss in gravitational potential energy (GPE)

$$GPE = mg\Delta h = (640 \times 10^{-3}) \times 9.81 \times (0.7 - 0.7\cos(20))$$

$$GPE = 0.265 \text{ J}$$

Step 4: Determine the equation for the speed

The GPE = KE at the vertical position

$$KE = GPE = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2 \times GPE}{m}}$$

Step 5: Substitute in the values

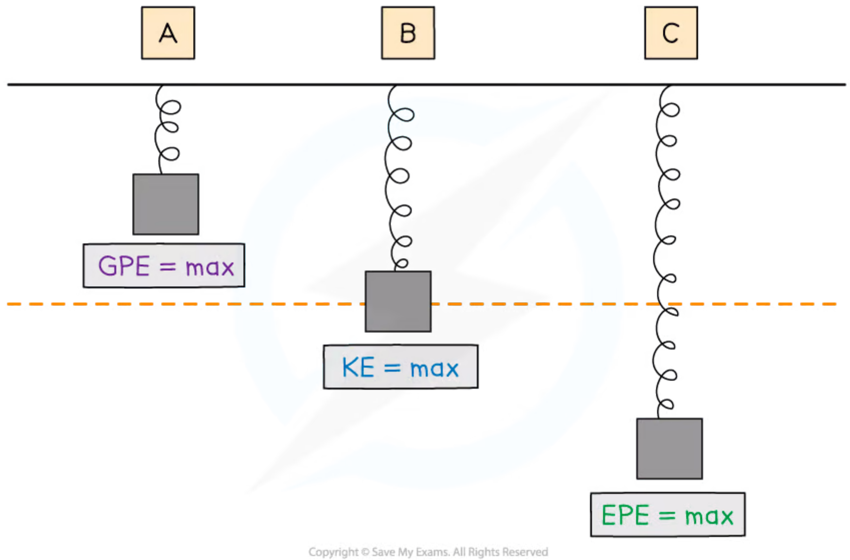
$$v = \sqrt{\frac{2 \times 0.265}{640 \times 10^{-3}}} = 0.91 \text{ m s}^{-1}$$

Spring Energy Conservation

- When a vertical spring is extended and contracted, its energy is converted into other forms
- Although the total energy of the spring will remain constant, it will have changing amounts of:

- **Elastic** potential energy (EPE)
- **Kinetic** energy (KE)
- **Gravitational** potential energy (GPE)

- When a vertical mass is hanging on a spring and it moves up and down, its energy will convert between the three in various amounts



- At position **A:**
  - The spring has some EPE since it is slightly compressed
  - Its KE is 0 since it is stationary
  - Its **GPE is at a maximum** because the mass is at its highest point
- At position **B:**
  - The spring has some EPE since it is slightly stretched
  - Its **KE is at a maximum** as it passes through the equilibrium position at its maximum speed
  - It has some GPE since the mass is still above the ground
- At position **C:**
  - The spring has its maximum EPE because it is at its maximum extension
  - Its KE is 0 since it is stationary
  - Its **GPE is at a minimum** because it is at its lowest point above the Earth's surface
- For a horizontal mass on a spring system, there is no gravitational potential energy to consider. The spring only converts between kinetic and elastic potential energy

YOUR NOTES
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2.3.7 Efficiency

YOUR NOTES
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Efficiency

- The efficiency of a system is a measure of how well energy is transferred in a system
- Efficiency is defined as:

The ratio of the useful power or energy transfer output from a system to its total power or energy transfer input

- If a system has **high** efficiency, this means most of the energy transferred is **useful**
- If a system has **low** efficiency, this means most of the energy transferred is **wasted**
- Determining which type of energy is useful or wasted depends on the system
    - When electrical energy is converted to light in a lightbulb, the light energy is **useful** and the heat energy produced is **wasted**
    - When electrical energy is converted to heat for a heater, the heat energy is **useful** and the sound energy produced is **wasted**

- Efficiency is represented as a percentage, and can be calculated using the equation:

$$\text{EFFICIENCY} = \frac{\text{USEFUL ENERGY OUTPUT}}{\text{TOTAL ENERGY INPUT}} \times 100\%$$

- The energy can be of any form e.g. gravitational potential energy, kinetic energy
- The efficiency equation can also be written in terms of power:

$$\text{EFFICIENCY} = \frac{\text{USEFUL POWER OUTPUT}}{\text{TOTAL POWER INPUT}} \times 100\%$$

- Where power is defined as the energy transferred per unit of time

$$\text{Power} = \frac{\text{Energy transferred}}{\text{Time}} = \frac{E}{t}$$

Worked Example

An electric motor has an efficiency of 35 %. It lifts a 7.2 kg load through a height of 5 m in 3 s. Calculate the power of the motor.

Step 1: Write down the efficiency equation

$$\text{Efficiency} = \frac{\text{Power output}}{\text{Power input}} \times 100$$

YOUR NOTES
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Step 2: Rearrange for the power input

$$\text{Power input} = \frac{\text{Power output} \times 100}{\text{Efficiency}}$$

Step 3: Calculate the power output

- The power output is equal to energy ÷ time
- The electric motor transferred electric energy into gravitational potential energy to lift the load

Gravitational potential energy = mgh = 7.2 × 9.81 × 5 = 353.16 J

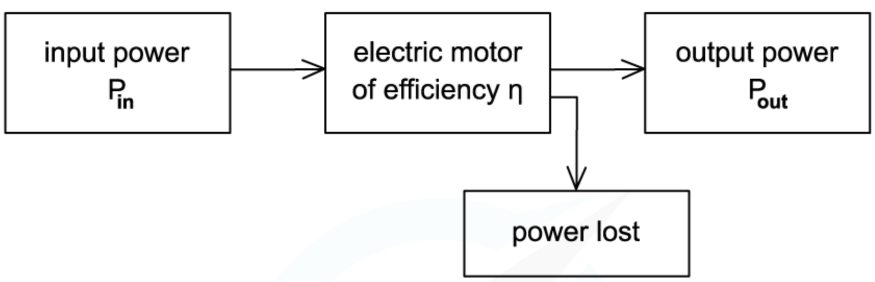
Power = 353.16 ÷ 3 = 117.72 W

Step 4: Substitute values into power input equation

$$\text{Power input} = \frac{117.72 \times 100}{35} = 336 \text{ W}$$

Worked Example

An electric motor has an input power P_{in}, useful output power P_{out} and efficiency η.



What is the output power P_{out} of the motor?

A. ηP_{in} **B.** $\frac{-\eta P_{lost}}{\eta - 1}$ **C.** ηP_{lost} **D.** $-\eta P_{lost}(\eta - 1)$

YOUR NOTES
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ANSWER: B

$$\text{EFFICIENCY} = \frac{\text{USEFUL POWER OUTPUT}}{\text{USEFUL POWER INTPUT}}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{lost}}$$

MULTIPLY BY $P_{out} + P_{lost}$ ON BOTH SIDES $\quad \eta(P_{out} + P_{lost}) = P_{out}$

EXPAND THE BRACKETS $\quad \eta P_{out} + \eta P_{lost} = P_{out}$

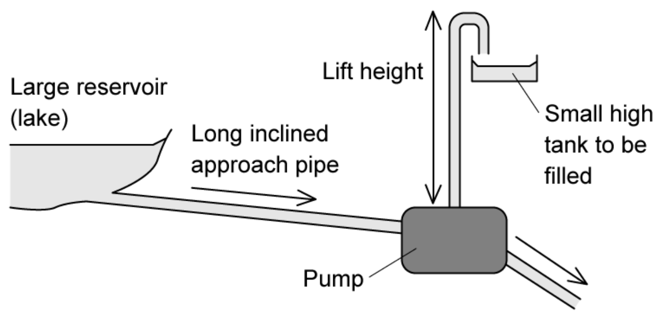
$-P_{out}$ FROM BOTH SIDES $\quad \eta P_{out} - P_{out} = -\eta P_{lost}$

TAKE POUT AS A FACTOR $\quad P_{out}(\eta - 1) = -\eta P_{lost}$

DIVIDE BY $\eta - 1$ $\quad P_{out} = \frac{-\eta P_{lost}}{\eta - 1}$

Worked Example

The diagram shows a pump called a hydraulic ram.



In one such pump, the long approach pipe holds 700 kg of water. A valve shuts when the speed of this water reaches 3.5 m s^{-1} and the kinetic energy of this water is used to lift a small quantity of water by height of 12m.

The efficiency of the pump is 20%.

Which mass of water could be lifted 12 m?

