# 3.7 Inverse & Reciprocal Trig Functions

## Question Paper

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| Course | DP IB Maths |
| Section | 3. Geometry & Trigonometry |
| Topic | 3.7 Inverse & Reciprocal Trig Functions |
| Difficulty | Very Hard |

**Time allowed:** 90

**Score:** /69

**Percentage:** /100

## Question 1a

a)

Show that $\frac{\cot x}{\sec x(1-\sin x)} \equiv 1 + \frac{1}{\sin x}$, $x \neq \frac{n\pi}{2}$, $n \in \mathbb{Z}$.

**[4 marks]**

## Question 1b

(b)

Hence determine the set of values for $k$ for which $\frac{\cot x}{\sec x(1-\sin x)} = k$ has no real solutions.

**[4 marks]**

## Question 2a

(a)

Find the exact value for the expression $\cot^2\left(\frac{\pi}{6}\right)+\operatorname{cosec}\left(\frac{2\pi}{3}\right)$.

Give your answer in the form $a+\frac{b\sqrt{c}}{d}$ where $a,b,c,d\in\mathbb{Z}$.

**[4 marks]**

## Question 2b

(b)

Find the exact value for the expression $\frac{\sec\left(\frac{2\pi}{3}\right)}{\cot\left(\frac{\pi}{4}\right)}$.

**[3 marks]**

## Question 3a

(a)

Sketch the graph of $y=\sec x$ for $-2\pi\leq x\leq\pi$

**[2 marks]**

## Question 3b

(b)

Given that $\sec 2\theta = \frac{8}{7}$ and $-2\pi \leq \theta \leq -\frac{3\pi}{2}$, find the value of $\sin \theta$.

**[5 marks]**

## Question 4a

(a)

Show that the equation

$$5\sin x - 9\cos x = \frac{2\operatorname{cosec} x}{1 + \tan^2 x}$$

can be rewritten as

$$(a\sin x + \cos x)(\sin x - b\cos x) = 0$$

**[4 marks]**

## Question 4b

(b)

Hence solve the equation $5\sin x - 9\cos x = \dfrac{2\,\text{cosec}\,x}{1+\tan^2 x},\ -\pi \le x \le \pi.$

**[3 marks]**

## Question 5a

Consider the function $f(x) = \cos^2(2\arcsin(x)), 0 \le x \le a$ where $a$ is a constant.

(a)

Given that $f$ has an inverse, find an expression for $f^{-1}(x)$.

**[4 marks]**

## Question 5b

(b)

Show that $\cos^2(2\arcsin(x)) = 4x^2 - 4x^2 + 1$ for $-1 \leq x \leq 1$.

**[4 marks]**

## Question 5c

(c)

Given that $f$ has an inverse, find the maximum value for $a$.

**[2 marks]**

## Question 6a

A function $f$ can be defined by $f(x) = \frac{1}{2}\tan(\arcsin(x)) + 5$.

(a)

Sketch the graph of $f$ indicating clearly any intercepts with the coordinate axes and asymptotes.

**[3 marks]**

## Question 6b

(b)

Solve the inequality $\frac{1}{2}\tan(\arcsin(x)) + 5 > 2$.

Give your answer in the form $\frac{a\sqrt{b}}{c} < x < d$ where $a, b, c, d \in \mathbb{Z}$.

**[5 marks]**

## Question 7a

Consider the equation $\frac{1-\cos^2\theta}{\cot\theta} = -\frac{1}{\operatorname{cosec}^2\theta}$.

(a)

Explain why the equation is undefined when $\theta = \frac{1}{2}k\pi$ for any $k \in \mathbb{Z}$.

**[2 marks]**

## Question 7b

(b)

Solve the equation $\frac{1-\cos^2\theta}{\cot\theta} = -\frac{1}{\operatorname{cosec}^2\theta}$, for $-\pi \leq \theta \leq 2\pi$.

**[6 marks]**

## Question 8

Show that $\sin^2\left(\frac{1}{2}\arctan(x)\right) = \frac{1+x^2-\sqrt{1+x^2}}{2(1+x^2)}, x \in \mathbb{R}$.

**[8 marks]**

## Question 9

Solve the equation $2\sin x + \frac{\cot x}{\sec x} = 3\cos x$ for the interval $-90° < x < 90°$.

**[6 marks]**