IB Physics DP

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11. Electromagnetic Induction (HL only)

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11.1 Electromagnetic Induction

11.1.1 Emf, Magnetic Flux & Magnetic Flux Linkage

Emf, Magnetic Flux & Magnetic Flux Linkage

Electromagnetic Induction

- When a conducting wire moves through a magnetic field, a **potential difference** is created along the wire
  - If the wire is part of a closed circuit then an e.m.f is induced
- We can produce a **current** in a wire simply by moving a magnet near to it
  - Electrical energy is produced by the system since work is done on the wire by moving the magnet relative to the free electrons within it
- Therefore, electromagnetic induction is the term applied when an e.m.f. is induced in a closed circuit conductor due to it moving through a magnetic field
  - Examples are a flat coil or a solenoid
- Electromagnetic induction happens when a conductor **cuts** through magnetic field lines
  - The amount of e.m.f induced is determined by the magnetic flux **and** the area on which the magnetic field acts

Magnetic Flux

- **Magnetic flux** is defined as:

The product of the magnetic flux density and the cross-sectional area perpendicular to the direction of the magnetic flux density

- Magnetic flux when the field and motion are at 90° can be calculated using the simple equation:

$$\Phi = BA$$

  - Where:
    - Φ = magnetic flux (Wb)
    - B = magnetic flux density(T)
    - A = cross-sectional area (m^2)

Changing Angle

- The flux is the total magnetic field that passes through a given area
  - It is a maximum when the magnetic field lines are **perpendicular** to the plane of the area
  - It is zero when the magnetic field lines are **parallel** to the plane of the area
- For a coil, the amount of magnetic flux varies as the coil rotates within the field

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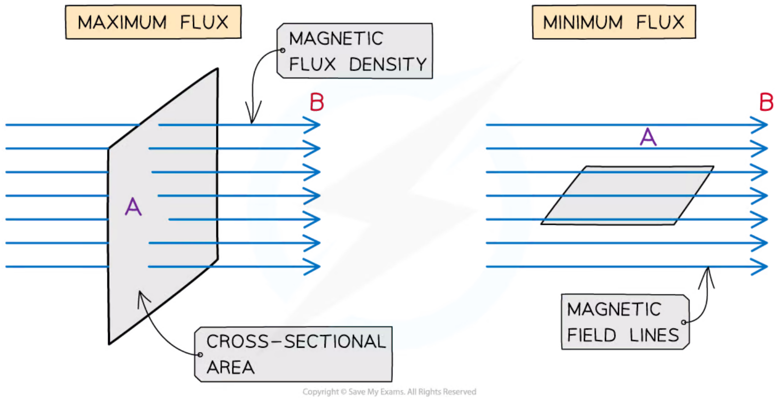

The magnetic flux is maximum when the magnetic field lines and the area they are travelling through are perpendicular

- In other words, magnetic flux is the **number of magnetic field lines through a given area**
- When the magnetic field lines are not completely perpendicular to the area A, then the component of magnetic flux density B is perpendicular to the area is taken
- The equation then becomes:

$$\Phi = BA\cos(\theta)$$

- Where:
  - Φ = magnetic flux (Wb)
  - B = magnetic flux density (T)
  - A = cross-sectional area (m^2)
  - θ = angle between magnetic field lines and the line perpendicular to the plane of the area (often called the normal line) (degrees)
- This means the magnetic flux is:
  - **Maximum** = BA when $\cos(\theta) = 1$ therefore $\theta = 0°$. The magnetic field lines are perpendicular to the plane of the area
  - **Minimum** = 0 when $\cos(\theta) = 0$ therefore $\theta = 90°$. The magnetic fields lines are parallel to the plane of the area

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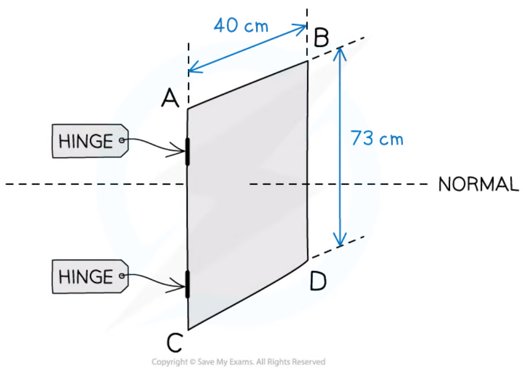
Worked Example

An aluminium window frame has width of 40 cm and length of 73 cm.



The frame is hinged along the vertical edge AC. When the window is closed, the frame is normal to the Earth's magnetic field with magnetic flux density 1.8×10^{-5} T.

a) Calculate the magnetic flux through the window when it is closed.

b) Sketch the graph of the magnetic flux against angle between the field lines and the normal when the window is opened and rotated by 180°

Part (a)

Step 1: Write out the known quantities

- Cross-sectional area, $A = 40 \text{ cm} \times 73 \text{ cm} = (40 \times 10^{-2}) \times (73 \times 10^{-2}) = 0.292 \text{ m}^2$
- Magnetic flux density, $B = 1.8 \times 10^{-5}$ T

Step 2: Write down the equation for magnetic flux

$$\Phi = BA$$

Step 3: Substitute in values

$$\Phi = (1.8 \times 10^{-5}) \times 0.292 = 5.256 \times 10^{-6} = 5.3 \times 10^{-6} \text{ Wb}$$

Part (b)

- The magnetic flux will be at a minimum when the window is opened by 90° and a maximum when fully closed or opened to 180°
- This is shown by the graph:

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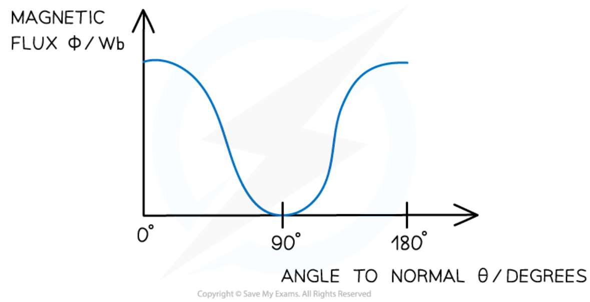


Magnetic Flux Linkage

- More coils in a wire mean a **larger** e.m.f is induced
- The **magnetic flux linkage** is a quantity commonly used for solenoids which are made of N turns of wire
- The flux linkage is defined as:

The product of the magnetic flux and the number of turns of the coil

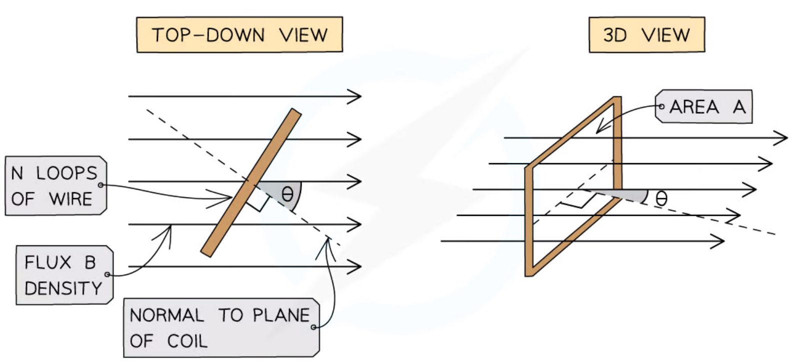
- It is calculated using the equation:

Magnetic flux linkage = ΦN = BAN

- Where:
  - Φ = magnetic flux (Wb)
  - N = number of turns of the coil
  - B = magnetic flux density (T)
  - A = cross-sectional area (m^2)

- The flux linkage ΦN has the units of **Weber turns (Wb turns)**

- An e.m.f is induced in a circuit when the magnetic flux linkage changes with respect to time
- This means an e.m.f is induced when there is:
  - A changing magnetic flux density B
  - A changing cross-sectional area A
  - A change in angle θ



The magnetic flux through a rectangular coil decreases as the angle between the field lines and plane decrease

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- Magnetic flux linkage also changes with the rotation of the coil
  - It is at a maximum when the field lines are perpendicular to the plane of the area they are passing through
- Therefore, the component of the flux density which is perpendicular is equal to:

$$\Phi N = BAN\cos(\theta)$$

- Where:
  - N = number of turns of the coil

Exam Tip

The vocabulary in this topic; induced emf, induced current, magnetic flux, magnetic flux density, can be confusing. Until you are absolutely clear which is which, you will struggle to answer questions using the correct equation. Spend some time with your revision notes making sure you are secure with using and understanding the terms.

