1.8 Complex Numbers

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.8 Complex Numbers
Difficulty	Medium

Time allowed: 110

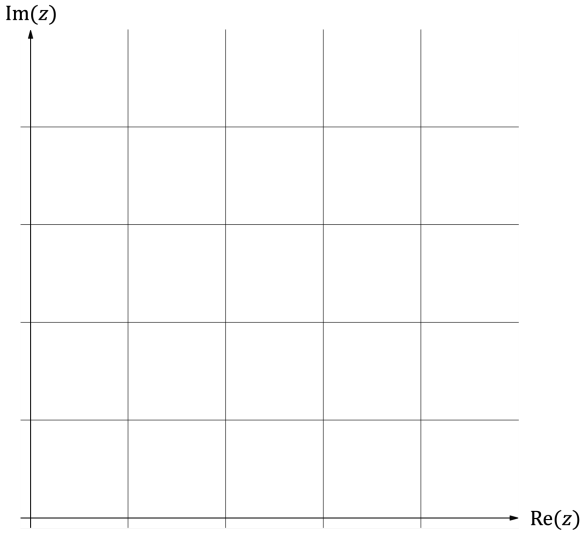
Score: /89

Percentage: /100

Question 1a

Consider the complex numbers $z_1 = 2 + 2\mathrm{i}$ and $z_2 = 2 + 2\sqrt{3}\,\mathrm{i}$.

a)
Sketch z_1 and z_2 on the Argand diagram below, be sure to include an appropriate scale.



[2 marks]

Question 1b

b)
Find the modulus of z_1 and z_2.

[3 marks]

Question 1c

c)
Find the argument of z_1 and z_2.

[3 marks]

Question 2

Solve the following equations for x

(i)
$x^2 + 4x + 5 = 0$

(ii)
$x^2 = -625$

(iii)
$x^4 = 24 - 2x^2$.

[7 marks]

Question 3a

Let $w_1 = z_1 z_2$, where $z_1 = 5 + \mathrm{i}$ and $z_2 = 1 + 2\mathrm{i}$.

a)
Express w in the form $w = a + b\mathrm{i}$.

[2 marks]

Question 3b

b)
Find the modulus and argument for w

[4 marks]

Question 4a

Let $z = \frac{w_1}{w_2}$, where $w_1 = 4 - \mathrm{i}$ and $w_2 = 1 - 2\mathrm{i}$.

a)
Express z in the form $z = a + b\mathrm{i}$.

[3 marks]

Question 4b

b)
Find the modulus and argument for z.

[4 marks]

Question 5a

Consider the complex numbers $z = 3 - 4\mathrm{i}$ and $w = 7 - 2\mathrm{i}$.

a)
Find

(i)
$z + w$

(ii)
$w - z$.

[2 marks]

Question 5b

Let z^* and w^* represent the complex conjugates of z and w, respectively.

b)
Write down z^* and w^*, giving your answers in the form $a + b\mathrm{i}$.

[2 marks]

Question 5c

c)
Find

(i)
$z^* w$

(ii)
$\frac{w^*}{z}$.

[4 marks]

Question 6

Find all possible real values for a and b such that

(i)
$a + b\mathrm{i} = 8\mathrm{i}$

(ii)
$(2 + 3\mathrm{i})(a + b\mathrm{i}) = 13$

(iii)
$(a + \mathrm{i})(2 + b\mathrm{i}) = -6 + 22\mathrm{i}.$

[7 marks]

Question 7

Consider the complex numbers $w = \mathrm{i}z$ and $w + 2z = 7 + 6\mathrm{i}$.

Find

(i)
$\mathrm{Re}(w)$

(ii)
$\mathrm{Im}(w)$

(iii)
$\mathrm{Re}(z)$

(iv)
$\mathrm{Im}(z)$.

[7 marks]

Question 8

It is given that $z_1 = 3 + 4\mathrm{i}$ and $z_2 = -2 + 2\mathrm{i}$.

Find

(i)
$\mathrm{i}z_1 + z_2$

(ii)
$\dfrac{z_1}{\mathrm{i}z_2}$

(iii)
$\mathrm{i}(z_1 z_2)$.

[7 marks]

Question 9

Find the complex numbers z and w such that

$$2z - \mathrm{i}w^* = 5 + 7\mathrm{i}$$

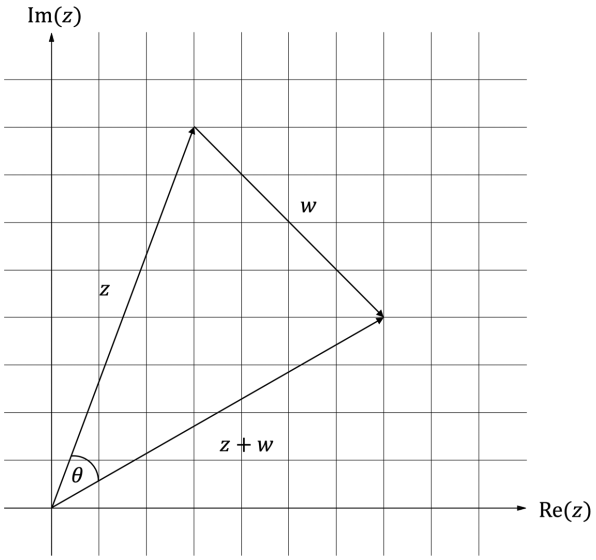
$$w + \mathrm{i}z^* = 5 + 16\mathrm{i}$$

[8 marks]

Question 10a

Let $z = 3 + 8\mathrm{i}$ and $w = 4 - 4\mathrm{i}$.

a)
Find θ, the angle shown on the diagram below.



[5 marks]

Question 10b

b)
Find the area of the triangle formed in the diagram above.

[3 marks]

Question 11a

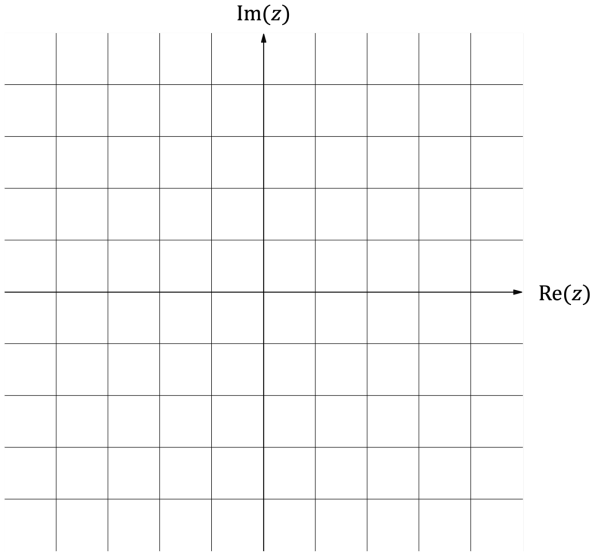
Let $z = -1-3i$ and $w = 1+i$.

a)
Find zw.

[2 marks]

Question 11b

b)
Sketch z, w and zw on the Argand diagram below.



[3 marks]

Question 11c

Let θ be the angle between z and zw and ϕ be the angle between w and zw.

c)
Find the angles θ and ϕ, giving your answers in degrees.

[4 marks]

Question 12a

Let $w = \dfrac{z+1}{z^* + 1}$, where $z = a + b\text{i},\ a,\ b \in \mathbb{R}$.

a)
Write w in the form $x + y\text{i},\ x,\ y \in \mathbb{R}$.

[4 marks]

Question 12b

b)
Determine the conditions under which w is purely imaginary.

[3 marks]