A. 6.2 kg **B.** 4.6 kg **C.** 7.3 kg **D.** 0.24 kg

YOUR NOTES
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ANSWER: C

THE KINETIC ENERGY OF THE WATER IS CONVVERTED TO GRAVITATIONAL POTENTIAL ENERGY WHEN LIFTED BY 12m

$$KE = GPE$$

$$\frac{1}{2}mv^2 = mgh$$

SINCE EFFICIENCY IS 20% ONLY 20% OF THE KINETIC ENERGY WILL BE CONVERTED.

$$0.2 \times \frac{1}{2}mv^2 = mgh$$

$$0.2 \times \frac{1}{2} \times 700 \times (3.5)^2 = m \times 9.81 \times 12$$

$$857.5 = m \times 117.72$$

$$\frac{857.5}{117.72} = m$$

$$m = 7.3 \text{ kg (2 s.f.)}$$

- The pump is what converts the water's **kinetic energy** into **gravitational potential energy**
- Since its efficiency is 20%, the kinetic energy can be multiplied by 0.2 since only 20% of the **kinetic energy** will be converted (**not** 20% of the gravitational potential energy)

Exam Tip

Efficiency can be in a ratio (between 0 and 1) or percentage format (between 0 and 100%)If the question asks for efficiency as a ratio, give your answer as a fraction or decimal.If the answer is required as a percentage, remember to multiply the ratio by 100 to convert it: if the ratio = 0.25, percentage = 0.25 × 100 = 25 %Remember that efficiency has **no units**

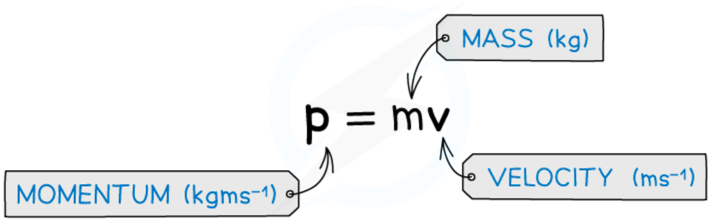
YOUR NOTES
⬇

2.4 Momentum & Impulse

2.4.1 Force & Momentum

Force & Momentum

- Linear momentum, *p*, is defined as the product of mass and velocity

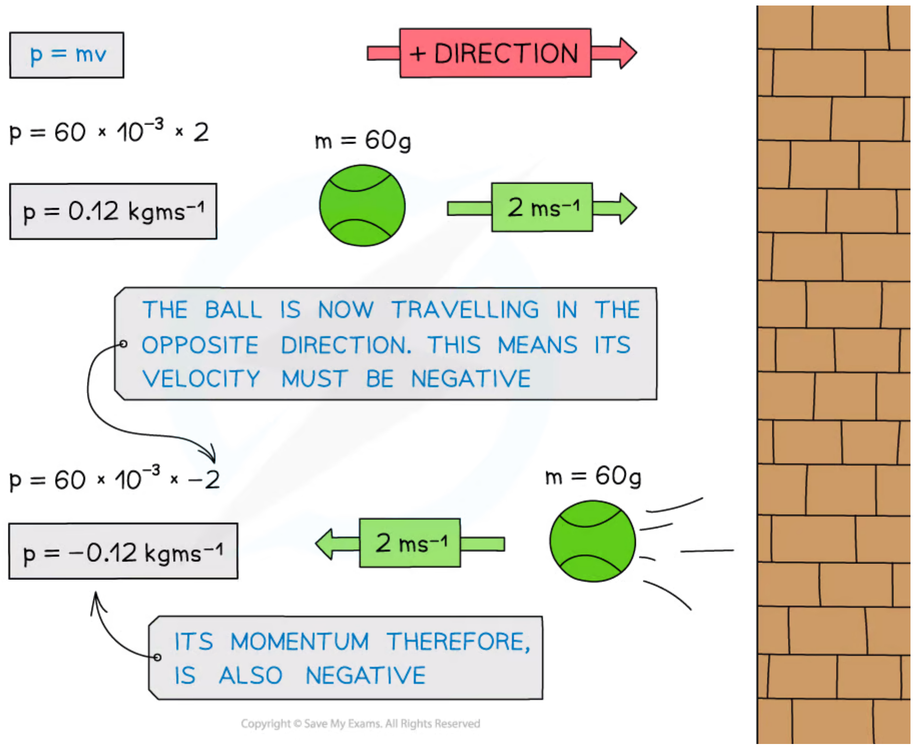


- Momentum is a **vector** quantity - it has both a **magnitude** and a **direction**
- This means it can have a negative or a positive value
  - If an object travelling to the **right** has **positive** momentum, an object travelling to the **left** (in the opposite direction) has a **negative** momentum
  - The negative or positive directions are **defined by the observer** on a case-by-case basis
- The SI unit for momentum is **kg m s^{-1}**

Direction of Momentum

- If a ball of mass 60 g travels at 2 m s^{-1}, it will have a momentum of 0.12 kg m s^{-1}
- If it then hits a wall and rebounds in the exact opposite direction, it will have a momentum of –0.12 kg m s^{-1}

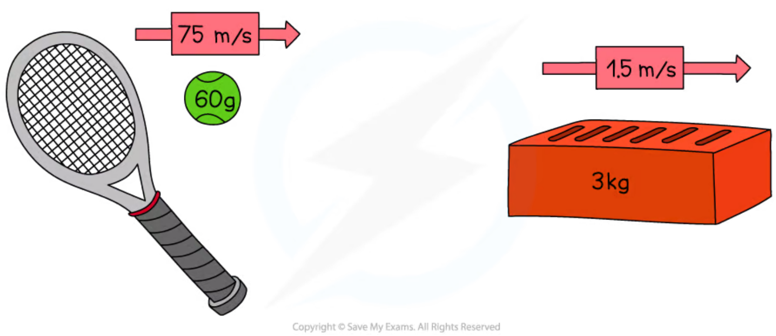
YOUR NOTES



When the ball is travelling in the opposite direction, its velocity is negative. Since momentum = mass × velocity, its momentum is also negative

Worked Example

Determine which object has the most momentum.



MOMENTUM = MASS × VELOCITY

MOMENTUM = 0.06 kg × 75 m/s = 4.5 kg m/s

MOMENTUM = MASS × VELOCITY

MOMENTUM = 3 kg × 1.5 m/s = 4.5 kg m/s

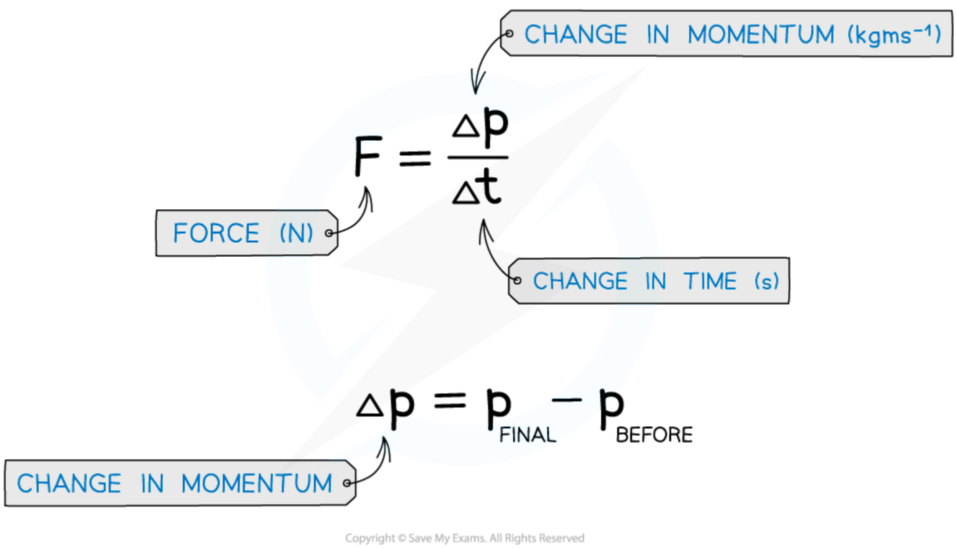
- Both the tennis ball and the brick have the **same momentum**
- Even though the brick is much **heavier** than the ball, the ball is travelling much **faster** than the brick

YOUR NOTES
↓

- This means that on impact, they would both exert a **similar force** (depending on the time it takes for each to come to rest)

Force & Momentum

- Force is defined as the **rate of change of momentum** on a body
- The change in momentum is defined as the final momentum minus the initial momentum
- These can be expressed as follows:



- It should be noted that the force in this situation is equivalent to Newton's second law:

$$F = m \times a$$

- Only when mass is constant
  - In situations where **mass** is **not constan**t, Newton's second law can **only** be considered to **assist descriptions** and not for calculations
- The force and momentum equation can be derived from Newton's Second law and the definition of acceleration

