2.3 Modelling with Functions

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.3 Modelling with Functions
Difficulty	Hard

Time allowed: 100

Score: /77

Percentage: /100

© 2015-2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 1a

A fence of length L is made to go around the perimeter of a rectangular paddock that borders a straight river. The cost of the fence along the river is \$15 per metre, while on the other three sides the cost is \$10 per metre. The total cost of the fence is \$2000.

(a) Calculate the maximum area of the paddock.

[3 marks]

Question 1b

(b) Using the value for the area from part (a), calculate

(i) the side lengths.

(ii) the total length L of the fence.

[3 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 2a

Henry is about to start a 25-game rugby season for his school. Teams receive 4 points for a win and 2 points for a draw. No points are awarded for a loss.

(a) (i) Using W, D and L as the number of wins, draws and losses respectively, write down an inequality relating the outcomes to the number of games played.

(ii) Explain why $W \geq 0, D \geq 0$ and $L \geq 0$ must also be conditions related to the problem.

[2 marks]

Question 2b

(b) Write down an equation for the number of points, P, a team receives from W wins and D draws.

[1 mark]

Question 2c

After 10 games Henry's team has lost 1 game and they have 32 points.

(c) Find the number of wins and draws Henry's team has had.

[2 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 3a

Antibiotics A and B are applied to a pure culture of bacteria. The number of bacteria present initially for both antibiotics is 6000. The number of bacteria present for antibiotic A, N_{A}, can be modelled by the function

$$N_{\mathrm{A}}(t) = a \times b^{-t}, \qquad t \geq 0,$$

where t is the elapsed time, in hours, since the start of the experiment.

(a) Find the value of a.

[1 mark]

Question 3b

The number of bacteria present for antibiotic A after two hours is 2160.

(b) Find the value of b. Give your answer as a fraction.

[2 marks]

Question 3c

The number of bacteria present for antibiotic B after four hours is 1185. The number of bacteria present for antibiotic B can be modelled using a similar function to antibiotic A.

(c) Write down the function $N_{\mathrm{B}}(t)$.

[3 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 3d

(d) Determine which antibiotic is more effective. Give a reason for your answer.

[2 marks]

Question 4a

In 1967 a house was bought for \$10 000. In 2021 the same house was sold for \$800 000.

The average annual growth percentage of the house from 1967 to 2021 is used to form the following model to estimate the value of the house.

$$\mathrm{V}(t) = a \times b^t, \qquad t \geq 0,$$

where t is the time in years.

(a) Find the value of a and b and write down the function $\mathrm{V}(t)$.

[2 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 4b

In 2026 the house is bought for $1 300 000.

(b) Calculate the percentage error between the actual value of the house and the estimated value approximated by the model.

[4 marks]

Question 5a

A factory produces cardboard boxes in the shape of a cuboid, with a fixed height of 25 cm and a base of varying area. The area, A, of each base can be modelled by the function

$$\text{A}(x) = x(50 - x), \qquad 10 \leq x \leq 40,$$

where x is the width of the base of the cardboard box in centimetres.

Cardboard box M has a width of 12 cm.

(a) Find the volume of cardboard box M.

[2 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 5b

(b) Find the possible dimensions of a cardboard box with volume of 15 400 cm^3.

[3 marks]

Question 5c

(c) (i) Find the value of x that makes the volume of the cardboard box a maximum.

(ii) Write down the maximum volume of the cardboard box.

(iii) State the mathematical shape of the carboard box when its volume is a maximum.

[3 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 6a

The downward speed, V, in metres per second, of a bird making a dive into the water to catch a fish can be modelled by the function

$$\mathrm{V}(t) = 62 - 62 \times 3.5^{-\frac{t}{2}}, \quad t \geq 0,$$

where t, in seconds, is the time the bird is diving.

(a) Write down the downward speed of the bird at $t = 0$.

[1 mark]

Question 6b

(b) Determine the equation of the horizontal asymptote for the graph of $\mathrm{V}(t)$.

[2 marks]

Question 6c

The bird's downward speed when it reaches the surface of the ocean is 14 ms^{-1}.

(c) Find the birds downward speed in kilometres per hour (km h^{-1}).

[2 marks]

© 2015-2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 6d

(d) Find the time, in seconds, for which the bird was diving.

