2.5 Reciprocal & Rational Functions

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.5 Reciprocal & Rational Functions
Difficulty	Very Hard

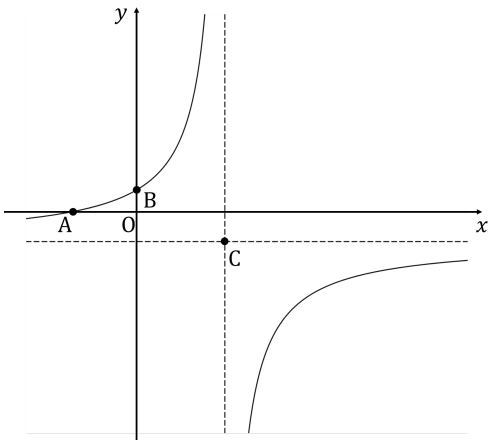
Time allowed: 110

Score: /91

Percentage: /100

Question 1

The graph of a function f is shown below. The graph crosses the x-axis at the point A and the y-axis at point B. The asymptotes intersect at the point C$\left(4,\ -\frac{8}{3}\right)$. The area of the triangle $\triangle \mathrm{AOB}$ is 3.



Find an equation for $f(x)$ in the form $\frac{ax+b}{cx+d}$, where $a,\ b,\ c,\ d \in \mathbb{Z}$. .

[7 marks]

Question 2a

Let the function f be defined by $f(x) = 3 - \frac{a}{2x-5}$, $x \in \mathbb{R}$, $x \neq p$, where a is a positive constant.

(a)
Write down the value of p.

[1 mark]

Question 2b

(b)
Sketch the graph of f. State the equations of the asymptotes and give the coordinates of the intersections with the coordinate axes in terms of a.

[4 marks]

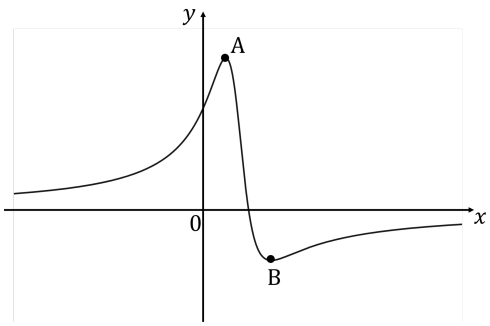
Question 2c

(c)
Given that the graph of f intersects the line given by $y = x$, find the set of possible values of a.

[4 marks]

Question 3a

Consider the function defined by $f(x) = \dfrac{4-2x}{x^2-3x+3}$. The graph of f is shown below. The graph has a maximum point at A and a minimum point at B.



(a)
Show that f is defined for all $x \in \mathbb{R}$.

[3 marks]

Question 3b

(b)
(i)
Given that the graph of f intersects the line $y = k$, show that $3k^2 - 4k - 4 \leq 0$.
(ii)
Hence, find the range of f.

[6 marks]

Question 3c

(c)
Hence, find the coordinates of A and B.

[4 marks]

Question 4a

Consider the function f defined by $f(x) = \dfrac{(x-p)(x-q)}{x-n}$, where n, p and q are positive constants and $p < q$.

(a)
In the case that p, q and n are distinct:
(i)
write down the coordinates of the points where the graph of f intersects the axes,

(ii)
write down the equation of the vertical asymptote,

(iii)
show that the equation of the oblique asymptote is $y = x + (n - p - q)$.

[5 marks]

Question 4b

(b)

Sketch the graph of f in that case that:

(i)

$n < p < q$,

(ii)

$p < n < q$,

(iii)

$n = p$.

Clearly indicate where the graph crosses the coordinate axes and any asymptotes or discontinuities.

[6 marks]

Question 5a

(a)

Sketch the graph of the function f be defined by $f(x) = \frac{4x-1}{2x^2-x-6}$. Clearly indicate the coordinates where the graph intersects the axes and state the equation of any asymptotes.