YOUR NOTES
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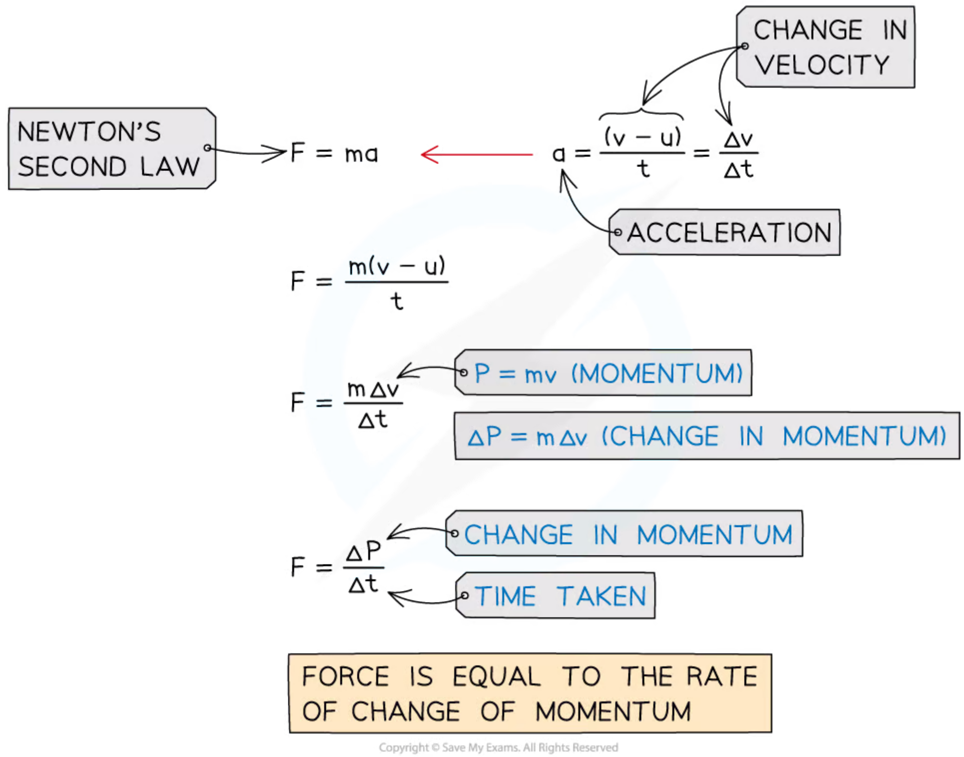
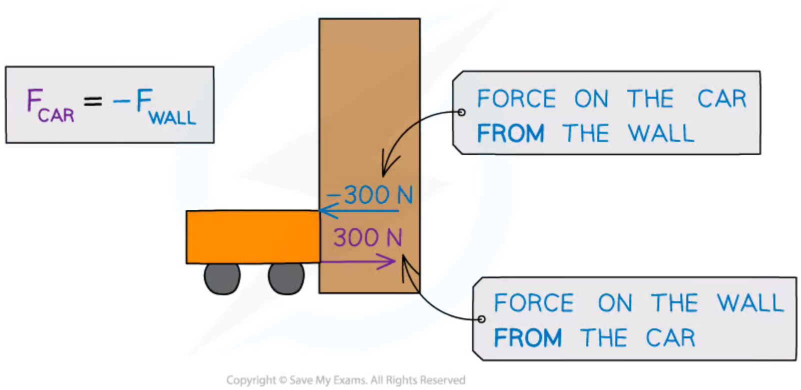


Direction of Forces

- Force and momentum are **vectors** so they can take either positive or negative values
- The force that is equal to the rate of change of momentum is still the **resultant force**
- A force on an object will be negative if it is directed in the opposite motion to its initial velocity
    - This means that the force is **produced by** the object it has collided with



The wall produces a force of -300N on the car and (due to Newton's Third Law) the car also produces a force of 300 N back onto the wall

YOUR NOTES
⬇

Worked Example

A car of mass 1500 kg hits a wall at an initial velocity of 15 m s^{-1}. It then rebounds off the wall at 5 m s^{-1} and comes to rest after 3.0 s. Calculate the average force experienced by the car.

YOUR NOTES ↓

STEP 1

FORCE IS EQUAL TO THE RATE OF CHANGE IN MOMENTUM

$$F = \frac{\Delta p}{\Delta t}$$

STEP 2

CHANGE IN MOMENTUM

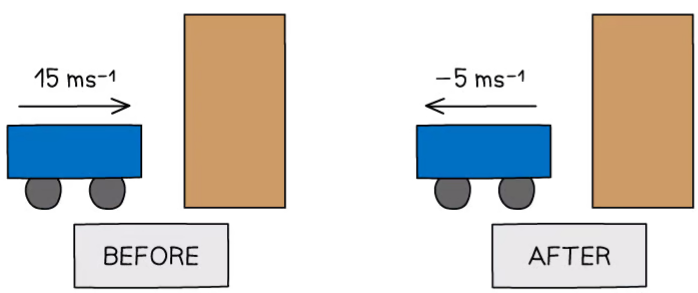
Δp = FINAL MOMENTUM − INITIAL MOMENTUM

STEP 3

INITIAL MOMENTUM

INITIAL MOMENTUM = MASS × INITIAL VELOCITY

$$P_i = m \times v_i$$
$$= 1500 \text{ kg} \times 15 \text{ ms}^{-1}$$
$$P_i = 22500 \text{ kgms}^{-1}$$



STEP 4

FINAL MOMENTUM

FINAL MOMENTUM = MASS × FINAL VELOCITY

$$P_f = m \times v_f$$
$$= 1500 \text{ kg} \times -5 \text{ ms}^{-1}$$
$$P_f = -7500 \text{ kgms}^{-1}$$

STEP 5

CALCULATE CHANGE IN MOMENTUM Δp

$$\Delta p = -7500 - 22500 = -30000 \text{ kgms}^{-1}$$

STEP 6

SUBSTITUTE THIS VALUE BACK INTO THE FORCE EQUATION

$$F = \frac{\Delta p}{\Delta t} = \frac{-30000}{3} = -10000\text{N}$$

YOUR NOTES

Exam Tip

The direction you consider positive is **your choice**, as long the signs of the numbers (positive or negative) are **consistent** with this throughout the questionIn an exam question, carefully consider what produces the force(s) acting. Look out for words such as '**from'** or '**acting on'** to determine this and don't be afraid to draw a force diagram to figure out what is going on.

2.4.2 Impulse

YOUR NOTES
⬇

Impulse

- The force and momentum equation can be rearranged to find the impulse
- Impulse, *I*, is equal to the **change in momentum:**

$$I = F\Delta t = \Delta p = mv - mu$$

- Where:
  - *I* = impulse (N s)
  - *F* = force (N)
  - *t* = time (s)
  - *p* = momentum (kg m s^{-1})
  - *m* = mass (kg)
  - *v* = final velocity (m s^{-1})
  - u = initial velocity (m s^{-1})

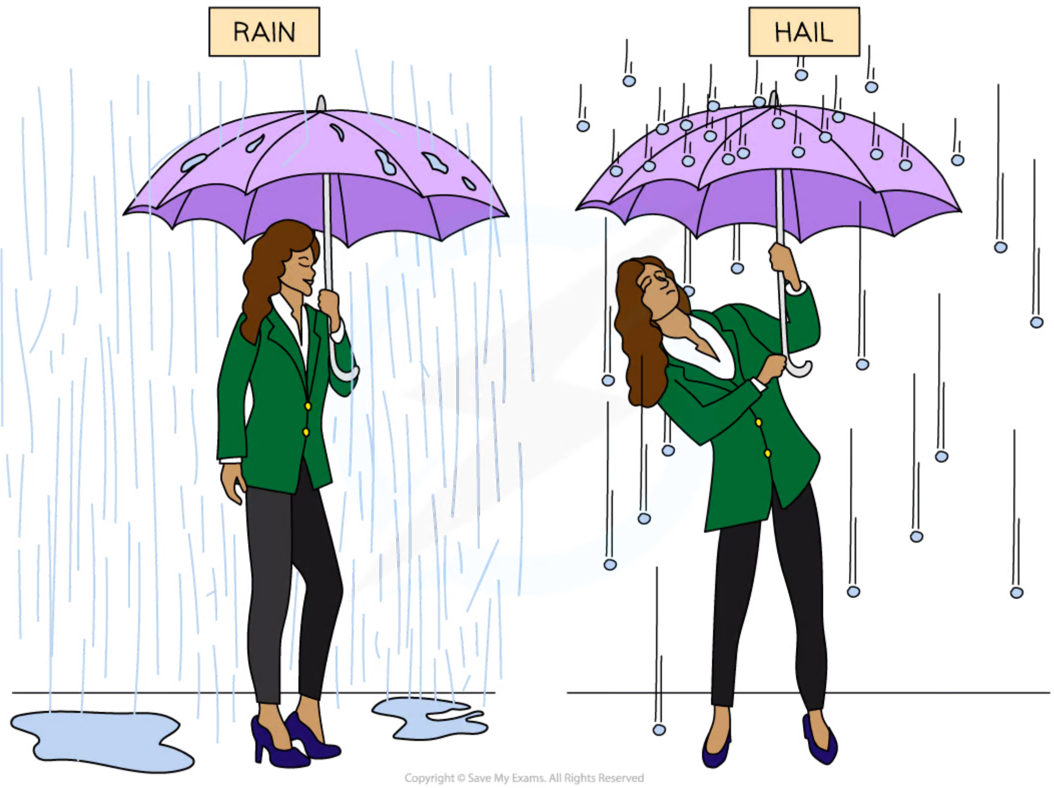
- This equation is only used when the force is **constant**
  - Since the impulse is proportional to the force, it is also a vector
  - The impulse is in the same direction as the force