[3 marks]

Question 7a

The temperature, T, of a cake, in degrees Celsius, °C, can be modelled by the function

$$T(t) = a \times 1.17^{-\frac{t}{4}} + 18, \qquad t \geq 0,$$

where a is a constant and t is the time, in minutes, since the cake was taken out of the oven.

(a) In the context of this model, state what the value of 18 represents.

[1 mark]

Question 7b

The cake was 180°C when it was taken out of the oven.

(b) Find the value of a.

[2 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 7c

(c) Find the temperature of the cake half an hour after being taken out of the oven.

[2 marks]

Question 7d

The cake is best eaten when its temperature is 75°C to 95°C.

(d) Calculate for how many minutes the cake's temperature is within this range.

[3 marks]

Question 8a

A hot winter soup has just been removed from the stove and is left outside to cool. The soup's temperature can be modelled by the function

$$\mathrm{T}(t) = a + b(k^{-t}), \qquad t \geq 0,$$

where t is the time, in minutes, since the soup was removed from the stove.

The temperature outside is 12°C.

(a) Write down the value of a and explain why it has this value.

© 2015-2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

[2 marks]

Question 8b

Initially the temperature of the soup is 95°C.

(b) Find the value of b.

[2 marks]

Question 8c

After two minutes the temperature of the soup is 60°C.

(c) Find the value of k.

[2 marks]

Question 8d

After 15 minutes the soup is put into the fridge.

(d) Calculate the temperature of the soup when it is put into the fridge.

[2 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 9a

The approximate quantity, N, of a decaying radioactive substance can be modelled by the function

$$N(t) = N_0 \times 2^{-\frac{t}{t_h}}, \qquad t \geq 0,$$

where N_0 is the initial quantity and t is the elapsed time, in years. t_h is the half-life of the substance and is a measure of how many years it takes for the quantity of the radioactive substance to decrease by half.

Radioactive substance A has $t_h = 1.5$ and after 2 years the quantity remaining is 124.

(a) Find N_0. Give your answer to the nearest integer.

[2 marks]

Question 9b

(b) Calculate the approximate quantity of the radioactive substance after five years. Give your answer to the nearest integer.

[2 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 9c

After five years the quantity remaining of the radioactive substance is 25.

(c) Calculate the percentage error between your approximate value found in part (b) and the exact value given above.

[2 marks]

Question 10a

Deserts are known for having high daily temperature ranges. Erica monitors the temperature, in °C, on a particular day in a desert. The table below shows some of the information she recorded.

	Temperature	Time
Maximum	41.3°C	
Minimum	0.9°C	2: 00 am

Erica uses her observations to form the following model for the temperature, T °C, during the day

$$\mathrm{T}(t) = a\cos\big(b(t+10)\big) + d, \quad 0 \le t \le 24,$$

where t is the elapsed time, in hours, since **midnight**.

(a) Calculate the value of t when the maximum temperature occurs and fill in the time in the table above in am/pm format.

[1 mark]

© 2015-2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 10b

(b) Find the values of

(i) a.

(ii) b.

(iii) d.

[4 marks]

Question 10c

Erica goes exploring in the desert at 6:30 am and leaves once the temperature reaches 32°C.

(c) (i) Calculate the temperature range Erica experiences whilst in the desert.

(ii) Find the time Erica leaves the desert. Give your time to the nearest ten minutes.

[3 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 11a

John is a contracted carpenter who earns an annual salary of \$55 000. Additionally, John sells innovative wooden art at a market. He goes to the market twice a month and he generally earns \$450 from the market.

(a) Estimate John's total annual income.

[1 mark]

Question 11b

John's actual total annual income over the year is \$68 000.

(b) Calculate the percentage error between your answer in part (a) and John's actual total annual income.

[2 marks]

Question 11c

The following table shows different tax rates for residents of the country John lives in.

Income thresholds, in € (x)	Rate, %	Tax payable on this income
$x \leq 9500$	0	0
$9500 < x \leq 60\,000$	15	15% of amounts over €9 500
$60\,000 < x \leq 275\,000$	25	€7575 + 25% of amounts over €60 000
$275\,000 < x$	40	€157 575 + 40% of amounts over 275 000

(c) Calculate the amount of tax John has to pay. Give your answer to the nearest euro (€).

[3 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

© 2015-2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers