5.12 Further Limits (inc l'Hôpital's Rule)

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.12 Further Limits (inc l'Hôpital's Rule)
Difficulty	Hard

Time allowed: 80

Score: /60

Percentage: /100

Question 1a

For each of the following limits,

(i)
determine whether or not l'Hôpital's rule may be used to evaluate the limit, giving a reason for your answer; and
(ii)
if l'Hôpital's rule may be used, then use the rule to evaluate the limit.
a)

$$\lim_{x \to 0} \frac{e^x - 1}{\sin x}$$

[4 marks]

Question 1b

(b)

$$\lim_{x \to 0} \frac{\cos x}{1 - e^x}$$

[2 marks]

Question 1c

(c)

$$\lim_{x \to \frac{\pi}{2}} \frac{1 - \cot 2x}{\operatorname{cosec} x \sec x}$$

[5 marks]

Question 2a

Consider the following limit:

$$\lim_{x \to 0} \frac{\cos x + 2 \sin x - \sin 2x - 1}{1 - \cos x}$$

(a)
Explain why it is appropriate to use l'Hôpital's rule to attempt to evaluate this limit.

[2 marks]

Question 2b

(b)
Use l'Hôpital's rule to evaluate the limit.

[4 marks]

Question 3a

Consider the function f defined by

$$f(x) = \frac{2 + 5e^{x^2}}{9e^{x^2} - 7}$$

a)
(i)
Use l'Hôpital's rule to evaluate $\lim_{x \to +\infty} f(x)$.

(ii)
Write down the value of $\lim_{x \to -\infty} f(x)$

[4 marks]

Question 3b

(b)
Hence write down the equation(s) of any horizontal asymptotes on the graph of $y = f(x)$, giving a reason for your answer.

[2 marks]

Question 3c

(c)

Show that $\lim_{x \to +\infty} f(x)$ may also be evaluated without the use of l'Hôpital's rule, and confirm that the limit found matches the answer from part (a)(i).

[3 marks]

Question 4a

(a)

Determine the Maclaurin series for $\ln(1 + x^2)$ in ascending powers of x up to the term in x^6.

[2 marks]

Question 4b

Consider the limit

$$\lim_{x \to 0} \frac{\ln(1 + x^2) + \cos x - 1}{x^2}$$

(b)

Use Maclaurin series to evaluate the limit.

[5 marks]

Question 4c

(c)
(i)
Show that it would also be appropriate to use l'Hôpital's rule to attempt to evaluate the limit.
(ii)
Evaluate the limit using l'Hôpital's rule, and confirm that this matches your answer in part (b).

[5 marks]

Question 5

Use Maclaurin series to evaluate the limit

$$\lim_{x \to 0} \frac{\sin(kx^2)}{x^2}$$

where k is a non-zero real constant. Give your answer in terms of k.

[5 marks]

Question 6

Use an appropriate method to evaluate the limit

$$\lim_{x \to 0} \frac{x - x \cos 3x}{x^3}$$

[6 marks]

Question 7a

Consider the function f defined by

$$f(x) = \frac{p - \arctan(\cos x)}{x^2}$$

where $p \in \mathbb{R}$ is a constant.

(a)
Given that $\lim_{x \to 0} f(x)$ converges to a finite value, determine the value of p.

[2 marks]

Question 7b

(b)
Using the value of p found in part (a), use l'Hôpital's rule to evaluate $\lim_{x \to 0} f(x)$.

[6 marks]

Question 7c

The first two terms, in ascending powers of x, of the Maclaurin series for $\arctan(\cos x)$ are $q + rx^2$, where $q, r \in \mathbb{R}$ are constants.

(c)
Write down the values of q and r, being sure to justify your answers.

[3 marks]