- The unit of impulse is **N s**
- The impulse quantifies the effect of a force acting over a time interval
  - This means a **small force acting over a long time** has the same effect as a **large force acting over a short time**

Rain vs Hail

- An example in everyday life of impulse is the effect of rain on an umbrella, compared to hail (frozen water droplets)
  - When rain hits an umbrella, the water droplets tend to splatter and fall off it and there is only a very **small** change in momentum
  - However, hailstones have a **larger mass** and tend to bounce back off the umbrella, creating a **greater** change in momentum
  - Therefore, the impulse on an umbrella is **greater** in hail than in rain
  - This means that **more force** is required to hold an umbrella upright in hail compared to rain

YOUR NOTES
⬇



Since hailstones bounce back off an umbrella, compared to water droplets from rain, there is a greater impulse on an umbrella in hail than in rain

Worked Example

A 58 g tennis ball moving horizontally to the left at a speed of 30 m s^{-1} is struck by a tennis racket which returns the ball back to the right at 20 m s^{-1}.(a) Calculate the impulse delivered to the ball by the racket(b) State the direction of the impulse

Part (a)

Step 1: **Write the known quantities**

- Taking the initial direction of the ball as positive (the left)
- Initial velocity, u = 30 m s^{-1}
- Final velocity, v = –20 m s^{-1}
- Mass, m = 58 g = 58 × 10^{-3} kg

Step 2: Write down the impulse equation

$$\text{Impulse } I = \Delta p = m(v - u)$$

Step 3: Substitute in the values

$$I = (58 \times 10^{-3}) \times (-20 - 30) = -2.9 \text{ N s}$$

Part (b)

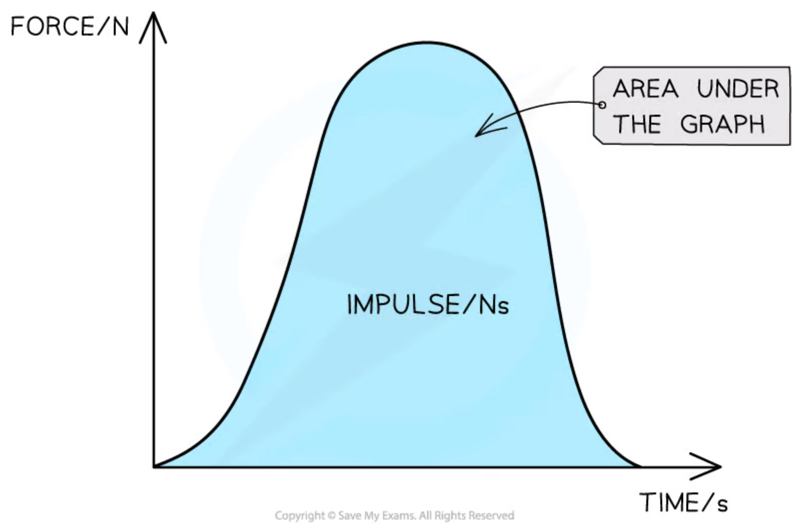
YOUR NOTES
⬇

Direction of the impulse

- Since the impulse is negative, it must be in the opposite direction to which the tennis ball was initial travelling (since the left is taken as positive)
- Therefore, the direction of the impulse is to the right

Impulse on a Force-Time Graph

- In real life, forces are often not constant and will vary over time
- If the force is plotted against time, **the impulse is equal to the area under the force-time graph**



When the force is not constant, the impulse is the area under a force-time graph

- This is because

$$\textbf{Impulse} = F\Delta t$$

- Where:
  - F = force (N)
  - Δt = change in time (s)

- The impulse is therefore equal whether there is
  - A small force over a long period of time
  - A large force over a small period of time

- The force-time graph may be a **curve** or a **straight line**
  - If the graph is a curve, the area can be found by counting the squares underneath
  - If the graph is made up of straight lines, split the graph into sections
  - The total area is the **sum of the areas** of each section

YOUR NOTES
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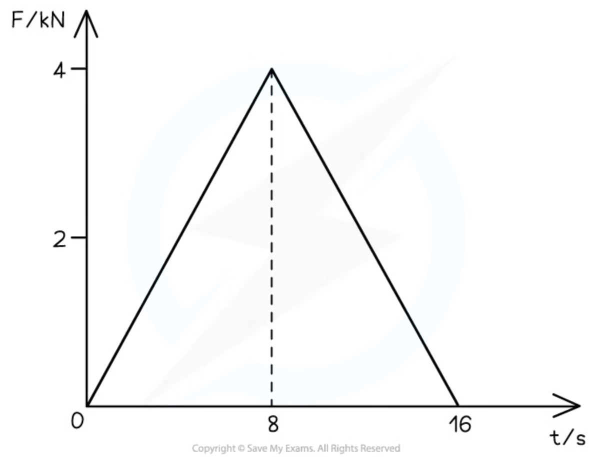
Worked Example

A ball of mass 3.0 kg, initially at rest, is acted on by a force *F* which varies with *t* as shown by the graph.



Calculate the magnitude of the velocity of the ball after 16 s.

Step 1: List the known quantities

- Mass, $m = 3.0$ kg
- Initial velocity, $u = 0$ m s^{-1} (since it is initially at rest)

Step 2: Calculate the impulse

- The impulse is the area under the graph
- The graph can be split up into two right-angled triangles with a base of 8 s and a height of 4 kN

$$\text{Area} = \left(\frac{1}{2} \times 8 \times (4 \times 10^3)\right) + \left(\frac{1}{2} \times (16 - 8) \times (4 \times 10^3)\right)$$

$$\text{Area} = \text{Impulse} = 32 \times 10^3 \text{ N s}$$

Step 3: Write the equation for impulse

$$\text{Impulse, } I = \Delta p = m(v - u)$$

Step 4: Substitute in the values

$$I = mv$$

YOUR NOTES

$$32 \times 10^3 = 3.0 \times v$$

$$v = (32 \times 10^3) \div 3.0$$

$$v = 10\,666 \text{ m s}^{-1} = 11 \text{ km s}^{-1}$$

Step 5: State the final answer

- The final magnitude of the velocity of the ball is:

$$\boldsymbol{v = 11 \text{ km s}^{-1}}$$

Exam Tip

Remember that if an object changes direction, then this must be reflected by the change in sign of the velocity. As long as the magnitude is correct, the final sign for the impulse doesn't matter as long as it is consistent with which way you have considered positive (and negative)For example, if the left is taken as positive and therefore the right as negative, an impulse of 20 N s to the right is equal to –20 N sSome maths tips for this section:**Rate of Change**

- ‘Rate of change’ describes how one variable changes with respect to another
- In maths, how fast something changes with **time** is represented as dividing by **Δ*t*** (e.g. acceleration is the rate of change in velocity)
- More specifically, **Δt** is used for finite and quantifiable changes such as the difference in time between two events