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11.1.2 Induced Emf

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Induced Emf

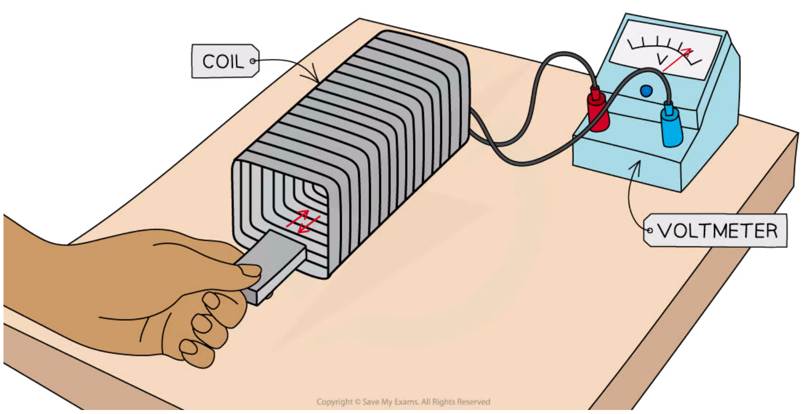
- Electromagnetic induction is a phenomenon which occurs when an e.m.f. is induced when a conductor moves through a magnetic field
- When the conductor cuts through the magnetic field lines:
  - This causes a change in magnetic flux ($\Delta\Phi$)
  - Which causes **work to be done**
  - This work is then transformed into **electrical energy**
- Therefore, if attached to a complete circuit, a current will be induced
- This is known as **electromagnetic induction** and is defined as:

The process in which an e.m.f is induced in a closed circuit due to changes in magnetic flux

- This can occur either when:
  - A conductor cuts through a magnetic field
  - The direction of a magnetic field through a coil changes
- Electromagnetic induction is used in:
  - Electrical **generators** which convert mechanical energy to electrical energy
  - **Transformers** which are used in electrical power transmission
- This phenomenon can easily be demonstrated with a magnet and a coil, or a wire and two magnets

Experiment 1: Moving a magnet through a coil

- When a coil is connected to a sensitive voltmeter, a bar magnet can be moved in and out of the coil to induce an e.m.f



A bar magnet is moved through a coil connected to a voltmeter to induce an e.m.f

The expected results are:

- When the bar magnet is **not moving**, the voltmeter shows a **zero reading**
  - When the bar magnet is held still inside, or outside, the coil, the rate of change of flux is zero, so, there is **no e.m.f induced**

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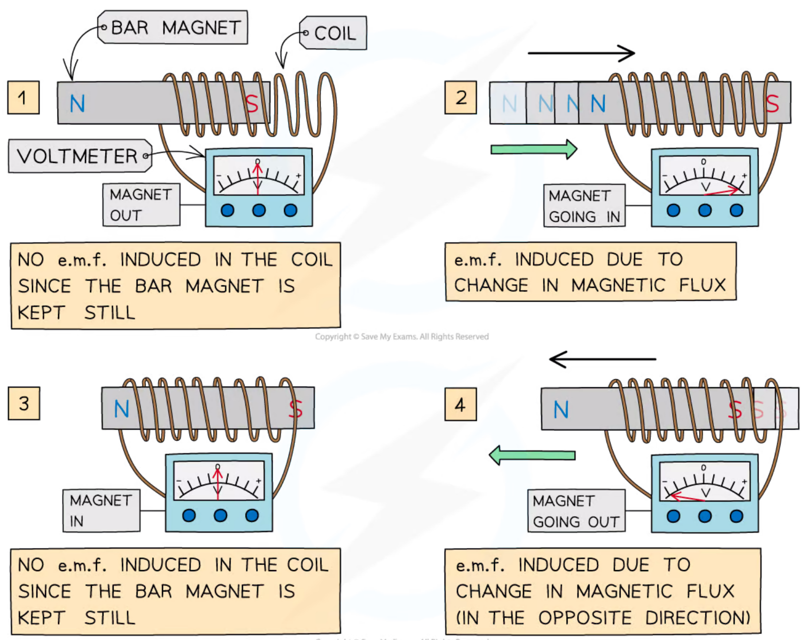
- When the bar magnet begins to move inside the coil, there is a reading on the voltmeter
  - As the bar magnet moves, its magnetic field lines 'cut through' the coil, generating a **change in magnetic flux** ($\Delta\Phi$)
  - This induces an **e.m.f** within the coil, shown momentarily by the reading on the voltmeter
- When the bar magnet is taken back out of the coil, an e.m.f is induced in the **opposite direction**
  - As the magnet changes direction, the direction of the current changes
  - The voltmeter will momentarily show a reading with the opposite sign
- Increasing the **speed** of the magnet induces an e.m.f with a **higher magnitude**
  - As the speed of the magnet increases, the rate of change of flux increases
- The direction of the electric current, and e.m.f, induced in the conductor is such that it **opposes** the change that produces it



An e.m.f is induced only when the bar magnet is moving through the coil

- Factors that will increase the induced e.m.f are:
  - Moving the magnet **faster** through the coil
  - Adding more **turns** to the coil
  - Increasing the **strength** of the bar magnet

Experiment 2: Moving a wire through a magnetic field

- When a long wire is connected to a voltmeter and moved between two magnets, an e.m.f is induced
  - **Note:** there is no current flowing through the wire to start with

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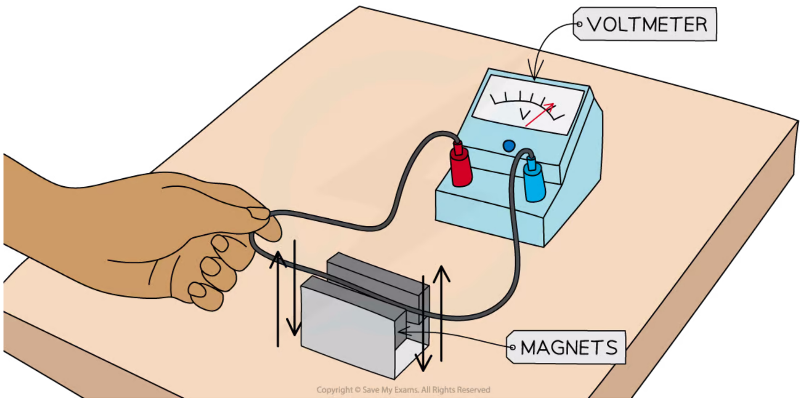

A wire is moved between two magnets connected to a voltmeter to induce an e.m.f

The expected results are:

- When the wire is **not moving**, the voltmeter shows a **zero reading**
  - When the wire is held still inside, or outside, the magnets the rate of change of flux is zero so there is **no e.m.f induced**

- As the wire is moved through between the magnets, an **e.m.f** is induced within the wire, shown momentarily by the reading on the voltmeter
  - As the wire moves, it 'cuts through' the magnetic field lines of the magnet, generating a **change in magnetic flux**

When the wire is taken back out of the magnet, an e.m.f is induced in the **opposite direction**

- As the wire changes direction, the direction of the current changes
- The voltmeter will momentarily show a reading with the opposite sign

- As before, the direction of the electric current, and e.m.f, induced in the conductor is such that it **opposes** the change that produces it

- Factors that will increase the induced e.m.f are:
  - Increasing the **length** of the wire
  - Moving the wire between the magnets **faster**
  - Increasing the **strength** of the magnets

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11.1.3 Faraday's Law

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Faraday's Law

- Faraday's Law connects the **rate** of change of flux linkage with induced e.m.f
- It is defined in words as:

The magnitude of the induced e.m.f is directly proportional to the rate of change of magnetic flux linkage

- Faraday's Law as an equation is defined as:

$$\varepsilon = \frac{\Delta(N\Phi)}{\Delta t}$$

- Where:
  - ε = induced e.m.f (V)
  - $\Delta(N\phi)$ = change in flux linkage (Wb turns)
  - Δt = time interval (s)

- If the interval of time becomes very small (i.e., in the limit of $\Delta t \to 0$) the equation for Faraday's Law can be written as:

$$\varepsilon = \frac{d(N\Phi)}{dt}$$

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Problems Involving Faraday's Law

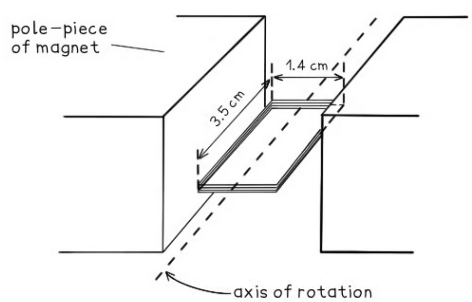
- Lenz's law combined with Faraday's law is given by the equation:

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

- This equation shows:
  - When a bar magnet goes through a coil, an e.m.f is induced within the coil due to a change in magnetic flux
  - A current is also induced which means the coil now has its own magnetic field
  - The coil's magnetic field acts in the **opposite direction** to the magnetic field of the bar magnet (shown by the minus sign)
- If a direct current (d.c) power supply is replaced with an alternating current (a.c) supply, the e.m.f induced will also be alternating with the same frequency as the supply

Worked Example

A small rectangular coil contains 350 turns of wire. The longer sides are 3.5 cm and the shorter sides are 1.4 cm.



The coil is held between the poles of a large magnet so that the coil can rotate about an axis through its centre. The magnet produces a uniform magnetic field of flux density 80 mT between its poles. The coil is positioned horizontally and then turned through an angle of 40° in a time of 0.18 s.

Calculate the magnitude of the average e.m.f induced in the coil.

