1.5 Further Proof & Reasoning

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.5 Further Proof & Reasoning
Difficulty	Very Hard

Time allowed: 90

Score: /67

Percentage: /100

Question 1

Prove that there are no real values of k such that the equation $kx(1-x) = 3x^2 - 1$ has no real solutions.

[4 marks]

Question 2

Prove that $2^{n+2} + 3^{3n}$ is divisible by 5 for $n \in \mathbb{Z}$, $n \geq 0$.

[6 marks]

Question 3

Prove that there are an infinite number of prime numbers.

[4 marks]

Question 4

523 and 541 are prime numbers.

Prove by exhaustion that these are consecutive prime numbers.

[3 marks]

Question 5

Three of the four statements below are false.

Eliminate the false statements by providing a counter example and thus deduce the true statement.

(i)

$(x-1)^2 \neq (x+1)^2, \ x \in \mathbb{R}$.

(ii)

Every $(4n)$th triangular number is even, $n \in \mathbb{N}$.

(iii)

$2 \ln x > \ln 2x, \ x \in \mathbb{R}, \ x > 0$.

(iv)

The product of any two distinct positive integers is greater than their sum.

[3 marks]

Question 6

Prove that the equation $5x^4 + 15x^3 - 20x^2 - 4 = 0$ has no integer solutions.

[5 marks]

Question 7a

The function $f(n)$ is given as $f(n) = n^3 + n^2 + 17$ where n is an integer.

(a)
Find $f(1)$, $f(2)$ and $f(3)$.

[2 marks]

Question 7b

(b)
Prove that $f(n)$ is not prime for all values of n.

[2 marks]

Question 8a

(a)

Write down a, b, c and d from smallest to largest, given $a, b, c, d \in \mathbb{R}$ and $c > d$, $a < d$ and $a > b$.

[2 marks]

Question 8b

(b)

Write down p, q, r and s from smallest to largest, given $p, q, r, s \in \mathbb{R}$ and

$$p > q$$
$$r - s < q - p$$
$$p + q = r + s.$$

[3 marks]

Question 8c

(c)

Prove $\frac{x}{1+x} < \frac{x}{1+y}$, $x, y \in \mathbb{R}$, given $0 \leq x \leq y$.

[3 marks]

Question 9a

(a)

Show that the derivative of $y = xe^{-x}$ is

$$\frac{dy}{dx} = e^{-x}(1 - x).$$

[3 marks]

Question 9b

(b)

Prove, by mathematical induction, that for $n \geq 1$,

$$\frac{d^n}{dx^n} = e^{-x}[(-1)^{n-1}n + (-1)^n x]$$

[7 marks]

Question 10

Given that the graph of $y = x^4 - 10x^3 + 37x^2 - 60x + 36$ touches the x-axis at the point with coordinates $(2,0)$, prove that $y \geq 0$ for all real values of x.

[6 marks]

Question 11

Prove that $[r(\cos\theta - i\sin\theta)]^n = r^n[\cos(n\theta) - i\sin(n\theta)]$, for all $n \in \mathbb{Z}^+$.

[7 marks]

Question 12

Proof by Induction that

$$\sum_{r=1}^{n} r(r^2-6) = \frac{1}{4}n(n+1)(n+4)(n-3)$$

for all positive integer values of n.

[7 marks]