Areas

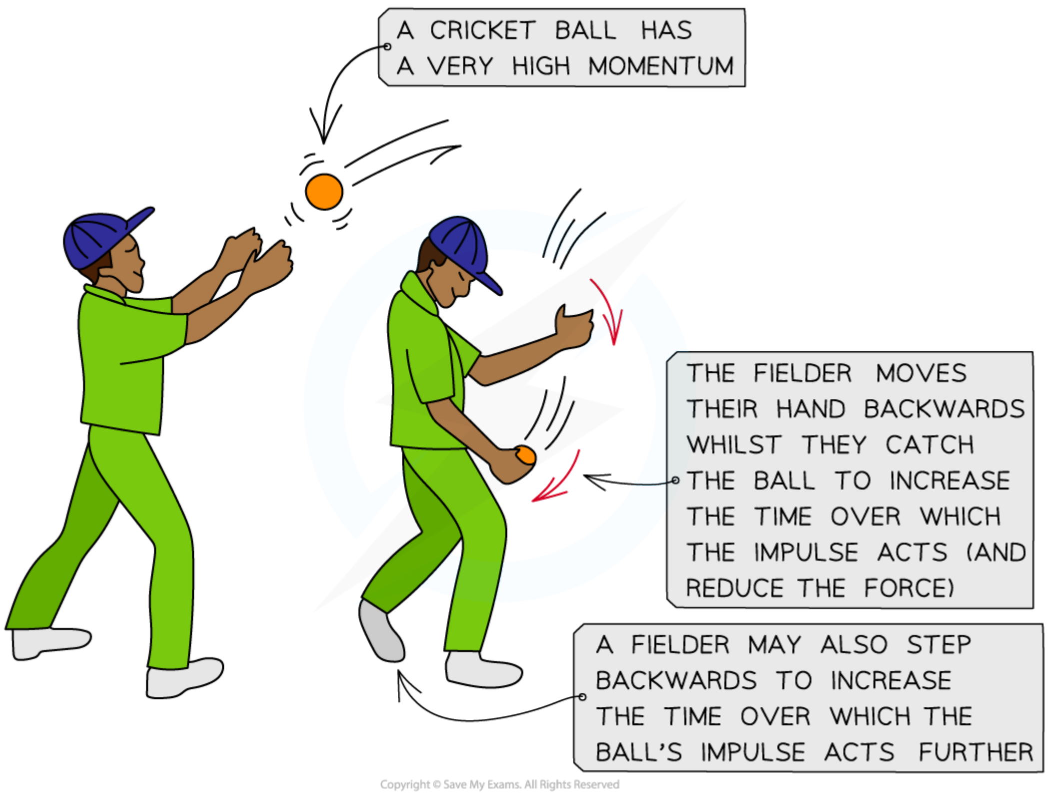
- The area under a graph may be split up into different shapes, so make sure you’re comfortable with calculating the area of squares, rectangles, right-angled triangles and trapeziums!

YOUR NOTES
⬇

Impulse in Context

In Sports

- For example, in cricket:
  - A cricket ball travels at very high speeds and therefore has a **high momentum**
  - When a fielder catches the ball, it exerts a force onto their hands
  - Stopping a ball with high momentum at once will cause a large force onto their hands
  - This is because a change in momentum (impulse) acts over a **short period of time** which creates a **large force** on the fielder's hands and could cause serious injury
  - A fielder moves their hands back when they catch the ball, which **increases the time** for its change in momentum to reduce
  - This means there will be **less force** exerted on the fielder's hands and therefore less chance of injury



A cricket fielder moves their hands backwards when catching a cricket ball to reduce the force it will exert on their hands

- In football:
  - Increasing the contact time is sometimes used to advantage, as the longer the contact time, the larger change in momentum
  - When kicking a football, after a strong kick the motion is followed through
  - This creates a **large impulse** and the ball then has a **higher velocity**

YOUR NOTES
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The follow through action of a football kick increases the change in momentum of the ball

Worked Example

A tennis ball hits two rackets with a change in momentum of 0.5 kg m s^{-1}. The first racket has a contact time of 2 s. The second racket has a contact time of 0.1 s. For the different contact times, which tennis racket experiences more force from the tennis ball?

YOUR NOTES
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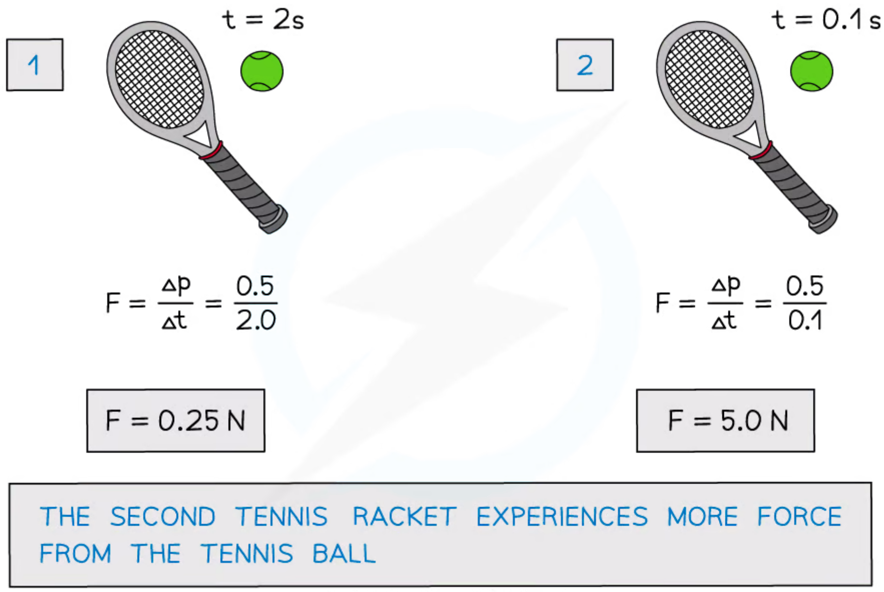

Momentum Conservation & Safety

- The force of an impact in a vehicle collision can be **decreased** by **increasing** the contact time over which the collision occurs
  - The contact time is the time in which the vehicle or the passenger is in contact with what it has collided with
- Vehicles have safety features such as **crumple zones**, **seat belts** and **airbags** to account for this
  - For a given force upon impact, these absorb the energy from the impact and **increase** the time over which the force takes place
  - This, in turn, **increases** the time taken for the change in momentum of the passenger and the vehicle to come to rest
  - The **increased** time **reduces** the force and risk of injury on a passenger

YOUR NOTES
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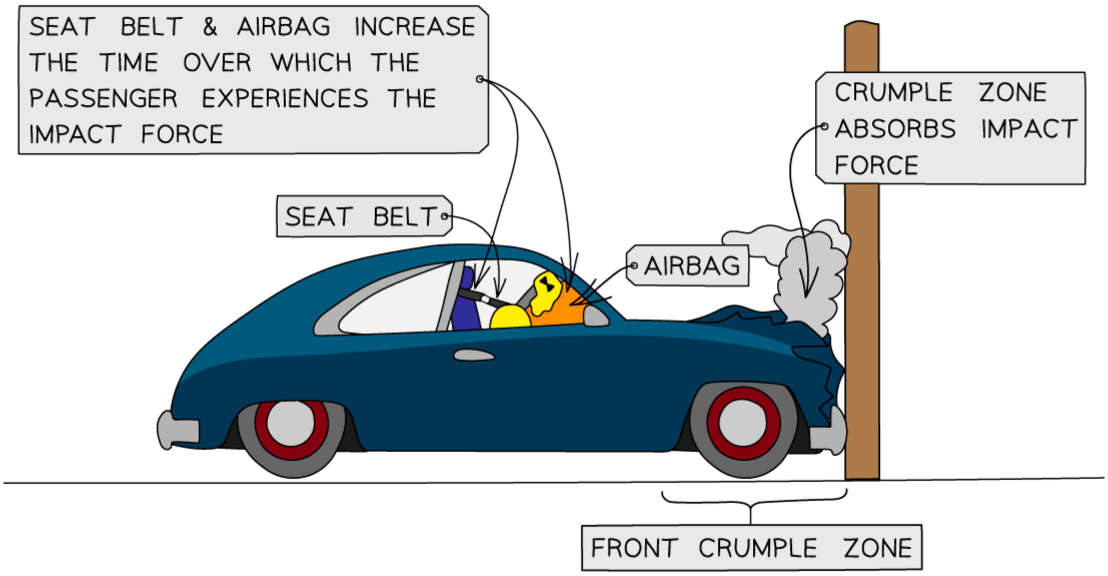

The seat belt, airbag and crumple zones help reduce the risk of injury on a passenger

Designing Safety Features

- Vehicle safety features are designed to absorb energy upon an impact by changing **shape**
- **Seat belts**
  - These are designed to stop a passenger from colliding with the interior of a vehicle by keeping them fixed to their seat in an abrupt stop
  - They are designed to stretch slightly to increase the time for the passenger's momentum to reach zero and reduce the force on them in a collision

- **Airbags**
  - These are deployed at the front on the dashboard and steering wheel when a collision occurs
  - They act as a soft cushion to prevent injury on the passenger when they are thrown forward upon impact

- **Crumple zones**
  - These are designed into the exterior of vehicles
  - They are at the front and back and are designed to crush or crumple in a controlled way in a collision
  - This is why vehicles after a collision look more heavily damaged than expected, even for relatively small collisions
  - The crumple zones increase the time over which the vehicle comes to rest, **lowering the impact force** on the passengers

- The effect of the increase in time and force can be shown on a force-time graph
  - For the same change in momentum, which depends on the mass and speed of a vehicle, the increase in contact time will result in a **decrease** in the maximum force exerted on the vehicle and passenger
  - This is demonstrated by a lower peak and wider base on a force-time graph

