

4.9 Further Normal Distribution (inc Central Limit Theorem)

Question Paper

Course	DPIB Maths
Section	4. Statistics & Probability
Topic	4.9 Further Normal Distribution (inc Central Limit Theorem)
Difficulty	Very Hard

Time allowed: 100
Score: /78
Percentage: /100

Question 1

At a cheese factory, cheese is made into 'wheels'. Their weights are normally distributed with a mean of 6.98 kg and a standard deviation 0.1 kg.

The cheese wheels are transported on pallets. Each pallet holds 9 cheese wheels and is labelled with "Weight: 63 kg".

The cheese inspector takes a random sample of 2 pallets.

Find the probability that the mean weight of the 2 pallets is less than labelled.

[4 marks]

Question 2a

A manufacturer producing bags of jellybeans claims that each bag contains an average of 60 jellybeans. Sebastian buys 35 bags of jellybeans check their claim.

Sebastian found that from his 35 bags

$$\sum x = 2081 \quad \sum x^2 = 123795$$

where x represents the number of jellybeans in each bag.

a)

Use the formula $s_{n-1}^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$ to help find a 95% confidence interval for the mean number of jellybeans per bag. State any assumptions you use.

[4 marks]

Question 2b

b)

Suggest, with justification,

i)

a conclusion that Sebastian could make about the manufacturer's claim

ii)

how the 95% confidence interval for the mean could be made more accurate.

[2 marks]**Question 2c**

The jellybean manufacturer would like to reward Sebastian for his research, and they decide to send him one million jellybeans. The manufacturer sends Sebastian 16820 bags.

c)

Using the data from Sebastian's sample, estimate the probability that the 16820 bags contain less than one million jellybeans in total. State any assumptions you use.

[4 marks]

Question 3a

It is known that the weights of male Border Terriers (a breed of dog) are normally distributed with a mean of 6.5 kg and a variance of 0.62 kg^2 .

A group of 68 male Border Terriers are sampled from the population.

a)

i)

Find the expected number of Border Terriers in this group who weigh less than 6.2 kg, rounded to the nearest integer.

ii)

Find the probability that the number of dogs in the sample who weigh less than 6.2 kg is exactly the same as the number found in part (i). State any assumptions that are needed.

[5 marks]

Question 3b

b)

Find the probability that the mean weight of the 68 Border Terriers in the sample is less than 6.2 kg.

[2 marks]

Question 3c

72 females are now added to the sample. The weights of female Border Terriers are normally distributed with mean 5.8 kg and variance 0.65^2 kg^2 . A random dog from the combined group of males and females is selected.

c)

Find the probability that the dog is a female, given that it weighs over 6 kg.

[4 marks]**Question 4a**

The table below shows the heights in cm of a class of 25 students, all aged 14.

Height, h cm	Frequency
$150 \leq h < 155$	3
$155 \leq h < 160$	5
$160 \leq h < 165$	9
$165 \leq h < 170$	7
$170 \leq h < 175$	1

A student who is 160 cm tall says "I think I am average height compared to other students our age in the country".

a)

Investigate this claim using a 95% confidence interval for the mean. Clearly state what the population is in this case.

[4 marks]

Question 4b

The tallest student in the class, at a height of 174.5 cm, says “I think my height is in the top 1% for students our age in the country”.

b)

Assuming the unbiased estimates for the population mean and variance are the actual population parameters, investigate this student’s claim. You may assume that the population is normally distributed.

[2 marks]

Question 5a

Andrew and Bob are inspecting the quality of two types of bolts which are going to be used to replace current bolts in two different parts of an aircraft.

Andrew is inspecting the replacements for Bolt A, which are used to help secure a compartment in the cabin containing the pilot's snacks. They should have a diameter of 1 cm. He takes a sample of 32 replacement bolts and produces a report about their suitability.

Bob is inspecting the replacements for Bolt B, which are used to help secure the engines to the wings. They should have a diameter of 8 cm. He takes a sample of 32 replacement bolts and produces a report about their suitability.

Bolt A (Andrew's report)

Unbiased estimate of mean: 0.995 cm

Unbiased estimate of the variance: 0.01^2 cm^2

Using a 99% confidence interval I found that the interval was entirely below 1 cm, so I have decided that Bolt A has FAILED.

Bolt B (Bob's report)

Unbiased estimate of mean: 7.99 cm

Unbiased estimate of the variance: 0.05^2 cm^2

Using an 80% confidence interval I found that the interval contained the value 8 cm, so I have decided that Bolt B has PASSED.

a)

Find the confidence intervals, correct to 4 decimal places, that Andrew and Bob used.

[2 marks]

Question 5b

b)

i)

Explain why it would have been more appropriate for Bob to use a higher confidence level.

ii)

Hence, comment on their final decisions about the two types of bolt.

[4 marks]

Question 6a

In the coastal town of Burnham-on-Sea, historical data from 1911 to 2001 shows that the sea flows over the flood defences and floods the road once every 5 years on average. It can be assumed that the number of times this happens in any length of time can be modelled by a Poisson distribution.

a)

Find the probability that the road is flooded by the sea at least 3 times in a 10-year period.