[7 marks]

Question 5b

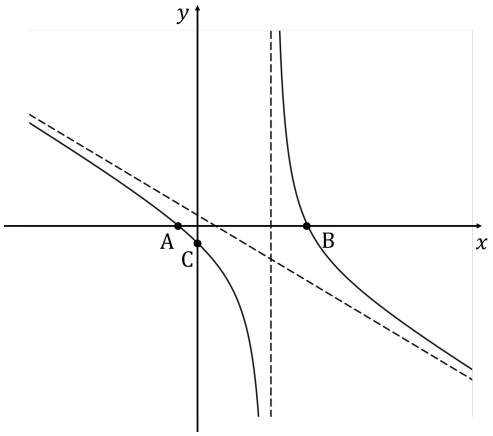
(b)

Sketch the graph of the function g defined by $g(x) = \frac{2x^2-x-6}{4x-1}$ Clearly indicate the coordinates where the graph intersects the axes and state the equation of any asymptotes.

[6 marks]

Question 6a

The graph of a function f is shown below. The graph crosses the x-axis at the points $\mathrm{A}(-0.5,\ 0)$ and $\mathrm{B}(3,0)$. The graph crosses the y-axis at $\mathrm{C}(0,\ -3)$. The equation of the vertical asymptote is $x = 2$.



The function f can be written in the form $f(x) = \dfrac{ax^2 + bx + c}{dx + e}$, where $a,\ b,\ c,\ d,\ e \in \mathbb{Z}$.

(a)
Given that $ce + 2 = b$, find the values of $a,\ b,\ c,\ d$ and e.

[6 marks]

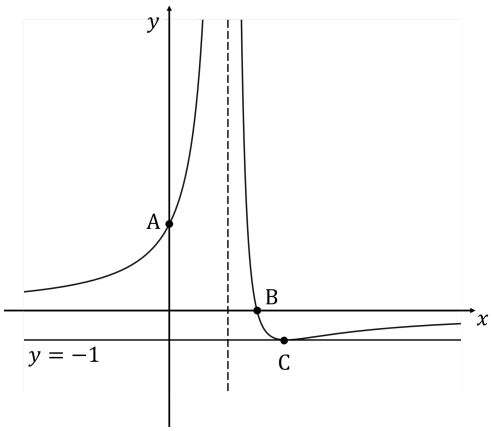
Question 6b

(b)
Hence find the equation of the oblique asymptote.

[4 marks]

Question 7a

Consider the function f defined by $f(x) = \dfrac{ax + b}{x^2 + cx + d}$, where $a, b, c, d \in \mathbb{Z}$. The line $x = 2$ is the only vertical asymptote of the graph of f as shown below. The graph crosses through the points $\mathrm{A}(0, k)$ and $\mathrm{B}(k, 0)$ where k is a positive constant. The line $y = -1$ is a tangent to the graph of f at the point C.



(a)
Find the values of a, b, c, d and k.

[8 marks]

Question 7b

(b)
Find the coordinates of C.

[2 marks]

Question 8a

Consider the function $f(x) = \dfrac{2x^2 + 5x - 12}{x^2 - 4}$, $x \in \mathbb{R}$, $x \neq \pm 2$.

a)
Solve $f(x) = 0$

[2 marks]

Question 8b

(b)
Show that $\dfrac{2x^2 + 5x - 12}{x^2 - 4} = A + \dfrac{Bx + C}{x^2 - 4}$, where A, B and C are constants to be found.

[3 marks]

Question 8c

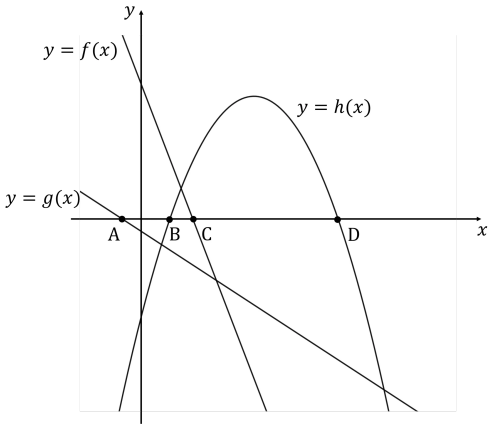
(c)

Using the answer to part (b), sketch the graph of f. Clearly indicate the coordinates of the points where the graph intersects the axes and state the equations of any asymptotes.

[6 marks]

Question 9a

The diagram below shows the graphs of two linear functions f and g and a quadratic function h.



(a)

Sketch the graph of $y = \dfrac{f(x)}{g(x)}$. Clearly indicate where the graph intersects the x-axis and the location of any vertical asymptotes.

[3 marks]

Question 9b

(b)

Sketch the graph of $y = \frac{g(x)}{h(x)}$. Clearly indicate where the graph intersects the x-axis and the location of any vertical asymptotes.

[4 marks]