Step 1: Write down the known quantities

- Magnetic flux density, $B = 80\text{ mT} = 80 \times 10^{-3}$ T
- Area, $A = 3.5 \times 1.4 = (3.5 \times 10^{-2}) \times (1.4 \times 10^{-2}) = 4.9 \times 10^{-4}$ m^2
- Number of turns, $N = 350$
- Time interval, $\Delta t = 0.18$ s

Step 2: Write out the equation for Faraday's law:

$$\varepsilon = \frac{\Delta(N\Phi)}{\Delta t}$$

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Step 3: Write out the equation for the change in flux linkage:

- The number of turns *N* and the coil area *A* stay constant
- The flux through the coil changes as it rotates
- Therefore, the change in flux linkage can be written as:

$$\Delta(N\Phi) = NA(\Delta B)$$

Step 4: Determine the change in magnetic flux linkage

- The initial flux through the coil is zero (flux lines are parallel to the coil face)
- The final flux through the coil is 80 mT (flux lines are perpendicular to the coil face)
- This is because the coil begins horizontally in the field and is rotated 90°
- Therefore, the change in flux linkage is:

$$\Delta(N\Phi) = NA(\Delta B) = 350 \times (4.9 \times 10^{-4}) \times (80 \times 10^{-3}) = 0.014 \text{ Wb turns}$$

Step 5: Substitute change in flux linkage and time into Faraday's law equation:

$$\varepsilon = \frac{0.014}{0.18} = 0.076 \text{ V}$$

Exam Tip

The 'magnitude' of the e.m.f just means its size, rather than direction. This is often what is required in exam questions, so the minus sign in Lenz's law is often not needed in calculations.

However, you may be expected to explain the significance of the minus sign. Be prepared to interpret it as an expression of Lenz's Law. You can find this is described on the next page.

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11.1.4 Lenz's Law

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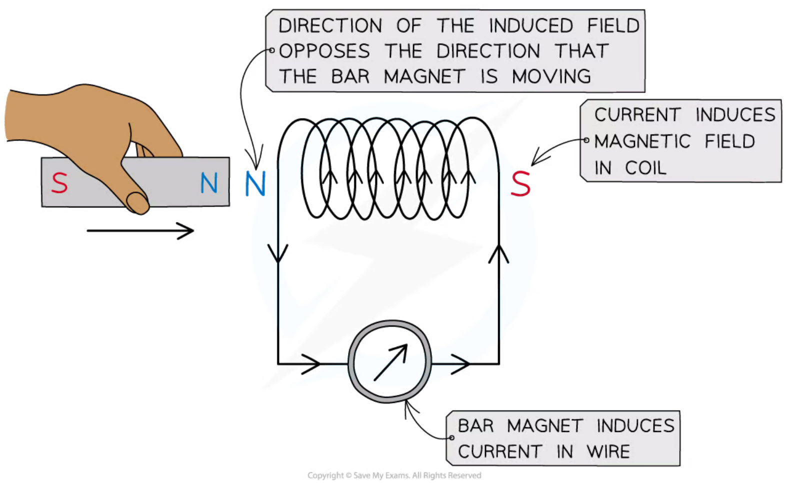
Lenz's Law

- Lenz's Law is used to predict the **direction** of an induced e.m.f in a coil or wire
- Lenz's Law is summarised below:

The induced e.m.f is set up in a direction to produce effects that oppose the change causing it

Experimental Evidence for Lenz's Law

- To verify Lenz's Law, the only apparatus needed is:
  - A bar magnet
  - A coil of wire
  - A sensitive ammeter
- Note, a cell is **not** required



Lenz's law can be verified using a coil connected in series with a sensitive ammeter and a bar magnet

- A known pole (either north or south) of a bar magnet is pushed into the coil
  - This induces an e.m.f in the coil
  - The induced e.m.f drives a current (because it is a complete circuit)
- Lenz's Law dictates:
  - The direction of the e.m.f, and hence the current, must be set up to **oppose** the incoming magnet
  - Since a **north pole** approaches the coil face, the e.m.f must be set up to create an induced **north pole**
  - This is because two north poles will **repel** each other
- The direction of the current is therefore as shown in the image above
  - The direction of current can be verified using the right hand grip rule
  - Fingers curl around the coil in the direction of current and the thumb points along the direction of the flux lines, from north to south

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  - Therefore, the current flows in an anti-clockwise direction in the image shown, in order to induce a north pole opposing the incoming magnet

- Reversing the magnet direction would give an opposite deflection on the voltmeter
  - Lenz's Law now predicts a south pole induced at the coil entrance
  - This would **attract** the north pole attempting to leave
  - Therefore, the induced e.m.f **always** produces effects to oppose the changes causing it

- Lenz's Law is a direct consequence of the **principle of conservation of energy**
  - Electromagnetic effects will not create electrical energy out of nothing
  - In order to induce and sustain an e.m.f, for instance, **work** must be done in order to overcome the repulsive effect due to Lenz's Law

Exam Tip

A typical exam question may ask you to explain the presence of the negative sign in Faraday's Law, which is the equation that tells you the size of the induced e.m.f ε as:

$$\varepsilon = -\frac{d(N\Phi)}{dt}$$

You should remember that the negative sign is representative of **Lenz's Law,** which says that the induced e.m.f ε is set up **to oppose the change causing it.** The negative indicates motion in an opposing direction.

The change causing the induced e.m.f is the changing flux linkage, which is represented by the quantity $\frac{d(N\Phi)}{dt}$.

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11.2 Power Generation & Transmission

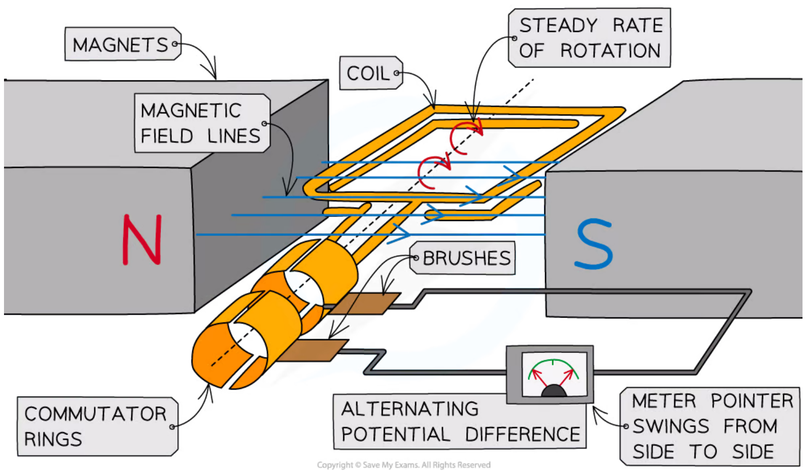
11.2.1 AC Generators

AC Generators

- If a coil of wire is rotated inside a magnetic field by an external force, an emf will be generated in the wire which causes current to flow within the coil
- The **generator effect** can be used to:
  - Generate **a.c.** in an **alternator**
  - Generate **d.c.** in a **dynamo**

Alternators

- A simple alternator is a type of generator that converts mechanical energy to electrical energy in the form of alternating current



An alternator is a rotating coil in a magnetic field connected to commutator rings

- A rectangular coil that is forced to spin in a **uniform magnetic field**
- The coil is connected to a centre-reading meter by **metal brushes** that press on two metal **slip rings** (or commutator rings)
  - The slip rings and brushes provide a continuous connection between the coil and the meter
- When the coil turns in one direction:
  - The pointer defects first one way, then the opposite way, and then back again
  - This is because the coil **cuts through** the magnetic field lines and a **potential difference,** and therefore current, is **induced** in the coil
- The pointer deflects in both directions because the current in the circuit repeatedly **changes direction** as the coil spins
  - This is because the induced potential difference in the coil repeatedly changes its direction
  - This continues on as long as the coil keeps turning in the **same** direction

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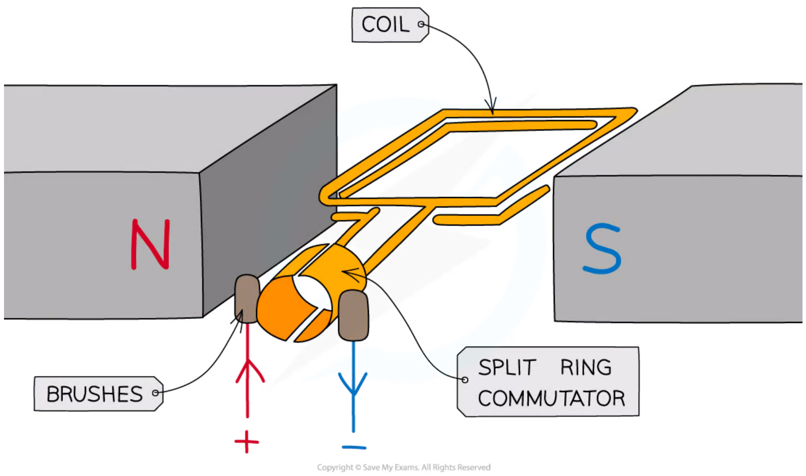
- The induced potential difference and the current **alternate** because they repeatedly **change direction**