[1 mark]

Question 6b

Clarissa, a climate scientist, collects data from other locations around the world with the same historical rate of flooding as Burnham-on-Sea. There are 32 different locations in her sample, and her data covers the 5-year period 2016 to 2020 in each location. Clarissa records the number of times that the roads are flooded in those 32 locations.

b)

The mean number of times the roads were flooded during the 5-year period was in the top 5% according to the model. Find the range of values for the mean.

[3 marks]

Question 6c

c)

Using the sample of the 32 locations, find the probability that the mean number of times that the roads were flooded during the 5-year period is more than 10% higher than the historical mean.

[2 marks]

Question 7a

A coffee shop; Küste Kaffee, has a loyalty card scheme which allows them to track customers' spending habits. They have found that adults have a mean spend of €6.80 and a standard deviation of €2. Teenagers have a mean spend of €4.75 with a standard deviation of €3.10. The spending of both adults and teenagers are modelled as normal distributions.

On Wednesday between 2pm and 3pm, 17 adults and 8 teenagers visit Küste Kaffee and they each make a single purchase.

a)

Find the probability that the coffee shop receives over €160 in the hour. State any assumptions you have made.

[4 marks]

Question 7b

On Thursday between 2pm and 3pm, the coffee shop experiments with a "half price hour" where all items are 50% off. They predict that 30 adults and 20 teenagers will each make a single purchase during this hour.

b)

Find the probability that Küste Kaffee receives over €160 on Thursday between 2pm and 3pm. State any assumptions you have made.

[4 marks]

Question 7c

- c)
- i)
Comment on the coffee shop's decision to try a "half price hour", explaining if it is likely to increase their income or not.
- ii)
Suggest a reason why the modelling in part (b) could be inaccurate.

[2 marks]

Question 8a

A game at a funfair involves throwing a ball at some coconuts and knocking them off their stand. There are 5 coconuts and players get three balls to throw in turn. To win, players must knock over 3 coconuts in a row. Players win a large stuffed toy unicorn as a prize if they win.

When there are n coconuts still standing, the probability of hitting one is $\frac{n}{8}$

- a)
On Friday night, 35 players play the game. Find the probability that at least 5 players win.

[3 marks]

Question 8b

A statistician, Stacey, visits the funfair and takes a random sample of 35 players each evening who play the game to see how many of them win. Stacey is really keen on gathering lots of statistics, so she does this over the course of 38 days.

- b)
Find the probability that over 38 days, the mean number of winners out of the 35 players in a sample is less than 2.84.

[5 marks]**Question 8c**

- c)
Explain why it is valid to use the central limit theorem for part (b).

[1 mark]

Question 9a

A class of primary school children are painting a wall with pictures of themselves. They are going to measure their heights, and then make the paintings an enlargement of scale factor 1.5; so that they cover more of the wall.

- a)
Explain how the 1.5 scale factor will affect
- the mean of the paintings' heights
 - the variance of the paintings' heights.

[2 marks]

Question 9b

A child in the class thinks that this still won't cover enough of the wall. They suggest that once the enlarged pictures are painted on the wall, they should draw a 30 cm hat on every picture's head.

- b)
Explain how adding the hats to all the paintings will affect
- the mean of the paintings' heights
 - the variance of the paintings' heights.

[2 marks]

Question 9c

In the class there are 20 children. Their heights can be modelled as a normal distribution with mean 128 cm and standard deviation 6 cm. They all paint their pictures onto the wall, which is 2.25 m high, using a scale factor of 1.5. Once the pictures are painted, 12 students are randomly selected to have a 30 cm hat drawn on top of their paintings' heads.

- c)
Find the probability that at least 1 student's painting is too tall to fit on the wall.

[5 marks]

Question 10a

Caleb owns a fishing lake. He creates an advert for social media

Come to Caleb's Fishing Lake!

Average catch size: a huge 37.8 cm!

(10 fish sampled with standard deviation of 18 cm)

£5 per day entry

Refund if you catch 10 fish and their mean is less than 35 cm!

Caleb models the length of fish in the lake using a normal distribution.

It costs Caleb £80 per day to run the fishing lake. On a given day, 28 people pay the entry fee to come fishing.

a)

Assuming that all 28 people each catch exactly 10 fish, find the probability that Caleb does not make a profit on this day. You may assume that the unbiased estimates for the mean and variance can be used as the actual mean and variance.

[5 marks]

Question 10b

Caleb decides he wants to put a different statistic in his next advert.

b)

Find a 90% confidence interval for the mean using Caleb's sample of 10 fish.

[1 mark]

Question 10c

Stacey the statistician visits the fishing lake and determines that the lengths of fish in the lake are not normally distributed.

c)

Decide whether Caleb should use the confidence interval, that was calculated in part (b), in his next advert. Justify your answer.

[1 mark]