2.4 Further Functions & Graphs

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.4 Further Functions & Graphs
Difficulty	Very Hard

Time allowed: 90

Score: /70

Percentage: /100

Question 1a

A function is defined by $f(x) = e^{x^2 + bx + 4}$. The graph of f has an axis of symmetry of $x = 2$.

(a) Find the value of b.

[2 marks]

Question 1b

(b) Find the range of f.

[1 mark]

Question 1c

Another function is defined by $g(x) = -\frac{(x^2 - 25)}{5}$. The graph of f and g intersect at points A and B.

(c) Find the equation of the line passing through points A and B. Give your answer in the form $y = mx + c$.

[3 marks]

Question 1d

(d) Find the distance of the line AB.

[2 marks]

Question 2a

Consider the function $f(x) = 5 - \log(6 - 4x)$. The line l_1 intersects the graph of f at point $A(-1, y)$ and $B(x, 5)$.

(a) Find the value of x and y.

[2 marks]

Question 2b

(b) Find the equation of l_1. Give your answer in the form $y = mx + c$, where m and c are fractions.

[2 marks]

Question 3a

The function f is a quadratic in the form $f(x) = ax^2 + bx - 2$, for $-10 \leq x \leq 10$.

The graph of f has x-intercepts $\left(\frac{1+\sqrt{5}}{2}, 0\right)$ and $\left(\frac{1-\sqrt{5}}{2}, 0\right)$.

(a) Find the values of a and b.

[4 marks]

Question 3b

Another function can be defined by $g(x) = 6(0.8)^{-x} - 1$, for $-10 \leq x \leq 10$.

The graph of f and g intersect at points A and B.

(b) Find the coordinates of A and B.

[2 marks]

Question 3c

(c) Solve the inequality $f(x) < g(x)$.

[2 marks]

Question 4a

(a) Write down the domain and range of the logarithmic function $y = \log_b x$, where $b > 0$ and $b \neq 1$.

[2 marks]

Question 4b

(b) Given that $\log_{y^2} x = 16 \log_x (y^2)$, find all the expressions for x in terms of y.

[6 marks]

Question 5a

Let $f(x) = 2x^4 - 2x^3 - 4x^2 + x + 1$, where $x \in \mathbb{R}$.

(a) Solve the inequality $f(x) < 0$.

[3 marks]

Question 5b

(b) For the graph of f, find the coordinates of the

    (i)   local maximum point.

   (ii)   local minimum points.

[3 marks]

Question 5c

(c) Write down the possible domains of f for which f has an inverse and explain why the domain must be restricted.

[3 marks]

Question 6a

Consider the function f defined by $f(x) = \ln(x^2 - 64)$, for $x > 8$.

The following diagram shows part of the graph of f which crosses the x-axis at point A, with coordinates $(a, 0)$. The line L is the tangent to the graph of f at the point B.
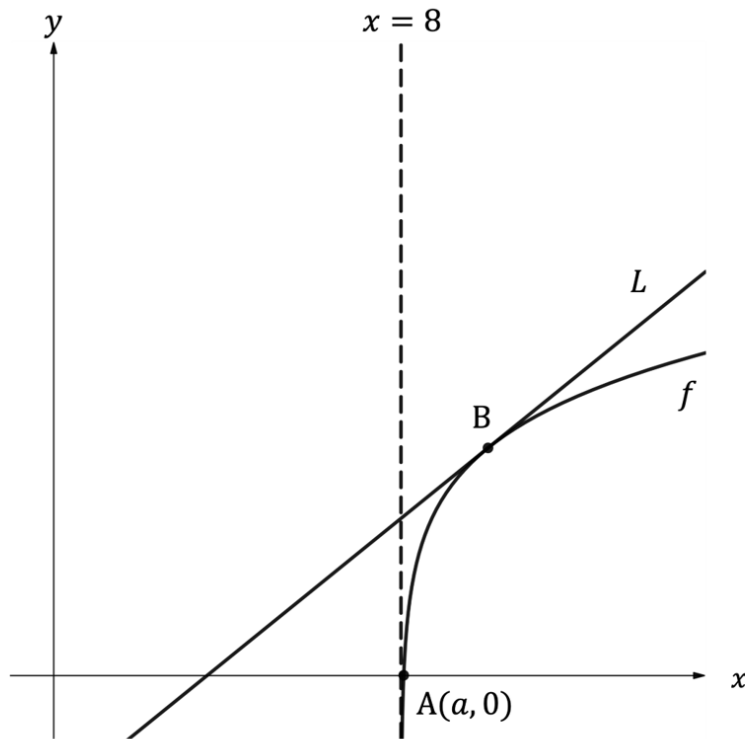
(a) Find the exact value of a.

[3 marks]

Question 6b

The x-coordinate of B is 10. The y-coordinate of B can be written in the form $p \ln q$, where $p, q \in \mathbb{Z}$.

(b) Find the value of p and the value of q.

[4 marks]

Question 6c

The gradient of L is $\dfrac{5}{9}$. The equation of L can be written in the form $y = \dfrac{5}{9}x - u(v - \ln w)$.

(c) Find the values of u, v and w.

[5 marks]

Question 7a

A population of endangered birds, P, can be modelled by the equation

$$P_t = P_0 e^{kt},$$

where P_0 is the initial population and t is measured in years.

After three years, it is estimated that $\dfrac{P_3}{P_0} = 0.87$.

(a) Find the value of k and interpret its meaning.

[3 marks]

Question 7b

(b) Find the least number of whole years for which $\dfrac{P_t}{P_0} < 0.45$.

[5 marks]

Question 8a

The intensity of light, I, is assumed to be 100% at the surface of the ocean and decreases with depth, d, and can be estimated by the function

$$I(d) = k(1.08)^{-d}$$

where I is expressed as a percentage, d is the depth below the surface, in metres, and k is a constant.

(a) Calculate the value of k.

[2 marks]

Question 8b

(b) State the domain and range of I.

[2 marks]

Question 8c

(c) Calculate the intensity of light 6.2 m below the surface.

[2 marks]

Question 9a

Let $f(x) = \dfrac{6-8x}{cx-12}$, for $x \neq \dfrac{12}{c}$, where $c \neq 0$.

(a) The line $x = 2$ is a vertical asymptote to the graph of $y = f(x)$.

   (i)   Find the value of c.

   (ii) Write down the equation of the horizontal asymptote to the graph of $y = f(x)$.

[4 marks]

Question 9b

(b) The line $y = h$, where $h \in \mathbb{R}$, intersects the graph of $y = |f(x)|$ at exactly one point. Find possible values of h.

[3 marks]