# 4.6 Normal Distribution

## Question Paper

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|---|---|
| Course | DP IB Maths |
| Section | 4. Statistics & Probability |
| Topic | 4.6 Normal Distribution |
| Difficulty | Medium |

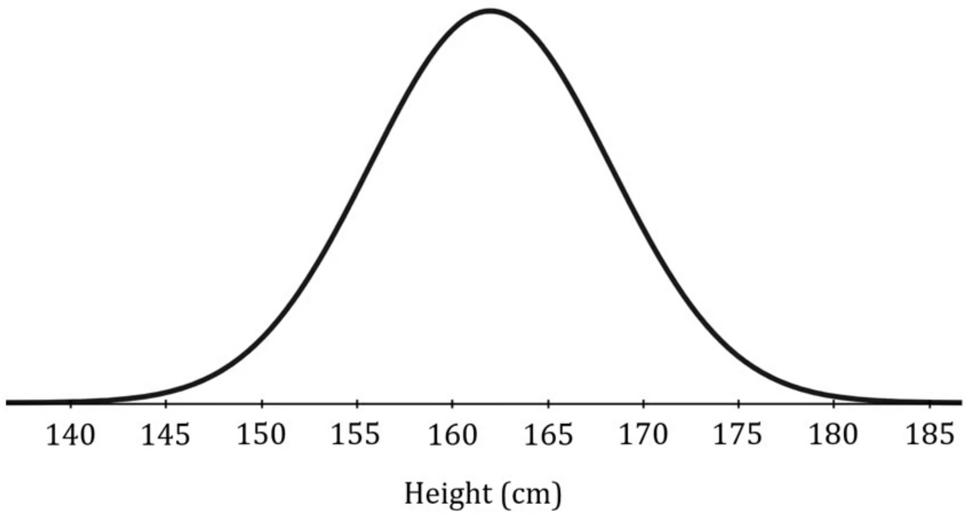
**Time allowed:** 80

**Score:** /61

**Percentage:** /100

## Question 1a

The random variable, $X$, is seen on the following diagram which shows the distribution of heights, in cm, of adult women in the UK:




The distribution of heights follows a normal distribution, with a mean of 162 cm and a standard deviation of 6.3 cm.

(a) On the diagram above, shade in the region representing $P(X > 155)$.

**[2 marks]**

## Question 1b

(b) (i) Find the probability that a randomly selected woman has a height of more than 155cm.

(ii) Use your answer from part (b)(i) to find the probability that a randomly selected woman has a height of more than 169cm.

**[4 marks]**

### Question 1c

(c) Suggest a range of heights within which the height of approximately

(i) 68%

(ii) 95%

(iii) 99.7%

of adult women in the UK will fall.

**[3 marks]**

## Question 2a

(a) For the random variable $X \sim \mathrm{N}(23,\ 4^2)$ find the following probabilities:

(i) $\mathrm{P}(X < 20)$

(ii) $\mathrm{P}(X \geq 29)$

(iii) $\mathrm{P}(20 \leq X < 29)$

**[3 marks]**

## Question 2b

(b) For the random variable $Y \sim \mathrm{N}(100,\ 225)$ find the following probabilities:

(i) $\mathrm{P}(Y \leq 90)$

(ii) $\mathrm{P}(Y > 140)$

(iii) $\mathrm{P}(85 \leq Y \leq 115)$

**[3 marks]**

## Question 3a

The weight, $W$ g, of a chocolate bar produced by a certain manufacturer is modelled as $W \sim \mathrm{N}(200, 1.75^2)$.

(a) Find:

(i) $\mathrm{P}(W < 195)$

(ii) $\mathrm{P}(W > 203)$

**[2 marks]**

## Question 3b

Heledd buys a pack containing 12 of the chocolate bars. It may be assumed that the 12 bars in the pack represent a random sample.

(b) Find the probability that all of the bars in the pack have a weight of at least 195 g.

**[2 marks]**

## Question 4a

The random variable $X \sim \mathrm{N}(330,\ 10^2)$.

(a) Find the value of $a$, to 2 decimal places, such that:

(i) $\mathrm{P}(X < a) = 0.25$

(ii) $\mathrm{P}(X > a) = 0.25$

(iii) $\mathrm{P}(315 \leq X \leq a) = 0.5$

**[4 marks]**

## Question 4b

The random variable $Y \sim \mathrm{N}(10,\ 10)$.

(b) Find the value of $b$ and the value of $c$, each to 2 decimal places, such that:

(i) $\mathrm{P}(Y < b) = 0.4$

(ii) $\mathrm{P}(Y > c) = 0.25$

**[2 marks]**

## Question 4c

(c) Use a sketch of the distribution of $Y$ to explain why $\mathrm{P}(b \leq Y \leq c) = 0.35$.