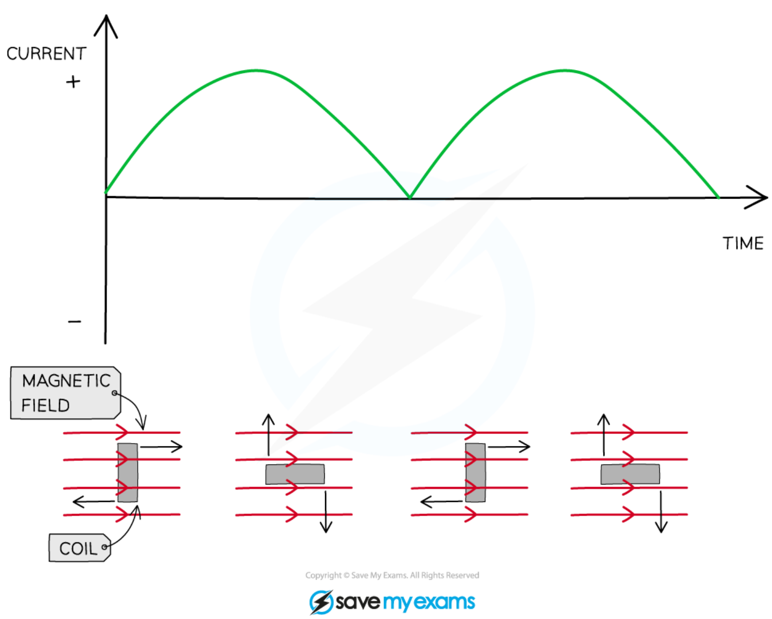
Dynamos

- A dynamo is a direct-current generator
- A simple dynamo is the **same** as an **alternator** except that the dynamo has a **split-ring commutator** instead of two separate slip rings



A dynamo is a rotating coil in a magnetic field connected to a split ring commutator

- As the coil rotates, it **cuts** through the field lines
  - This **induces a potential difference** between the end of the coil
- The split ring commutator changes the connections between the coil and the brushes every half turn in order to keep the current leaving the dynamo in the **same direction**
  - This happens each time the coil is perpendicular to the magnetic field lines



D.C output from a dynamo – the current is only in the positive region of the graph

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- Therefore, the induced potential difference **does not reverse** its direction as it does in the alternator
- Instead, it varies from zero to a maximum value twice each cycle of rotation, and never changes polarity (positive to negative)
    - This means the current is always **positive** (or always **negative**)

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11.2.2 Root-Mean-Square Current & Voltage

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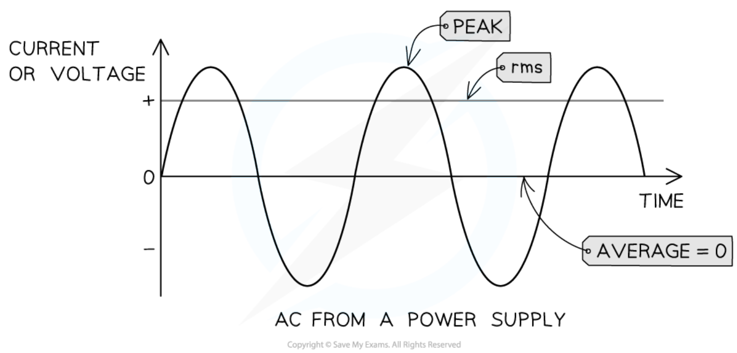
Root-Mean-Square Current & Voltage

- Direct current sources provide a constant voltage and current over time, making it easy to measure
- In situations involving alternating voltage and current, the average values of voltage and current will **always be zero**
  - This can make it difficult to measure



The mean value for alternating current and voltage is always zero

- The use of **root mean square** values gets around this problem
  - First remove all the negative signs by simply squaring the peak current, or voltage
  - Find the average of the squared value
  - And finally, take the square root

- Root-mean-square (rms) values of current, or voltage, are a useful way of **comparing** a.c current, or voltage, to its equivalent direct current (d.c), or voltage
  - The rms values represent the direct current, or voltage, values that will produce the same **heating effect**, or power dissipation, as the alternating current, or voltage

- The rms value of an alternating current is defined as:

  The square root of the mean of squares of all the values of the current in one cycle

- An alternate definition is:

  The equivalent direct current that produces the same power

- The rms current I_{rms} is defined by the equation:

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

- Where:
  - I_0 = peak current (A)

- The rms value of an alternating voltage is defined as:

  The square root of the mean of squares of all the values of the voltage in one cycle

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- An alternate definition is:

  The equivalent dc voltage that produces the same power

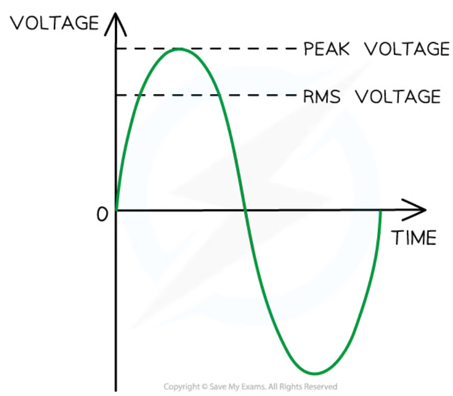
- The rms voltage V_{rms} is defined by the equation:

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

- Where:
  - V_0 = peak voltage (V)
- Rms current is equal to 0.707 × I_0, which is about 70% of the peak current I_0
  - This is also the case for rms voltage
- The rms value is therefore defined as:

  The steady direct current, or voltage, that delivers the same average power in a resistor as the alternating current, or voltage

- A resistive load is any electrical component with resistance eg. a lamp



V_{rms} and peak voltage. The rms voltage is about 70% of the peak voltage

Worked Example

An electric oven is connected to a 230 V root mean square (rms) mains supply using a cable of negligible resistance.

Calculate the peak-to-peak voltage of the mains supply.

Step 1: Write down the V_{rms} equation

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

Step 2: Rearrange for the peak voltage, V_0

$$V_0 = \sqrt{2} \times V_{rms}$$

Step 3: Substitute in the values

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$$V_0 = \sqrt{2} \times 230$$

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Step 4: Calculate the peak-to-peak voltage

- The peak-to-peak voltage is the peak voltage (V_0) × 2
- Peak-to-peak voltage = ($\sqrt{2}$ × 230) × 2 = 650.538 = **651 V (3 s.f)**

Exam Tip

Remember to double-check the units on the alternating current and voltage graphs. These are often shown in milliseconds (ms) instead of seconds (s) on the x-axis.

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Average Power Calculations

- The average **power** of a supply is the product of the rms current and voltage:

$$\text{Average power} = I_{rms} \times V_{rms}$$

Worked Example

What is the maximum current supplied to a 2300 W kettle which is connected to an a.c. supply of peak voltage 325 V?

Step 1: Write down the V_{rms} equation

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

Step 2: Substitute in the values and calculate V_{rms}

$$V_{rms} = \frac{325}{\sqrt{2}} = 230 \text{ V}$$

Step 3: Write down the equation for average power

$$\text{Average power} = I_{rms} \times V_{rms}$$

Step 4: Rearrange the equation for I_{rms}

$$I_{rms} = \frac{P_{average}}{V_{rms}}$$

Step 5: Substitute in the values and calculate I_{rms}

$$I_{rms} = \frac{2300}{230} = 10 \text{ A}$$

Step 6: Write down the equation for I_0

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

Step 7: Rearrange for I_0 and substitute in the values

$$I_0 = I_{rms} \times \sqrt{2}$$

$$I_0 = 10 \times \sqrt{2} = 14.1 \text{ A}$$

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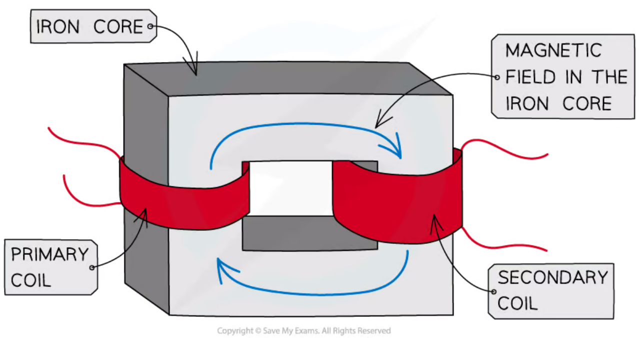
Step-Up & Step-Down Transformers

- A transformer is

A device that changes high alternating voltage at low current to low alternating voltage at high current, and vice versa

- This is designed to reduce heat energy lost whilst electricity is transmitted down electrical power lines from power stations to the national grid
- A transformer is made up of:
  - A primary coil
  - A secondary coil
  - An iron core
- The primary and secondary coils are wound around the soft iron core
  - The soft iron core is necessary because it focuses and directs the magnetic field from the primary to the secondary coil
  - Soft iron is used because it can easily be magnetised and demagnetised



A step-up transformer has more turns in the secondary coil than primary

- In the primary coil, an alternating current producing an alternating voltage is applied
  - This creates an **alternating magnetic field** inside the iron core and therefore a changing magnetic flux linkage
- A changing magnetic field passes through to the secondary coil through the iron core
  - This results in a changing magnetic flux linkage in the secondary coil and from Faraday's Law, an **e.m.f is induced**
- An e.m.f produces an alternating output voltage from the secondary coil
- The output alternating voltage is at the **same** frequency as the input voltage
- A **step-up** transformer has **more coils** in the **secondary** than the primary and the **secondary voltage** is **larger** than the primary voltage
- A **step-down** transformer has **more coils** in the **primary** than the secondary and the **secondary voltage** is **smaller** than the primary voltage

