4.5 Binomial Distribution

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.5 Binomial Distribution
Difficulty	Very Hard

Time allowed: 120

Score: /92

Percentage: /100

Question 1a

In London, England, the probability that a football player, who plays club football, is left-footed is 0.24. One particular squad, Raiders FC, has 25 players.

(a) Find the probability that Raiders FC has exactly the expected number of left-footed players.

[3 marks]

Question 1b

(b) Find the probability that there are no left-footed players on the Raiders FC squad.

[2 marks]

Question 1c

(c) Find the probability that there are more right-footed players than left-footed players on the Raiders FC squad.

[2 marks]

Question 1d

(d) Suggest one assumption that has been made in this question.

[1 mark]

Question 2a

When a pizza store receives a delivery order via their app the customer is told a predicted delivery time. The probability that the pizza store delivers the order before the predicted delivery time is $\dfrac{9}{20}$. During a given week, the pizza store makes 160 deliveries.

(a) Find the expected number of deliveries delivered before the predicted time during that week.

[2 marks]

Question 2b

(b) Calculate the probability that they deliver over 100 orders before the predicted delivery time during that week. Give your answer in the form $a \times 10^{k}$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

[2 marks]

Question 2c

The manager, Ciro, decides that it is more important that they deliver the delivery within 5 minutes of the predicted delivery time, either 5 minutes before or 5 minutes after the predicted time. As he believes a delivery being too early is equally as inconvenient as being too late. He finds that the constant probability for this measure is $\frac{3}{4}$.

(c) Calculate the probability that over 100 orders were delivered within 5 minutes of the predicted time during the week with 160 deliveries.

[2 marks]

Question 3a

At a university, it is known from previous cohorts that the probability that a student is taller than 1.8 m is $\frac{3}{10}$ and the probability that a student weighs less than 80 kg is $\frac{9}{10}$. The university currently has 820 students.

(a) Find the mean and variance for the number of students that

    (i)   are 1.8 m or shorter.

    (ii) weigh 80 kg or more.

[5 marks]

Question 3b

(b) Assuming that a student's height and weight are independent, find the probability that 250 students or more are taller than 1.8 m and weigh less than 80 kg.

[3 marks]

Question 3c

(c) Comment on the assumption that a student's height and weight are independent.

[1 mark]

Question 4a

A swimming training session for Ben includes swimming 12 lengths of 100 m. Ben can swim 100 m in less than 60 s with a constant probability of $\frac{5}{6}$. Let X be the number of times Ben completes the 100 m length in less than 60 s during a training session.

(a) Calculate the probability that Ben completes the 100 m in less than 60 s for all the 12 lengths.

[2 marks]

Question 4b

(b) Find the probability that Ben completes the 100 m in less than 60 s at least twice but no more than 9 times during a training session.

[3 marks]

Question 4c

Ben's younger brother, Zack, is doing the session with Ben and can complete the 100 m in less than 60 s with a constant probability of $\frac{2}{5}$.

(c) Find the probability that both Ben and Zack complete the 100 m in less than 60 s more than four times during a training session.

[3 marks]

Question 4d

(d) Suggest a reason as to why the probability for Ben and Zack to complete the 100 m in less than 60 s is not constant.

[1 mark]

Question 5a

A multiple-choice test consists of 50 questions. Each question has 5 options of which only one is correct.

Henry takes the exam and randomly chooses one of the five answers for each question.

(a) Find the expected number of questions Henry answers incorrectly.

[1 mark]

Question 5b

Henry must get at least 25 of the questions correct to pass the test.

(b) Find the probability Henry passes the test. Give your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

[2 marks]

Question 5c

As Henry is going through the test, he realizes he is certain that he knows the answer to 12 of the questions.

(c) Find the probability Henry passes, given he gets these 12 questions all correct.

