

# 1.4 Proof & Reasoning

## Question Paper

Course	DPIB Maths
Section	1. Number & Algebra
Topic	1.4 Proof & Reasoning
Difficulty	Very Hard

**Time allowed:** 60  
**Score:** /44  
**Percentage:** /100

**Question 1a**

(a) (i) Prove that

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

(ii) Specify any cases for which the relation in part (a)(i) is **not** valid.

[3 marks]

**Question 1b**

(b) Prove that  $(p - q)^2 = (q - p)^2$  for all numbers  $p$  and  $q$ .

[2 marks]

**Question 2**

Prove that the product of two odd numbers is odd.

[4 marks]

**Question 3**

The sum of squares of two consecutive integers is 313. Find the possible values of the integers.

[5 marks]

**Question 4**

Prove that the sum of the cubes of any two consecutive odd integers is divisible by four.

[5 marks]

**Question 5a**

(a) Prove that  $\frac{a^2 - a - 6}{a + 4} \times \frac{a^2 - 16}{a^2 + 2a} = a - 7 + \frac{12}{a}$ .

[4 marks]

**Question 5b**

(b) State any values of  $a$  for which this mathematical statement does **not** hold true.

[1 mark]

**Question 6**

Prove that there are no integers  $p$  and  $q$  that satisfy the equation

$$4p^2 - q^2 = 49$$

[4 marks]

**Question 7**

Prove the binomial coefficient identity

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}.$$

**[8 marks]**

**Question 8**

Prove that the sum of all integers between 600 and 1400 (inclusive) that are not divisible by 7 is equal to 685885.

**[8 marks]**