Energy losses

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- **Eddy currents** are small currents created in the iron core that come from the **changing magnetic fields**
  - These current move **free electrons** within the core causing **heating** of the core and therefore energy dissipation
- By replacing the solid iron core with a **laminated core**, power **losses** are **decreased** from eddy currents
- If there is **flux leakage** from the transformer, there could be **further** eddy currents and **power losses** in the surrounding metallic structure of the transformer device
- When switching the magnetic field it changes the alignment of the magnetic dipoles, this requires some work which is known as **magnetic hysteresis**

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11.2.4 Transformer Calculations

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Transformer Calculations

- The transformer equation is:

$$\frac{\varepsilon_p}{\varepsilon_s} = \frac{N_p}{N_s}$$

- Where:
  - N_s = number of turns in the secondary coil
  - N_p = number of turns in the primary coil
  - ε_s = output voltage from the secondary coil (V)
  - ε_p = input voltage in the primary coil (V)

There are two types of transformers:

- **Step-up** transformer (increases the voltage of the power source) where **Ns > Np**
- **Step-down** transformer (decreases the voltage of the power source) where **Np > Ns**
- For an ideal transformer, there is no electrical energy lost and it is 100% efficient
- This means the power in the primary coil equals the power in the second coil;

$$\frac{\varepsilon_p}{\varepsilon_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p}$$

- Where:
  - I_p = current in the primary coil (A)
  - I_s = output current from the secondary coil (A)

Worked Example

A step-down transformer turns a primary voltage of 0.5 kV into a secondary voltage of 100 V. Calculate the number of turns needed in the secondary coil if the primary coil contains 3000 turns of wire.

Step 1: List the known quantities

- Primary voltage, $\varepsilon_p = 0.5 \text{ kV} = 0.5 \times 10^3 \text{ V}$
- Secondary voltage, $\varepsilon_s = 100 \text{ V}$
- Number of turns in the primary coil, $N_p = 3000$ turns

Step 2: Write down the transformer equation

$$\frac{\varepsilon_p}{\varepsilon_s} = \frac{N_p}{N_s}$$

Step 3: Rearrange for number of turns in the secondary coil

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$$N_s = \frac{N_p \times \varepsilon_s}{\varepsilon_p}$$

Step 4: Substitute in the values

$$N_s = \frac{3000 \times 100}{500} = 600 \text{ turns}$$

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11.2.5 AC Electrical Power Distribution

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AC Electrical Power Distribution

- Energy losses due to the heating of transmission lines in national power grids are **significant**
  - This is because the electrical energy is transmitted across long distances from power stations to buildings
- Inefficiencies in a transformer appear not from just the core, but also in the wires
- The coils of wire have **resistance**
  - This causes heat energy to be lost from the current flowing through the coils
  - The larger the current, the greater the amount of heat energy lost
- In the core, the **inefficiencies** appear from:
  - Induced eddy currents
  - The reversal of magnetism
  - Poor insulation between the primary and secondary coil
- Ways to **reduce energy loss** in a transformer are:
  - Making the core from soft iron or iron alloys to allow easy magnetisation and demagnetisation and reduce hysteresis loss
  - Laminating the core
  - Using thick wires, especially in the secondary coil of step-down transformers
  - Using a core that allows all the flux due to the primary coil to be linked to the secondary coil
- Power losses from the current are calculated using the equation:

$$P = I^2R$$

- Where:
  - P = power (W)
  - I = current (A)
  - R = resistance (Ω)
- The equation shows that:
  - $P \propto I^2$
  - This means **doubling** the current produces **four** times the power loss
- Therefore, step-up transformers are used to **increase** the **voltage** which **decreases** the **current** through transmission lines
  - This **reduces** the overall heat **energy lost** in the wires during transmission
- A step-down transformer is then used to decrease the voltage to that required in homes and buildings

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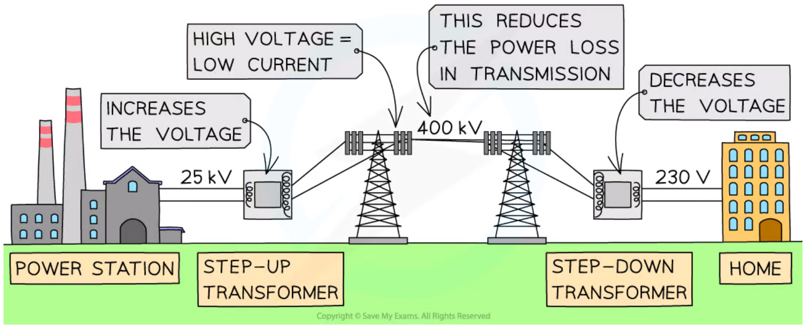

The use of step-up and step-down transformers in the National Grid

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Worked Example

A current of 2500 A is transmitted through 150 km of cables. The resistance of the transmission cable is 0.15 Ω per km.

Calculate the power wasted.

Step 1: List the known quantities

- Current, I = 2500 A
- Length of cables, L = 150 km = 150×10^3 m
- Resistance of the cables, R = 0.15 Ω km^{-1}

Step 2: Write out the power equation

$$P = I^2R$$

Step 3: Determine the total resistance, *R*

$$R = \text{Resistance of the wires} \times \text{Length of wires}$$

$$R = 0.15 \times 150$$

Step 4: Substitute values into the power equation

$$\text{Power lost} = (2500)^2 \times (0.15 \times 150) = \mathbf{141\ MW}$$

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11.2.6 Diode Bridges

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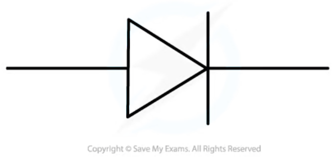
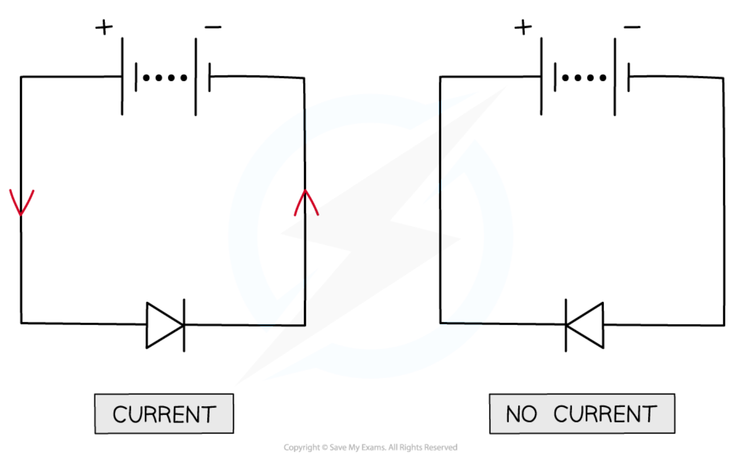
Diode Bridges

- Most electronic devices cannot use alternating current, they require direct current instead
  - This is possible by use of a **rectifier**
- Rectifiers convert alternating current (ac) to direct current (dc)
- There are two types of rectifier:
  - The **half-wave** rectifier
  - The **full-wave** rectifier
- A commonly used rectifier is a **diode bridge**
- A diode is a component which only allows charge to flow through it in one direction



A diode is a component which only allows a current when the potential difference is in the direction of the arrow

- If a power source is connected from the negative to the positive terminal, then there will be no current



In order to have a current, the diode must point around the circuit from positive to negative

- If a diode is connected to an a.c. (alternating current) power supply, it will only supply a current half of the time leading to the production of direct current
  - This is the process of **half-wave rectification**

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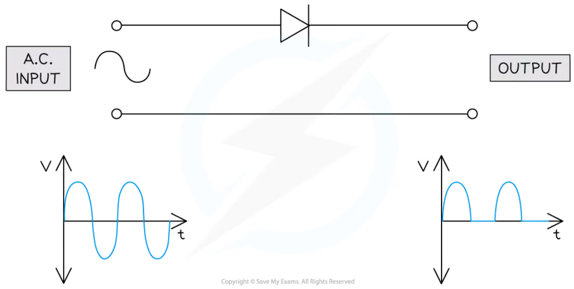
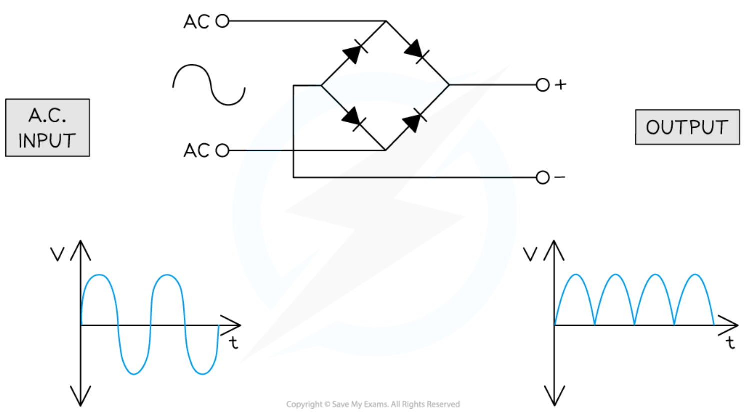


A diode can be used to rectify an alternating current into a direct current

- A diode bridge is an arrangement of four (or sometimes more) diodes in a bridge circuit configuration
- When diodes are set up in this way and connected to an a.c. supply, this enables the positive half of each cycle to pass through while the negative half of each cycle is reversed
  - This is the process of **full-wave rectification**