**[2 marks]**

## Question 5a

The test scores, $X$, of a group of RAF recruits in an aptitude test are modelled as a normal distribution with $X \sim \mathrm{N}(210, 27.8^2)$.

(a) (i) Find the values of $a$ and $b$ such that $\mathrm{P}(X < a) = 0.25$ and $\mathrm{P}(X > b) = 0.25$.

(ii) Hence find the interquartile range of the scores.

**[3 marks]**

## Question 5b

Those who score in the top 30% on the test move on to the next stage of training.

(b) One of the recruits, Amelia, achieves a score of 231. Determine whether Amelia will move on to the next stage of training.

**[2 marks]**

### Question 6a

(a) For the standard normal distribution $Z \sim \mathrm{N}(0, 1^2)$, find:

(i) $\mathrm{P}(Z < 1.5)$

(ii) $\mathrm{P}(Z > -0.8)$

(iii) $\mathrm{P}(-2.1 < Z < -0.3)$

**[4 marks]**

### Question 6b

The random variable $X \sim \mathrm{N}(2, 0.1^2)$.

(b) By using the coding relationship between $X$ and $Z$, re-express the probabilities from parts (a) (i), (ii) and (iii) in the forms $\mathrm{P}(X < a)$, $\mathrm{P}(X > b)$ and $\mathrm{P}(c < X < d)$ respectively, where $a$, $b$, $c$ and $d$ are constants to be found.

**[3 marks]**

## Question 7a

The table below shows the percentage points of the normal distribution. The values $z$ in the table are those which a random variable $Z \sim \mathrm{N}(0,1)$ exceeds with probability $p$.

| $p$ | $z$ | $p$ | $z$ |
|---|---|---|---|
| 0.5000 | 0.0000 | 0.0500 | 1.6449 |
| 0.4000 | 0.2533 | 0.0250 | 1.9600 |
| 0.3000 | 0.5244 | 0.0100 | 2.3263 |
| 0.2000 | 0.8416 | 0.0050 | 2.5758 |
| 0.1500 | 1.0365 | 0.0010 | 3.0902 |
| 0.1000 | 1.2816 | 0.0005 | 3.2905 |

(a) (i) Use the percentage points table for the standard normal distribution to find the value of $z$ for which $\mathrm{P}(Z > z) = 0.2$.

(ii) Use your answer to part (a)(i) along with the properties of the normal distribution to work out the values of $a$ and $b$ for which $\mathrm{P}(Z < a) = 0.2$ and $\mathrm{P}(Z < b) = 0.8$.

**[3 marks]**

## Question 7b

The weights, $W$ kg, of coconuts grown on the Coconutty As They Come coconut plantation are modelled as a normal distribution with mean 1.25 kg and standard deviation 0.38 kg. The plantation only considers coconuts to be exportable if their weight falls into the 20% to 80% interpercentile range.

(b) Use your answer to part (a)(ii) to find the range of possible weights, to the nearest 0.01 kg, for an exportable coconut.

**[2 marks]**

## Question 8a

A machine is used to fill cans of a particular brand of soft drink. The volume, $V$ ml, of soft drink in the cans is normally distributed with mean 330 ml and standard deviation $\sigma$ ml. Given that 15% of the cans contain more than 333.4 ml of soft drink, find:

(a) the value of $\sigma$

**[2 marks]**

## Question 8b

(b) $\mathrm{P}(320 \leq V \leq 340)$.

**[1 mark]**

## Question 8c

Six cans of the soft drink are chosen at random.

(c) Find the probability that all of the cans contain less than 329 ml of soft drink.

**[3 marks]**

## Question 9a

The random variable $X \sim \mathrm{N}(\mu, \sigma^2)$. It is known that $\mathrm{P}(X > 36.88) = 0.025$ and $\mathrm{P}(X < 27.16) = 0.1$

(a) Find the values of $a$ and $b$ for which $\mathrm{P}(Z > a) = 0.025$ and $\mathrm{P}(Z < b) = 0.1$, where $Z$ is the standard normal variable. Give your answers correct to 4 decimal places.

**[2 marks]**

## Question 9b

(b) Use your answers to part (a), along with the relationship between $Z$ and $X$, to show that the following simultaneous equations must be true:

$$\mu + 1.96\sigma = 36.88$$

$$\mu - 1.2816\sigma = 27.16$$

**[2 marks]**

### Question 9c

(c) By solving the simultaneous equations in (b), determine the values of $\mu$ and $\sigma$. Give your answers correct to 2 decimal places.

**[2 marks]**

### Question 10

The ages, $A$, in years, that Liverpool players have made their debuts over the past 20 years are normally distributed with a mean of 22.5 years and a standard deviation of $\sigma$ years.

Given that 10% of Liverpool players make their debuts before turning 20 years old, find:

(i) the value of $\sigma$,

(ii) the probability that a randomly selected player made his debut before his $18^{\text{th}}$ birthday.

**[5 marks]**