[4 marks]

Question 6a

A mall has a "clothing" section for clothes and an "other" section for everything else. The probability that a randomly observed customer goes to the clothing section is 0.78, and the probability that a randomly observed customer goes to the other section is 0.42. Assume that each customer goes to at least one section.

(a) Find the probability that a randomly observed customer

    (i)   goes to both sections,

    (ii)   only goes to the clothing section.

[4 marks]

Question 6b

On a given day the mall has 341 customers.

(b) On this day, find

    (i)   the expected number of customers that go to the other section,

    (ii)   the probability that at least 250 customers visit the clothing section.

[4 marks]

Question 7a

360 students at a school are divided into three groups. The probability of a student being put into group A is $\frac{1}{12}$. A student is twice as likely to be put into group B than group C.

(a) Find the expected number of students put into each group.

[4 marks]

Question 7b

20 students are chosen at random to form a sample.

(b) Find the probability that at least four of the students are in group A.

[2 marks]

Question 7c

(c) Find the probability that less than 15 of the students are in groups A or B.

[2 marks]

Question 7d

(d) The headteacher of the school now wants exactly $\frac{1}{12}$ of the students to be in group A. Explain why the number of group A students in the sample can not be modelled by a binomial distribution.

[1 mark]

Question 8a

A bag of 500 marbles are divided into five colours: red, orange, yellow, green and blue. There are x red marbles, $3x$ orange marbles, $5x$ yellow marbles, $7x$ green marbles and $9x$ blue marbles.

(a) Find the value of x.

[2 marks]

Question 8b

Maeve selects a marble at random, records its colour and then returns it to the bag. Maeve follows this process 56 times.

(b) Find the probability that none of the selected marbles are red.

[2 marks]

Question 8c

(c) Find the probability that at least a quarter of the selected marbles are red or blue.

[4 marks]

Question 8d

(d) Maeve decides not to return the marbles to the bag after she records their colours. State, with a reason, whether this would change the answers to part (b) and (c).

[1 mark]

Question 9a

In a restaurant, it is known that there is an 8 out of 15 chance that a guest will ask for water with their meal.

A random sample of 25 guests are sampled.

(a) Find the probability that

    (i)   over half of the guests ask for water,

    (ii)   between 24% and 92%, inclusive, of the guests ask for water.

[4 marks]

Question 9b

A second random sample of 100 guests is selected. Let X be the random variable that represents the number of guests that ask for water.

(b) Find the smallest value of n such that

$$P(X \leq n) \geq 0.95.$$

[3 marks]

Question 10a

In a game, players select pieces of string from a box containing a large number of pieces of string of different lengths. The length, in cm, of a randomly chosen piece of string has uniform distribution over the interval [4, 9].

A piece of string is selected at random from the box.

(a) Find the probability that the piece of string is shorter than 7.5 cm.

[2 marks]

Question 10b

If a player selects five pieces of string, then they win a chocolate bar if the length of the longest piece of string is more than 7.5 cm.

(b) A player selects five pieces of string, find the probability of winning a chocolate bar.

[2 marks]

Question 10c

If a player selects eight pieces of string, they then win a box of chocolates if five or more of the pieces of string are shorter than 5.2 cm

(c) A player selects eight pieces of string, find the probability of winning a box of chocolates.

[2 marks]

Question 11a

A biased dice is thrown 44 times and the expected number of times it will land on a six is 33.

The dice is thrown another eight times.

(a) Find the expected number of sixes seen from the eight throws.

[2 marks]

Question 11b

(b) Calculate the variance of the number of sixes seen from the eight throws. Give your answer as a fraction.

[2 marks]

Question 11c

(c) Find the probability that exactly four sixes are seen from the eight throws. Give your answer as a fraction.

[4 marks]

Question 12

A box contains many marbles, $\frac{2}{5}$ of which are blue. The rest are white. Marbles are picked at random from the box.

From drawing three marbles, with replacement, the probability of getting x blue marbles is $\frac{36}{125}$. Find the value of x.

[6 marks]