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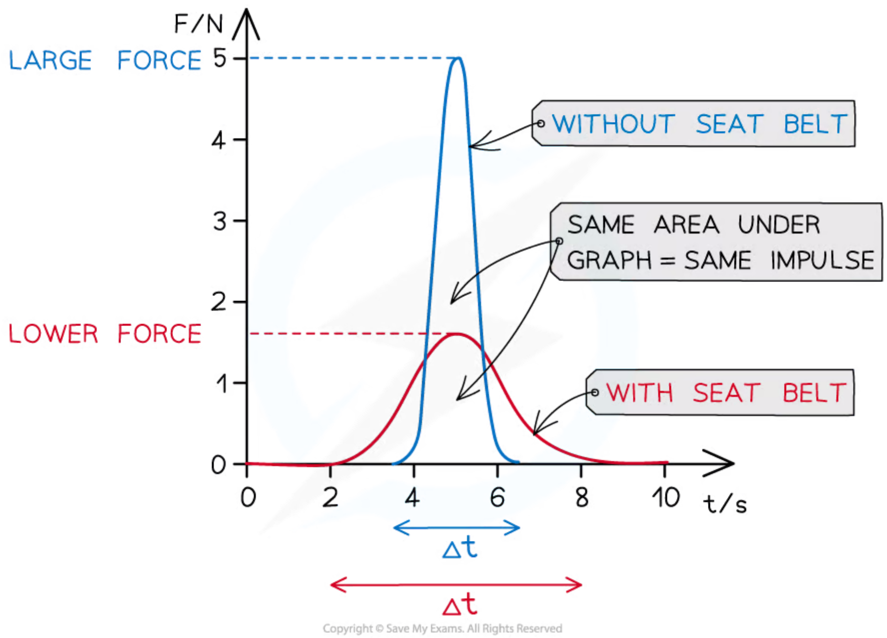

The increase in contact time Δt decreases the force for the same impulse

Worked Example

A 7 kg bowling ball has an impulse of 84 N s act upon it. The bowling ball was initially at rest and sitting on a flat frictionless surface.Predict the distance moved by the bowling ball in the first 3 seconds after the impulse was delivered.

Step 1: List the known quantities

- Mass of the bowling ball, m = 7 kg
- Impulse acting on the bowling ball, I = 84 N s
- Bowling ball initial velocity (at rest), u = 0 m s^{-1}
- Time of movement, t = 3 s

Step 2: Find the velocity caused by the impulse

- The velocity caused by the impulse can be found from the equation linking mass, velocity, and impulse:

$$\text{Impulse, } I = \Delta p = m(v - u)$$

Step 3: Rearrange and solve for *v*

$$I = m \times v \text{ (since } u = 0 \text{ m s}^{-1}\text{)}$$

$$v = I \div m = 84 \div 7 = 12 \text{ m s}^{-1}$$

Step 4: Find the distance travelled

- This can be found using time and velocity

$$v = d \div t$$

$$d = v \times t$$

$$d = 12 \times 3 = 36 \text{ m}$$

Step 5: State the final answer

- The bowling ball moved **36 m**

YOUR NOTES
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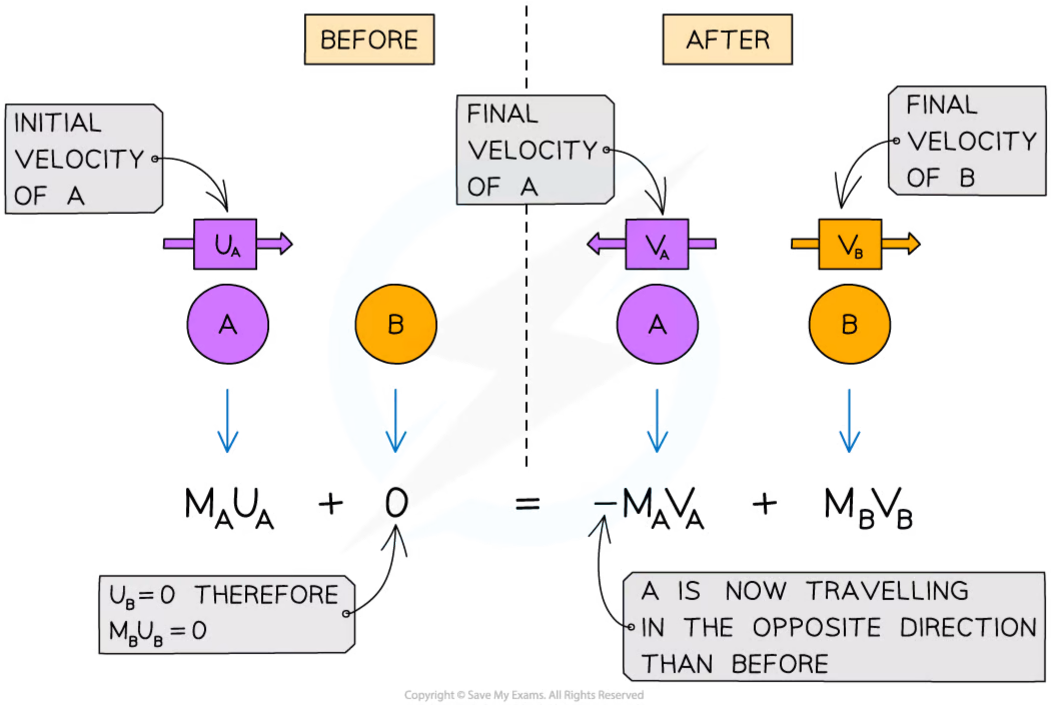
Conservation of Linear Momentum

- The principle of conservation of linear momentum states:

  The total momentum before a collision = the total momentum after a collision provided no external force acts

- Linear momentum is the momentum of an object that only moves in one dimension
- Momentum is a **vector** quantity
  - This means oppositely-directed vectors can cancel each other out resulting in a net momentum of zero
  - If after a collision an object starts to move in the opposite direction to which it was initially travelling, its velocity will now be **negative**
- Momentum, just like energy, is **always conserved**



The conservation of momentum for two objects A and B colliding then moving apart

External & Internal Forces

- **External forces** are forces that act on a system from outside of it e.g. friction and weight
- **Internal forces** are forces exchanged by the particles in the system e.g. tension in a string
- Forces that are internal or external will depend on the system itself
- For example, in a mass-spring system:
  - The internal force is the spring force
  - The external force is the weight

YOUR NOTES
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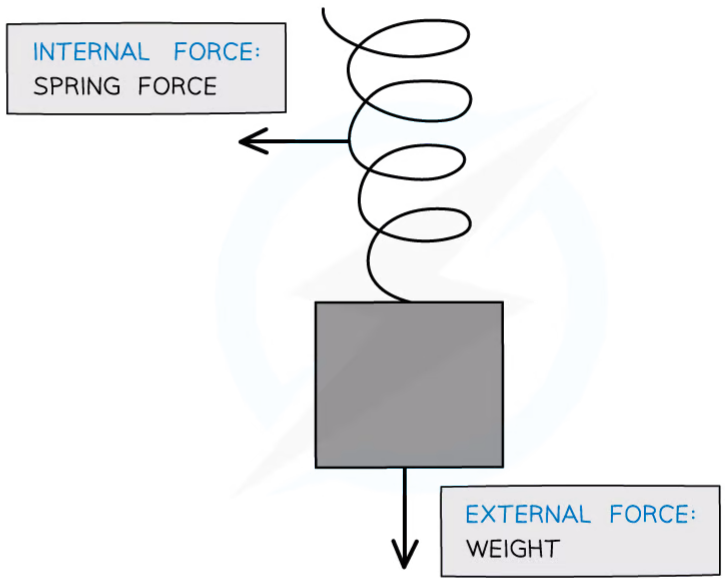

Internal and external forces on a mass on a spring

- Systems with no external forces may be described as '**closed**' or '**isolated**'
  - These are keywords that refer to a system that is not affected by external forces
- For example, a swimmer diving from a boat:
  - The diver will move **forwards**, and, to conserve momentum, the boat will move **backwards**
  - This is because the momentum beforehand was zero and no **external forces** were present to affect the motion of the diver or the boat

Worked Example

Trolley **A** of mass 0.80 kg collides head-on with stationary trolley **B** whilst travelling at

3.0 m s^{-1}. Trolley **B** has twice the mass of trolley **A**. On impact, the trolleys stick together.

Using the conversation of momentum, calculate the common velocity of both trolleys after the collision.