A diode bridge can be used in order to achieve full-wave rectification

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Investigating Diode Bridges

Aim of the Experiment

- The aim of the experiment is experimentally verify a half-bridge rectifier will produce partial DC output from AC input and a full wave bridge rectifier will produce near complete DC output from AC input

Variables:

- Independent variable = Input voltage (Alternating Current)
- Dependent variable = Output voltage (Direct Current)

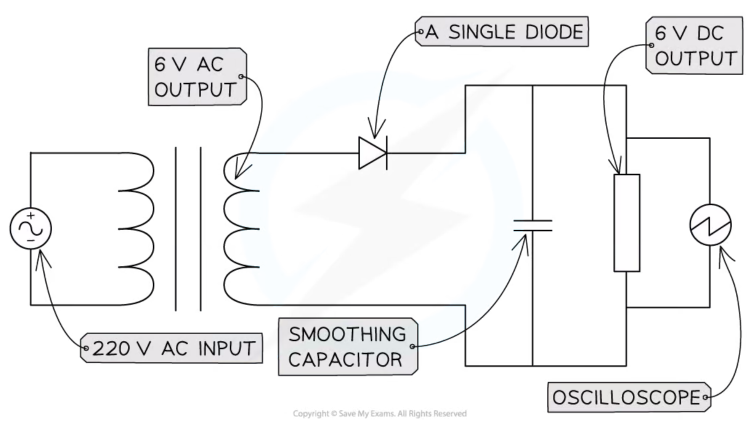
Equipment List

- Step down transformer
- Diode bridge
- Single diode
- Capacitor (330 μF or 50 μF)
- Resistor
- Multimeter
- Oscilloscope
- Connecting wires etc.

- Resolution of measuring equipment:
  - Oscilloscope resolution = 8-bit resolution or better
  - Step-down transformer = 220V to 6V AC
  - Multimeter
    - Ammeter setting = 0.01 A
    - Voltmeter setting = 0.01 V

Methods

Half Wave Rectification



Experimental set-up for investigating half-wave rectification

1. Study the above circuit diagram **carefully** before setting up the appropriate circuit

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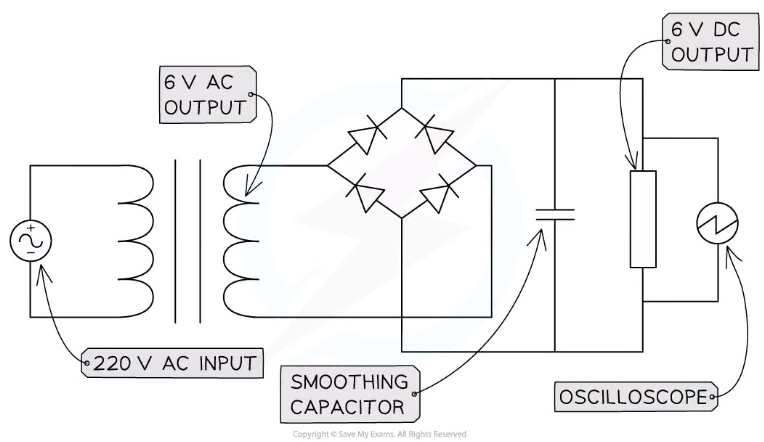
2. Connect the single diode to the step-down transformer and resistor in circuit
3. Connect the resistor to the step-down transformer and the oscilloscope in parallel across the resistor
4. **Before** connecting and turning on the 220 V alternating current (AC) source, voltmeters and ammeters may be set up in appropriate locations to investigate points of interest such as across the diode or the output of the step-down transformer
   - However, the oscilloscope is the most crucial measuring equipment for this experiment and is the only truly necessary measurement required for this practical if the circuit is set-up appropriately
5. Connect the step-down transformer to the 220 V AC source
6. **Observe** the **oscilloscope** and look at the voltage reading to confirm partial Direct Current (DC) output (half-wave rectification)

Full Wave Rectification



Experimental set-up for investigating full-wave rectification

1. Study the above circuit diagram **carefully** before setting up the appropriate circuit
2. Connect the components of the diode bridge ensuring that the necessary parallel and series diodes are connected so that they run alternatively during the opposite phases of alternating current
3. Connect the opposite edges of the diode bridge to the transformer making sure that there is an individual diode facing towards and one facing away from both junctions
4. Connect the capacitor and resistor in parallel with their outer connections to the points of the diode bridge where one point of the diode bridge has both diodes facing towards the junction and the opposite point where both of the diodes are facing away from the junction
5. Connect the oscilloscope in parallel across the resistor to measure the voltage across that component
6. **Before** connecting and turning on the 220 V AC source, voltmeters and ammeters may be set up in appropriate locations to investigate points of interest such as across the diode bridge, over the smoothing capacitor or the output of the step-down transformer
   - However, the oscilloscope is the most crucial measuring equipment for this experiment and is the only truly necessary measurement required for this practical if the circuit is set-up appropriately
7. Connect the 220 V AC source to the step-down transformer
8. **Observe** the **oscilloscope** and look at the voltage reading to confirm DC output

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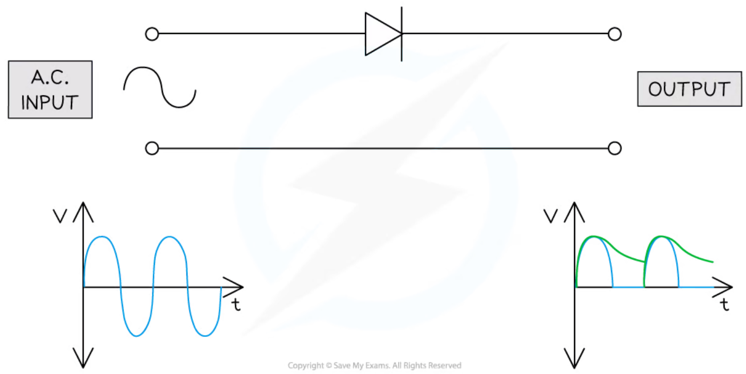
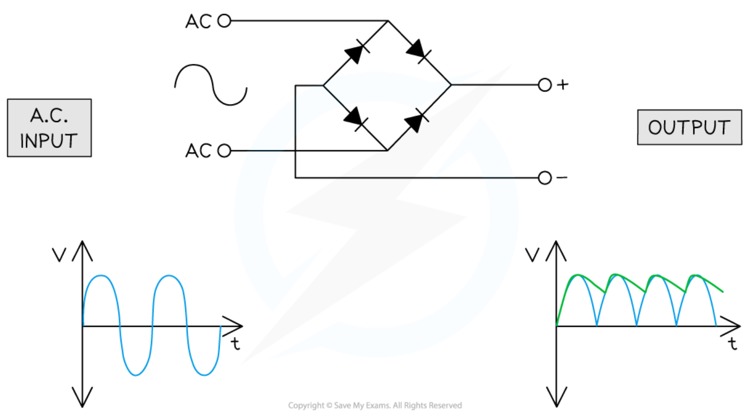
Analysis of Results

- The output of the oscilloscope for half-wave rectification should be similar to the following output:



The green line shows how the capacitor creates a smoothing effect that brings the output of the single diode closer to DC output

- The output for the oscilloscope for full-wave rectification should be similar to the following output:



The green line shows how the capacitor creates a smoothing effect that brings the output of the diode bridge closer to DC output

- Notice the smoothing effect which comes from the capacitor in the circuit and makes the output more like DC in nature

Evaluating the Experiment

Systematic Errors:

- The voltmeter and ammeters should start from zero, to avoid zero error in the readings

Random Errors:

- In practice, the voltmeter and ammeter will still have some resistance, therefore the voltages and currents displayed may be slightly inaccurate
- The temperature of the equipment could affect its resistance. This must be controlled carefully

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- Taking multiple readings of the current for each component will provide a more accurate result and reduce uncertainties

Safety Considerations

- When there is a high current and a thin wire, the wire will become **very hot**
  - Make sure never to touch the wire directly when the circuit is switched on
- **Switch off** the power supply right away if **burning** is smelled
- Make sure there are **no liquids** close to the equipment, as this could damage the electrical equipment
- The components will get hot especially at higher voltages

Alternatives and Variations

- In the case that an oscilloscope is not available, it may be possible to use a multimeter to check that the output is indeed similar to DC and able to be detected as such
  - While the actual voltage pattern will not be visible, this would still allow the DC-like output to be verified and confirm that the diode bridge (full-wave rectification) does work experimentally
- If access to a diode bridge and complete circuit is not possible, then there are often simulations available online which might allow the possibility to do this investigation virtually

