3.6 Matrix Transformations

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.6 Matrix Transformations
Difficulty	Very Hard

Time allowed: 90

Score: /68

Percentage: /100

Question 1a

A geometric transformation $T:(x\ y) \mapsto (x'y')$ is defined by

$$T:(x\ y) = \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)(x\ y)$$

a)

Given that T is a composite function comprising a transformation defined by the matrix A followed by a rotation of $\frac{\pi}{4}$ rad clockwise:

(i)

Find A.

(ii)

Describe fully the single geometric transformation represented by A.

[3 marks]

Question 1b

After being transformed by T, an additional transformation B is undergone. The final position of the points is a reflection of their initial position in the line $y = x$.

b)

Find B.

[4 marks]

Question 2

A trapezoid $ABCD$ is shown in the diagram below.

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By first transforming the trapezoid so that the base is parallel to the y-axis, calculate the area of the trapezoid.

[8 marks]

Question 3

Points in a plane are subject to a transformation AB that transforms a point (x, y) to the point (x', y'), where A and B are defined by

$$A = (4\ 6\ 1\ -3),\ B = (-2\ 0\ 0\ -2)$$

The position matrix of a series of transformed points is $X' = (2\ 4\ 0\ 1\ 2\ 0\ 1\ 8\ -9)$.

Given that the final position matrix of the points X'' is a reflection in the y-axis from their original position, find X''.

[7 marks]

Question 4

A triangle ABC undergoes a transformation represented by the matrix $(-3\ -1\ 3\ 5)$ after which it has an area of 48 cm^2.

The original triangle ABC is then transformed by A, where A is defined as a stretch with scale factor 3 parallel to the x-axis and a stretch with scale factor p parallel to the y-axis.

Given that the area of triangle ABC after being transformed by A is 84 cm^2, find p.

[6 marks]

Question 5

Consider the functions $y = f(x)$ and $y = g(x)$ defined by $f(x) = 2x^2$ and $g(x) = 3x + 5$.

Let the transformation of a point be represented by $T{:}(x\ f(x)) \mapsto (x\ g \circ f(x))$.

Use a matrix method to determine the coordinates of a point P after undergoing the transformation T, given that $P'(-2,\ 29)$.

[5 marks]

Question 6

T is a 2×2 matrix $\begin{pmatrix} 2 & b \\ c & d \end{pmatrix}$ that represents the transformation of points $A(-1,6)$ and $B(4,2)$ to $A'(2p, -18)$ points and $B'(10, -3p)$ respectively.

Given that point $C(-2,5)$ is transformed by T^2, find the coordinates of the image point C'.

[6 marks]

Question 7

Show that a transformation matrix representing a reflection is a self-inverting matrix

[4 marks]

Question 8a

Consider the general complex number $z = x + yi$ with position vector $\begin{pmatrix} x & y \end{pmatrix}$ and a second complex number z_1.

a)
State the two transformations that occur when z is multiplied by z_1.

[2 marks]

Question 8b

b)
Hence write down the single matrix, T that represents the two transformations from part (a).

[2 marks]

Question 8c

c)
Given that $z = 1 + 2i$ and $z_1 = 3\sqrt{3} - 3i$ and using the result from part (b), find the position vector of the result of zz_1.

[4 marks]

Question 8d

d)
By multiplying the two numbers together in their complex form, verify that your answer to part (c) is correct.

[2 marks]

Question 9

The matrices $R,$ S and T are defined by R, a stretch with scale factor 4 and y-axis invariant, S, a reflection in the line $y = -x$ and $T = (a\ b\ c\ d)$.

Triangle X is mapped onto triangle Y by the transformation represented by $R^3 TS$.

Given triangle Y is a rotation of triangle X by $\frac{\pi}{4}$ rad clockwise about the origin, find T.

[5 marks]

Question 10a

The triangle PQR with position matrix T_0 has vertices $P(0,\ 4)$, $Q(8, -3)$, and $R(7,7)$.

The triangle is transformed by a matrix M comprising a counter-clockwise rotation of $\frac{\pi}{3}$ about the point $(-2,1)$.

a)
Given that T_n denotes the position matrix of the image triangle after PQR has been transformed n times by matrix M, find T_3
.

[6 marks]

Question 10b

The same original triangle PQR is transformed by a matrix N comprising a clockwise rotation of $\frac{\pi}{3}$ about the origin followed by a translation of $(3\ 4)$.

R_n denotes the position matrix of the image triangle after PQR has been transformed n times by matrix N.

b)
Find the distance between the vertices corresponding to the initial point P of the triangle with position matrix T_3 and the triangle with position matrix R_1.

[4 marks]