YOUR NOTES
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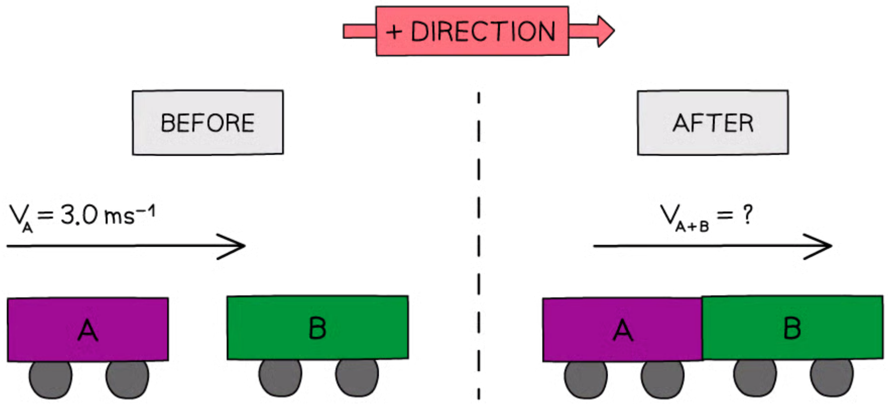

MOMENTUM BEFORE $= (M_A \times V_A) + (M_B \times V_B)$

$= (0.8\ kg \times 3.0\ ms^{-1}) + 0$

$= 2.4\ kgms^{-1}$

SINCE TROLLEY B IS STATIONARY, V = 0 THEREFORE ITS MOMENTUM IS 0

MOMENTUM AFTER $= (M_A + M_B) \times V_{A+B}$

$= (0.8\ kg + 1.60\ kg) \times V_{A+B}$

$= 2.4\ kg \times V_{A+B}$

TROLLEY B HAS TWICE THE MASS OF TROLLEY A

THE PRINCIPLE OF CONVERSATION OF MOMENTUM STATES THAT **THE TOTAL MOMENTUM OF A SYSTEM REMAINS CONSTANT PROVIDED NO EXTERNAL FORCE ACTS ON IT**

→ MOMENTUM BEFORE = MOMENTUM AFTER

$2.4\ kgms^{-1} = 2.4\ kg \times V_{A+B}$

$V_{A+B} = \dfrac{2.4\ kgms^{-1}}{2.4\ kg}$ ← REARRANGING FOR V_{A+B}

$V_{A+B} = 1.0\ ms^{-1}$

YOUR NOTES
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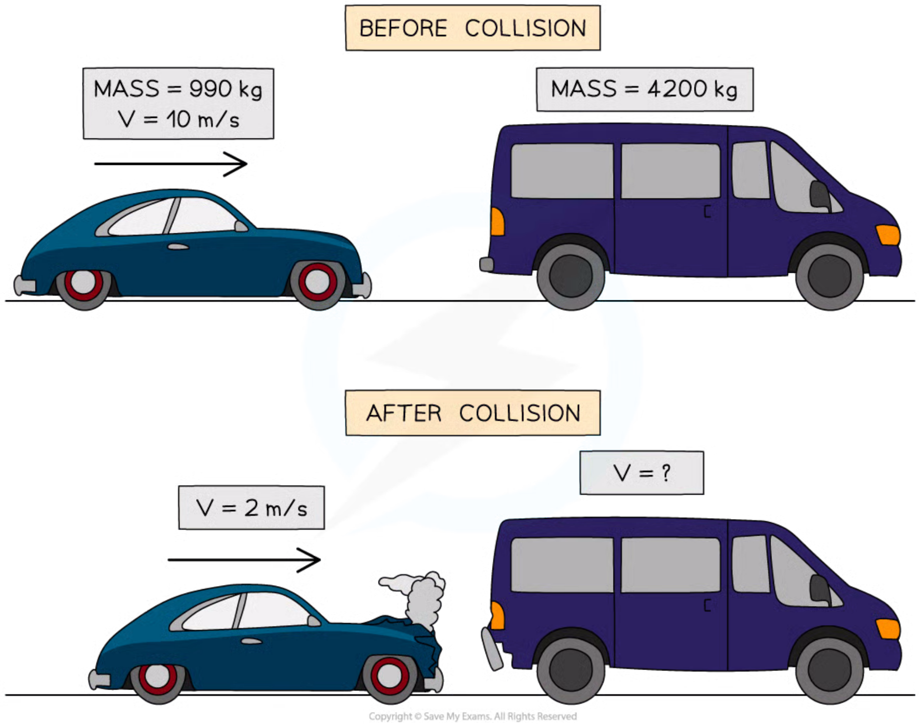
Worked Example

The diagram shows a car and a van, just before and just after the car collided with the van, which is initially at rest.



Use the idea of conservation of momentum to calculate the velocity of the van when it is pushed forward by the collision.

YOUR NOTES
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Step 1: State the principle of conversation of momentum

- In a closed system, the total momentum before an event is equal to the total momentum after the event

Step 2: Calculate total momentum before the collision

$$p = mv$$

- Momentum of the car:

$$p = 990 \times 10 = 9900 \text{ kg m/s}$$

- Momentum of the van:

The van is at rest, therefore v = 0 m/s and p = 0 kg m/s

- Total momentum before:

$$p_{before} = 9900 + 0 = 9900 \text{ kg m/s}$$

Step 3: Calculate the momentum after the collision

- Momentum of the car:

$$p = 990 \times 2 = 1980 \text{ kg m/s}$$

- Momentum of the van:

$$p = 4200 \times v$$

- Total momentum after

$$p_{after} = 1980 + 4200v \text{ kg m/s}$$

Step 4: Rearrange the conversation of momentum equation for the velocity of the van

$$p_{before} = p_{after}$$

$$9900 = 1980 + 4200v$$

$$9900 - 1980 = 4200v$$

$$v = \frac{9900 - 1980}{4200} = 1.9 \text{ m/s}$$

Exam Tip

If it is not given in the question already, drawing a diagram of before and after helps keep track of all the masses and velocities (and directions) in the conversation of momentum questions.

YOUR NOTES
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YOUR NOTES
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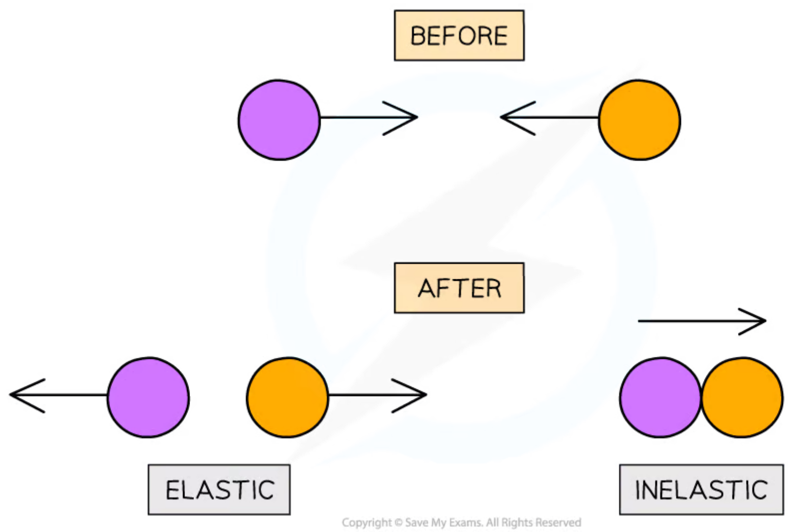
Collisions & Explosions

- In both collisions and explosions, **momentum is always conserved**
  - However, **kinetic energy** might not always be
- A collision (or explosion) is either:
  - **Elastic** – if the kinetic energy **is** conserved
  - **Inelastic** – if the kinetic energy is **not** conserved
- Collisions are when objects strike against each other
  - **Elastic** collisions are commonly those where objects colliding do not stick together and then **move in opposite directions**
  - **Inelastic** collisions are commonly those where objects collide and **stick together** after the collision



Elastic collisions are where two objects move in opposite directions. Inelastic collisions are where two objects stick together

- An explosion is commonly to do with **recoil**
  - For example, a gun recoiling after shooting a bullet or an unstable nucleus emitting an alpha particle and a daughter nucleus
- To find out whether a collision is elastic or inelastic, **compare the kinetic energy before and after the collision**
- The equation for kinetic energy is:

YOUR NOTES
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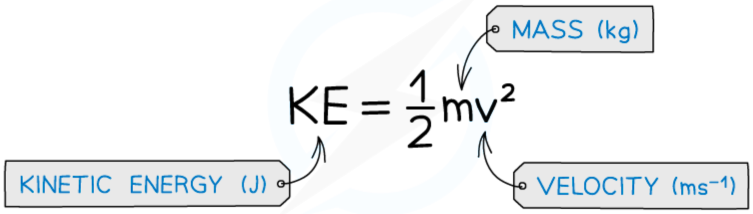

Worked Example

Two similar spheres, each of mass *m* and velocity *v* are travelling towards each other. The spheres have a head-on elastic collision. What is the total kinetic energy after the impact?

A. $\frac{1}{2}mv^2$ **B.** 0 **C.** mv^2 **D.** 2mv

ANSWER: C

IN AN ELASTIC COLLISION, KINETIC ENERGY IS CONSERVED.

THIS MEANS KINETIC ENERGY BEFORE = KINETIC ENERGY AFTER.

KINETIC ENERGY BEFORE = $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$.

IN AN ELASTIC COLLISION, KINETIC ENERGY AFTER WILL ALSO EQUAL mv^2.

Worked Example

Trolley **A** of mass 0.80 kg collides head-on with stationary trolley **B** at speed 3.0 m s^{-1}. Trolley **B** has twice the mass of trolley **A**. The trolleys stick together and travel at a velocity of 1.0 m s^{-1}. Determine whether this is an elastic or inelastic collision.