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11.2.8 Rectification

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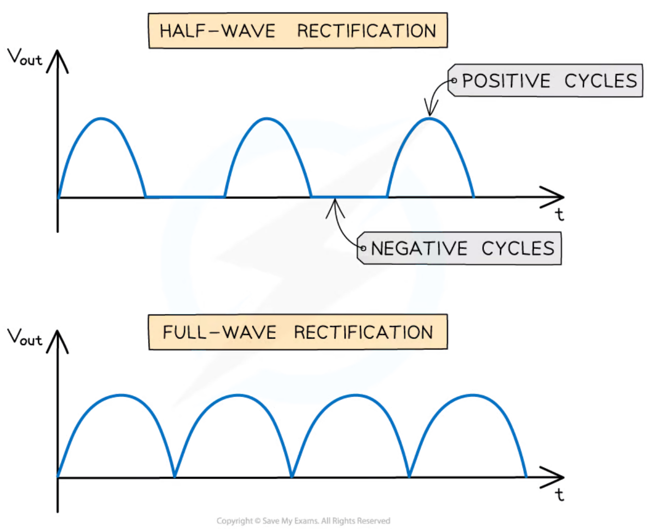
Rectification

- Rectification is defined as:

  The process of converting alternating current and voltage into direct current and voltage

- Rectification is used in electronic equipment which requires a direct current
  - For example, mains voltage must be rectified from the alternating voltage produced at power stations
- There are two types of rectification:
  - Half-wave rectification
  - Full-wave rectification
- For **half-wave** rectification:
  - The graph of the output voltage V_{out} against time is a sine curve with the positive cycles and a flat line (V_{out} = 0) on the negative cycle
  - This is because the diode only conducts in the positive direction
- For **full-wave** rectification:
  - The graph of the output voltage V_{out} against time is a sine curve where the positive cycles and the negative cycles are both curved 'bumps'



The difference between the graphs of full-wave and half-wave rectification

Half-Wave Rectification

- Half-wave rectification consists of a single **diode**
  - An alternating input voltage is connected to a circuit with a load resistor and diode in series
- The diode will only conduct during the positive cycles of the input alternating voltage,
  - Hence there is only current in the load resistor during these positive cycles

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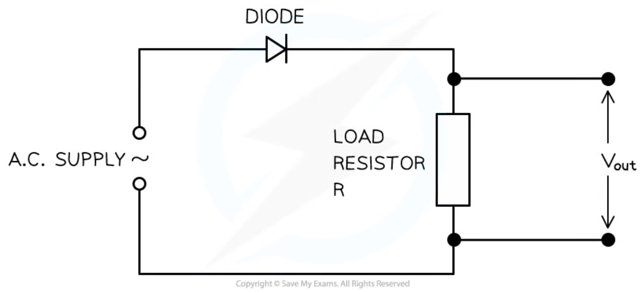
- The output voltage V_{out} across the resistor will fluctuate against time in the same way as the input alternating voltage **except** there are no negative cycles

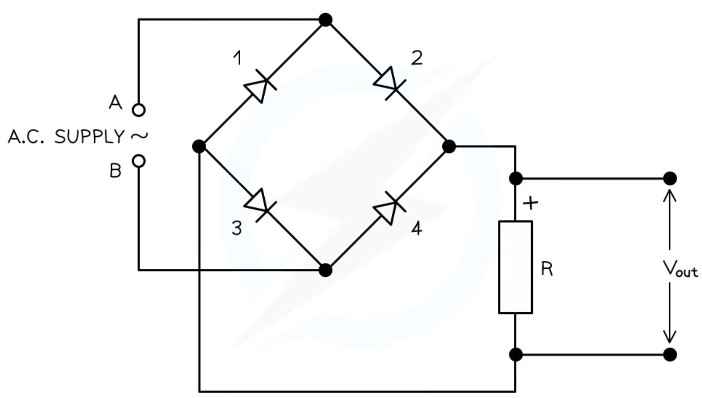


Half-wave rectification requires a single diode and the graph is represented by only the positive cycles

- This type of rectification means half of the time the voltage is zero
- The power available from a half-wave rectified supply is reduced

Full-Wave Rectification

- Full-wave rectification requires a bridge rectifier circuit
  - This consists of **four** diodes connected across an input alternating voltage supply
- The output voltage V_{out} is taken across a load resistor
- During the **positive** cycles of the input voltage, one terminal if the voltage supply is positive and the other negative
  - Two diodes opposite each other that are in forward bias will conduct
  - The other two in reverse bias will not conduct
  - A current will flow in the load resistor with the positive terminal at the top of the resistor
- During the **negative** cycles of the input voltage, the positive and negative terminals of the input alternating voltage supply will swap
  - The two diodes that were in forward bias will now be in reverse bias and not conduct
  - The other two in reverse bias will now be in forward bias and will conduct
  - The current in the load resistor will still flow in the same direction as before



When A is positive and B is negative, diodes 2 and 3 will conduct and 1 and 4 will not. When A is negative and B is positive, diodes 1 and 4 will conduct and diodes 2 and 3 will not. The current in the load resistor R will flow downwards

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- In both the positive and negative cycles, the current in the load resistor is the **same**
- Each diode pair is the same as in half-wave rectification
  - Since there are two pairs, this equates to full-wave rectification overall
- The main advantage of full-wave rectification compared to half-wave rectification is that there is **more power** available
  - Therefore, greater power is supplied on every half cycle

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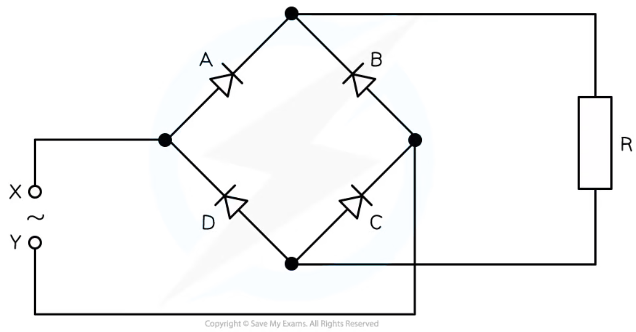
Worked Example

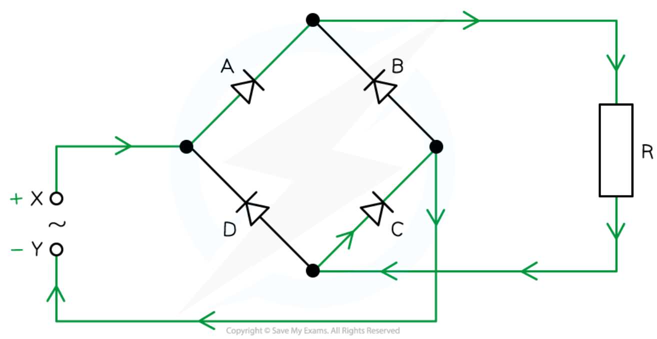
A bridge rectifier consists of four ideal diodes A, B, C and D as connected in the figure shown below.



An alternating supply is applied between the terminal **X** and **Y**.

Identify which diodes are conducting when terminal **X** of the supply is positive.

- Draw path of the current direction with diodes in **forward bias**
- Remember that conventional current flow is from **positive to negative** and only travels through the paths with diodes in **forward bias**



- Therefore, the answer is: diodes **A** and **C**

Exam Tip

Being able to reproduce the diode bridge correctly and explain the cycles of full-wave rectification are important physics concepts that may occur during an examination. It is worth learning to draw and explain full-wave rectification accurately

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11.2.9 Capacitors & Diode Bridges

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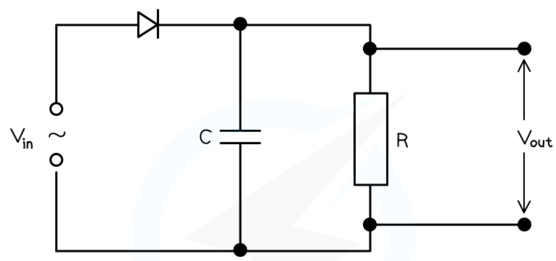
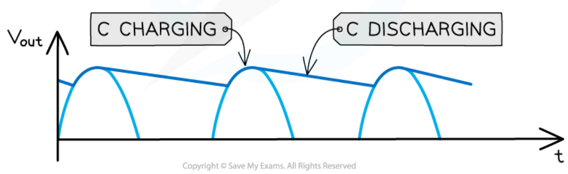
Capacitors & Diode Bridges

- In rectification, to produce a steady direct current or voltage from an alternating current or voltage, a **smoothing capacitor** is necessary
- Smoothing is defined as:

The reduction in the variation of the output voltage or current

- This works in the following ways:
  - A single capacitor with capacitance C is connected in parallel with a load resistor of resistance *R*
  - The capacitor charges up from the input voltage and maintains the voltage at a high level
  - It discharges gradually through the resistor when the rectified voltage drops but the voltage then rises again and the capacitor charges up again





A smoothing capacitor connected in parallel with the load resistor. The capacitor charges as the output voltage increases and discharges as it decreases

- The resulting graph of a smoothed output voltage V_{out} and output current against time is a 'ripple' shape

A smooth, rectified current graph creates a 'rippling' shape against time

- The amount of smoothing is controlled by the capacitance C of the capacitor and the resistance *R* of the load resistor
  - The smaller the rippling effect, the smoother the rectified current and voltage output

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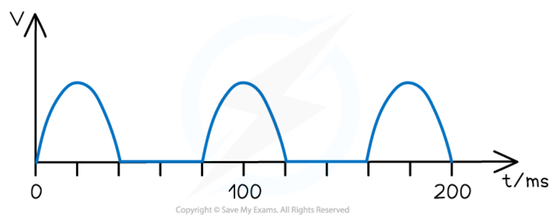
- The slower the capacitor discharges, the more the smoothing that occurs ie. smaller ripples
- This can be achieved by using:
  - A capacitor with **greater** capacitance C
  - A resistor with **larger** resistance *R*
- Recall that the product *RC* is the **time constant** τ of a resistor
- This means that the time constant of the capacitor must be **greater than the time interval** between the adjacent peaks of the output signal

