1.2 Exponentials & Logs

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.2 Exponentials & Logs
Difficulty	Very Hard

Time allowed: 70

Score: /57

Percentage: /100

Question 1a

Let $f(x) = \ln\left(\dfrac{x}{3} - 1\right)$.

(a) Find the values of x for which $f(x)$ is undefined.

[2 marks]

Question 1b

(b) Given that point A has coordinates $(a, 0)$, find the value of a.

[3 marks]

Question 2

Solve $27^{4x+2} = 81^{8x-3}$.

[6 marks]

Question 3

Solve $5 \ln 2 - \ln 8 = -\ln x$.

[6 marks]

Question 4

Solve the equation $216^{k+2} = 12^{3k}$ for k. Express your answer in terms of $\ln 6$ and $\ln 2$.

[6 marks]

Question 5

Solve the equation $2 \times 25^x - 30 \times 5^{x-1} = 1$.

[5 marks]

Question 6a

Simplify the following expressions, giving your answers in the form ax^n where a and n are rational numbers and any fractions are in lowest terms.

(a) $(8x^2)^{-\frac{1}{3}} \times \frac{1}{4}x^{-\frac{1}{3}}$

[3 marks]

Question 6b

(b) $\left(\frac{2}{9}x^{\frac{1}{2}} \times \frac{1}{18}x^{-\frac{3}{4}}\right)^{-\frac{1}{4}}$

[3 marks]

Question 6c

(c) $\dfrac{\left(8x^{-\frac{2}{3}}\right)^{\frac{2}{3}}}{\left(64x^{-\frac{1}{3}}\right)^{\frac{1}{3}}}$

[3 marks]

Question 7a

Given that $y = \dfrac{81}{16} x^{-12}$, express each of the following in the form ax^n, where a and n are constants.

(a) $y^{\frac{3}{4}}$

[2 marks]

Question 7b

(b) $y^{-\frac{1}{2}}$

[2 marks]

Question 7c

(c) $\left(y^{\frac{1}{2}} \right)^{-3}$

[3 marks]

Question 8a

A company has conducted some product research and believes that their profit (P) made over time (t weeks after opening) can be modelled by the equation

$$P(t) = R(t) - C(t),$$

where

$$R(t) = C(t)^{\frac{b}{100}t} \quad \text{and} \quad C(t) = a^{-bt} + 1000$$

The company collects data on their revenue, R, and costs, C, at the end of each week. After exactly one week the company's costs are \$3000 and they make a loss of \$1000.

(a) Find the values of a and b.

[4 marks]

Question 8b

(b) Find the week in which the company first makes a positive profit.

[2 marks]

Question 8c

(c) Suggest a limitation to the company's model.

[1 mark]

Question 9

The rate, R, of increase of the volume of a cloud created in a science lab is related to the change in air temperature, T, and air pressure, P, by the equation

$$R = kT^x P^y, \text{ where } x, y, k \in \mathbb{R}.$$

A meteorologist takes measurements at three intervals and records the data as follows.

Measurement	R $(\text{cm}^3\text{s}^{-1})$	T (°C)	$P(\text{kPa})$
1	48.75	17.1	101.2
2	46.13	15.9	101.8
3	43.47	14.7	102.5

Find x, y and k.

[6 marks]