3.10 Vector Equations of Lines

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.10 Vector Equations of Lines
Difficulty	Medium

Time allowed: 90

Score: /73

Percentage: /100

Question 1a

The points A and B are given by $\mathrm{A}(4, 2, -3)$ and $\mathrm{B}(0, 5, 1)$.

a)
Find a vector equation of the line L that passes through points A and B.

[3 marks]

Question 1b

b)
Determine if the point $\mathrm{C}(-1, 3, 2)$ does not lie on the line L.

[3 marks]

Question 2

Find the Cartesian equations of a line that is parallel to the vector $\boldsymbol{a} = 3\boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k}$ and passes through the point $\mathrm{X}(3, -2, 0)$.

[5 marks]

Question 3

Find the equation of the line that is normal to the vector $4\boldsymbol{i}+5\boldsymbol{j}$ and passes through the point $\mathrm{P}(7,-1)$, leaving your answer in the form $ax+by+c=0$, where $a,\ b$ and $c\in\mathbb{Z}$.

[6 marks]

Question 4a

Consider the two lines l_1 and l_2 defined by the equations:

$$l_1 : \boldsymbol{a}=\begin{pmatrix}4\\1\\6\end{pmatrix}+\lambda\begin{pmatrix}1\\-3\\-5\end{pmatrix}$$

$$l_2 : \boldsymbol{b}=\begin{pmatrix}5\\-11\\10\end{pmatrix}+\mu\begin{pmatrix}-1\\6\\2\end{pmatrix}$$

a)

Find the scalar product of the direction vectors.

[2 marks]

Question 4b

b)
Hence, find the angle, in radians, between the l_1 and l_2.

[4 marks]

Question 5a

Consider the lines l_1 and l_2 defined by:

$$l_1:\begin{cases} x = 3 - \mu \\ y = -2 + 5\mu \\ z = 4 + 2\mu \end{cases}$$

$$l_2:\ \mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}.$$

a)
Show that the lines are not parallel.

[2 marks]

Question 5b

b)
Hence, show that the lines l_1 and l_2 are skew.

[5 marks]

Question 6a

Consider the lines l_1 and l_2 defined by the equations $\boldsymbol{r_1} = \begin{pmatrix} t \\ -2 \\ 5 \end{pmatrix} + \alpha \begin{pmatrix} -5 \\ 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{r_2} = \begin{pmatrix} -3 \\ 6 \\ 9 \end{pmatrix} + \beta \begin{pmatrix} 15 \\ 3k \\ -3 \end{pmatrix}$.

a)
Given that l_1 and l_2 are coincident, find the value of k.

[2 marks]

Question 6b

b)
Find the value of t.

[4 marks]

Question 7a

Two ships A and B are travelling so that their position relative to a fixed point O at time t, in hours, can be defined by the position vectors $\boldsymbol{r}_{\mathrm{A}} = (2-t)\boldsymbol{i} + (4+3t)\boldsymbol{j}$ and $\boldsymbol{r}_{\mathrm{B}} = (t-8)\boldsymbol{i} + (29-2t)\boldsymbol{j}$.

The unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ are a displacement of 1 km due East and North of O respectively.

a)
Find the coordinates of the initial position of the two ships.

[2 marks]

Question 7b

b)
Show that the two ships will collide and find the time at which this will occur.

[3 marks]

Question 7c

c)
Find the coordinates of the point of collision.

[2 marks]

Question 8a

The lines l_1 and l_2 can be defined by:

$$l_1 : \boldsymbol{r} = \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} + \alpha \begin{pmatrix} 3 \\ 2 \\ k \end{pmatrix}$$

$$l_2 : \boldsymbol{s} = \begin{pmatrix} -3 \\ -4 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} -11 \\ -3 \\ 5 \end{pmatrix}$$

a)
Write down the parametric equations for l_1.

[2 marks]

Question 8b

b)
Given that l_1 and l_2 intersect at point T,

(i)
find the value of k.

(ii)
determine the coordinates of the point of intersection, T.

[7 marks]

Question 9a

Consider the triangle ABC. The points A, B and C have coordinates $(4,\ 0,\ -3)$, $(2,\ -2,\ -1)$ and $(7,\ 1,\ 5)$ respectively.

M is the midpoint of $[\mathrm{AB}]$.

a)
Find the coordinates of the midpoint M.

[2 marks]

Question 9b

b)
Hence, find a vector equation of the line that passes through points C and M.

[2 marks]

Question 9c

The point P is the midpoint of $[\mathrm{BC}]$. The line passing through points A and P can be defined by $\boldsymbol{a} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 5 \end{pmatrix}$.

c) Show that the line AP intersects CM at the point $\left(\frac{13}{3}, -\frac{1}{3}, \frac{1}{3}\right)$.

[5 marks]

Question 10a

A car, moving at constant speed, takes 4 minutes to drive in a straight line from point $\mathrm{A}(-3, 5)$ to point $\mathrm{B}(7, 11)$.

At time t, in minutes, the position vector of the car relative to the origin can be given in the form $\boldsymbol{p} = a + tb$.

a)
Find the vectors a and b.

[3 marks]

Question 10b

A cat has decided to take a nap at point $X(4, 9)$.

b)
Show that the cat does not lie on the route along which the car drives.

[3 marks]

Question 10c

c)
Find the shortest distance between the car and the cat during the movement of the car.

[6 marks]