YOUR NOTES
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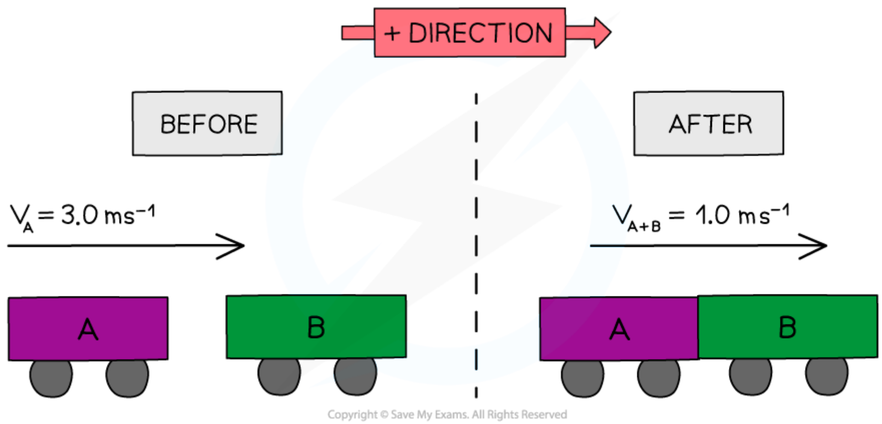


KINETIC ENERGY = 1/2 mv^2

KINETIC ENERGY BEFORE

$$= \frac{1}{2} \times M_A \times (V_A)^2 + \frac{1}{2} \times M_B \times (V_B^2)$$

$$= \frac{1}{2} \times 0.8 \times (3.0)^2 + 0$$

$V_B = 0$

$$= 3.6\ J$$

KINETIC ENERGY AFTER

$$= \frac{1}{2} \times M_{A+B} \times (V_{A+B})^2$$

$$= \frac{1}{2} \times 2.4 \times (1.0)^2$$

$$= 1.2\ J$$

THIS IS AN INELASTIC COLLISION SINCE KINETIC ENERGY IS **NOT** CONSERVED

Exam Tip

If an object is stationary or at rest, its velocity equals **0**, therefore, the momentum and kinetic energy are also equal to 0When a collision occurs in which two objects are stuck together, treat the final object as a single object with a mass equal to the **sum** of the two individual objectsDespite velocity being a vector, kinetic energy is a scalar quantity and therefore will **never** include a minus sign - this is because in the kinetic energy formula, mass is scalar and the v^2 will always give a positive value whether its a negative or positive velocity

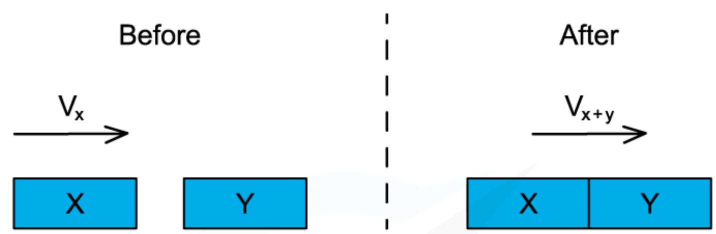
Applying Conservation of Momentum

YOUR NOTES ↓

- The principle of conservation of momentum can be used to solve various types of problems for isolated systems such as problems involving:
  - Collisions
  - Fluid jets
  - Conveyor belts
  - Explosions & many more
- Kinetic energy can also be used to check if a collision is elastic or inelastic
  - In elastic collisions, momentum and kinetic energy are conserved
  - In inelastic collisions, momentum is conserved, but kinetic energy is not
  - Inelastic includes explosions

Worked Example

Two trolleys **X** and **Y** are of equal mass. Trolley **X** moves towards trolley **Y** which is initially stationary. After the collision, the trolleys join and move off together.Show that this collision is inelastic.



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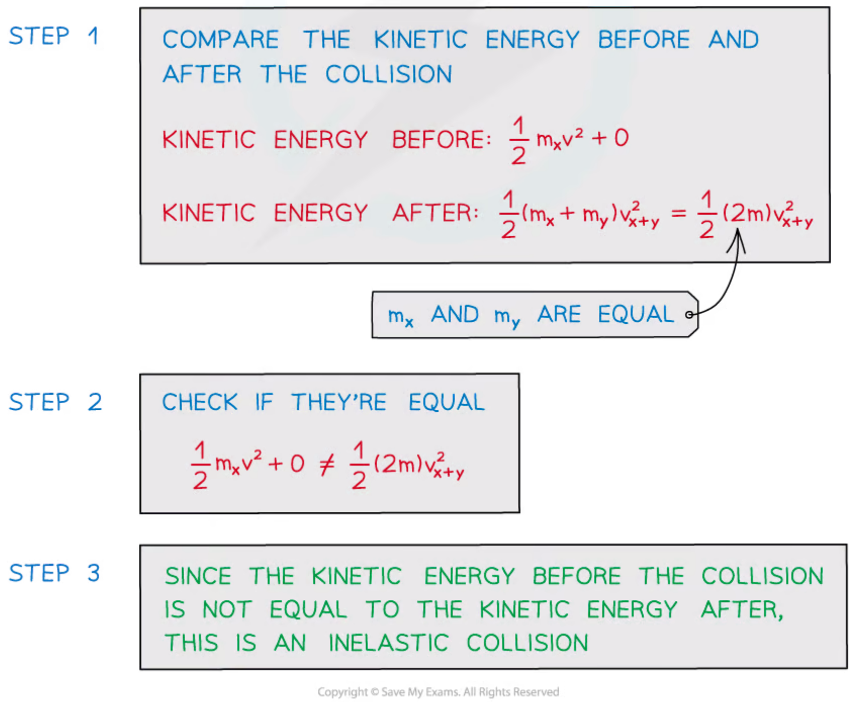
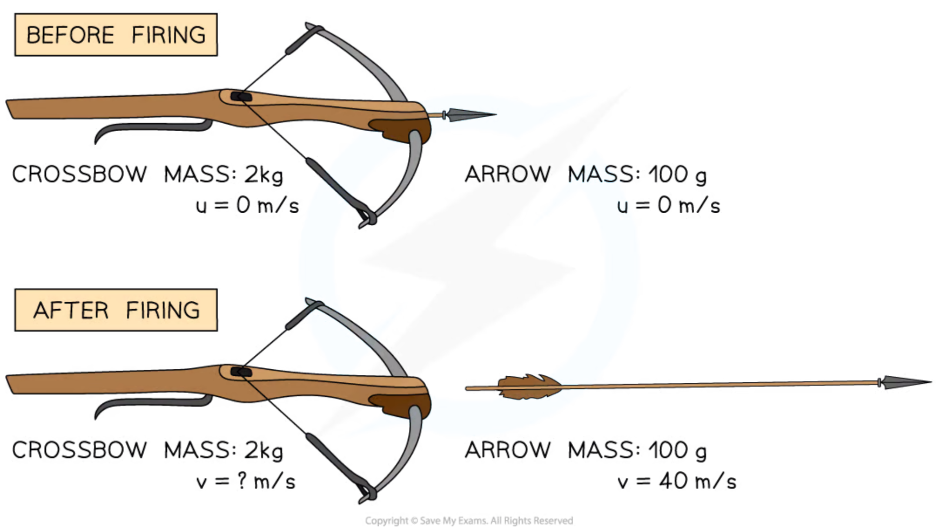


Worked Example

A 2 kg crossbow is fired and a 100 g arrow is fired horizontally. The arrow is released from the crossbow at 40 m s^{-1}. Determine the magnitude of the recoil velocity of the crossbow.



Step 1: List the known quantities

- Mass of the crossbow: 2 kg
- Mass of the arrow: 100 g = 0.1 kg
- Speed of arrow during release: 40 m s^{-1}

YOUR NOTES
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Step 2: Determine the momentum before release

- Since before release neither the arrow nor the crossbow was moving, their momentum together is **0 kg m s^{-1}**

Step 3: Determine the momentum of the arrow

- The momentum of the arrow can be found from the equation:

$$\boldsymbol{p = m \times v}$$

$$p_{arrow} = 0.1 \times 40 = 4 \text{ kg m s}^{-1}$$

Step 4: Determine the recoil velocity of the crossbow

- The arrow has a momentum of 4 kg m s^{-1} and the system had a total momentum of 0 kg m s^{-1} before the collision
- Therefore, the crossbow must have a momentum of 4 kg m s^{-1} in the opposite direction to the arrow
- Therefore:

$$p_{crossbow} = m \times v$$

$$p_{crossbow} \div m = v$$

$$v = 4 \div 2 = 2 \text{ m/s in opposite direction to arrow}$$

Step 5: State the final answer

- The magnitude of the recoil velocity of the crossbow is **2 m s^{-1}**.