Worked Example

The graph below shows the output voltage from a half-wave rectifier. The load resistor has a resistance of 2.6 kΩ. A student wishes to smooth the output voltage by placing a capacitor in parallel across the load resistor



Suggest if a capacitor of 60 pF or 800 μF would be suitable for this task

Step 1:

Calculate the time constant with the 60 pF capacitor

$$\tau = RC = (2.6 \times 10^3) \times (60 \times 10^{-12}) = 1.56 \times 10^{-7} = 156\text{ ns}$$

Step 2:

Compare time constant of 60 pF capacitor with interval between adjacent peaks of the output signal

- The time interval between adjacent peaks is 80 ms
- The time constant of 156 ns is too small and the 60 pF capacitor will discharge far too quickly
- There would be no smoothing of the output voltages
- Therefore, the 60 pF capacitor is **not suitable**

Step 3:

Calculate the time constant with the 800 μF capacitor

$$\tau = RC = (2.6 \times 10^3) \times (800 \times 10^{-12}) = 2.08\text{ s}$$

Step 4:

Compare time constant of 800 pF capacitor with interval between adjacent peaks of the output signal

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- The time constant of 2.08 s is much larger than 80 ms
- The capacitor will not discharge completely between the positive cycles of the half-wave rectified signal
- Therefore, the **800 μF capacitor** would be suitable for the smoothing task

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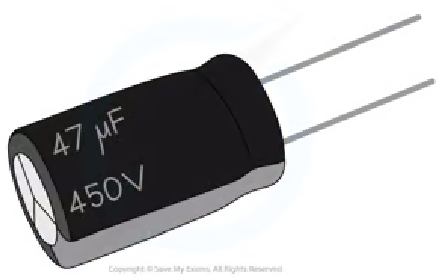
11.3 Capacitance

11.3.1 Capacitance

Capacitance

- Capacitors are electrical devices used to store energy in electronic circuits, commonly for a backup release of energy if the power fails
- They are in the form of two conductive metal plates connected to a voltage supply (parallel plate capacitor)
  - There is commonly a **dielectric** in between the plates, this is to ensure charge does not freely flow between the plates
- The capacitor circuit symbol is:

The capacitor circuit symbol is two parallel lines



A capacitor used in small circuits

- Capacitors are marked with a value of their **capacitance.** This is defined as:

The charge stored per unit potential difference (between the plates)

- The greater the **capacitance**, the greater the **energy stored** in the capacitor
- The capacitance of a capacitor is defined by the equation:

$$C = \frac{q}{V}$$

- Where:
  - C = capacitance (F)
  - q = charge (C)
  - V = potential difference (V)
- Capacitance is measured in the unit **Farad (F)**
  - In practice, 1 F is a very large unit
  - Often it will be quoted in the order of micro Farads (μF), nanofarads (nF) or picofarads (pF)

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- If the capacitor is made of parallel plates, Q is the charge on the plates and *V* is the potential difference across the capacitor
  - The charge Q is **not** the charge of the capacitor itself, it is the charge stored **on** the plates
- This capacitance equation shows that an object's capacitance is the **ratio of the charge stored by the capacitor to the potential difference between the plates**

? Worked Example

A parallel plate capacitor has a capacitance of 1 nF and is connected to a voltage supply of 0.3 kV.

Calculate the charge on the plates.

Step 1: Write down the known quantities

- Capacitance, $C = 1\text{ nF} = 1 \times 10^{-9}\text{ F}$
- Potential difference, $V = 0.3\text{ kV} = 0.3 \times 10^{3}\text{ V}$

Step 2: Write out the equation for capacitance

$$C = \frac{q}{V}$$

Step 3: Rearrange for charge Q

$$q = CV$$

Step 4: Substitute in values

$$q = (1 \times 10^{-9}) \times (0.3 \times 10^{3}) = 3 \times 10^{-7}\text{ C} = 300\text{ nC}$$

Electric Field in a Parallel Plate Capacitor

- A parallel plate capacitor is made up of two conductive metal plates connected to a voltage supply
  - An assumption made for these types of capacitors is that their length of the plates is much greater than their separation
- The negative terminal of the voltage supply pushes electrons onto one plate, making it negatively charged
- The electrons are repelled from the opposite plate, making it positively charged
  - There is commonly a dielectric in between the plates, this is to ensure charge does not freely flow between the plates

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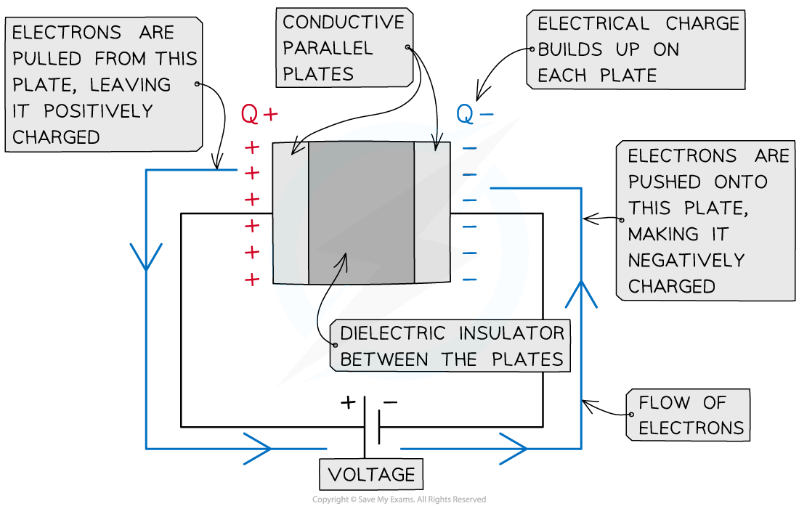
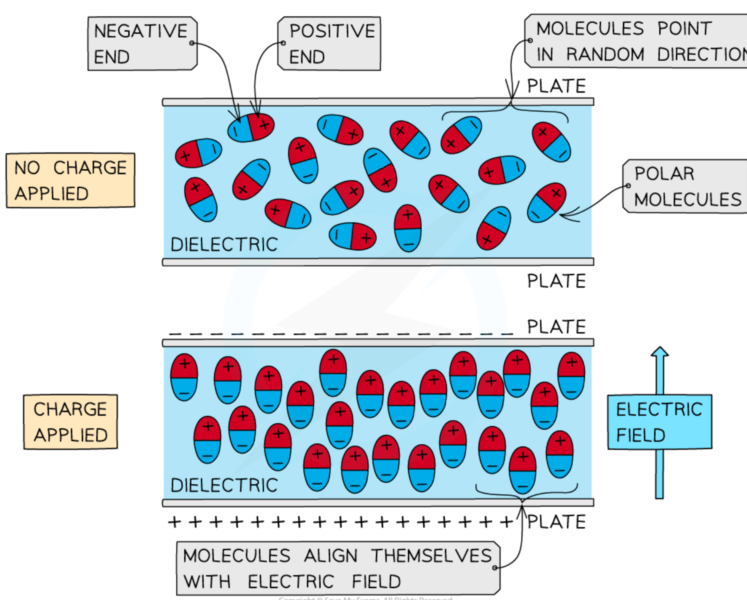


A parallel plate capacitor is made up of two conductive plates with opposite charges building up on each plate

- A dielectric is made up of many polar molecules
  - These are molecules that have a 'positive' and 'negative' end (poles)
- When **no charge** is applied to the capacitor:
  - There is no electric field between the parallel plates and the molecules are aligned in **random** directions
- When there is a **charge applied**:
  - One of the parallel plates becomes positively charged and the other negatively charged hence an electric field is generated between the plates (from positive to negative)
  - The negative ends of the polar molecules are attracted to the positive plate and vice versa
  - This means all the molecules rotate and align themselves **parallel to the electric field**



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Polar molecules align themselves when an electric field is between two parallel plates

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Exam Tip

The 'charge stored' by a capacitor refers to the magnitude of the charge stored **on** each plate in a parallel plate capacitor or **on** the surface of a spherical conductor. The capacitor itself **does not** store charge.

The letter 'C' is used both as the symbol for capacitance as well as the unit of charge (coulombs). Take care not to confuse the two!

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11.3.2 Dielectric Materials

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Dielectric Materials

- Permittivity is the measure of how easy it is to generate an electric field in a certain material
- The relativity permittivity ε_r is sometimes known as the **dielectric constant**
- For a given material, it is defined as:

The ratio of the permittivity of a material to the permittivity of free space

- This can be expressed as:

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

- Where:
  - ε_r = relative permittivity
  - ε = permittivity of a material (F m^{-1})
  - ε_0 = permittivity of free space (F m^{-1})
- The relative permittivity has **no** units because it is a ratio of two values with the same unit

Worked Example

Calculate the permittivity of a material that has a relative permittivity of 4.5×10^{11}.
State an appropriate unit for your answer.

Step 1: Write down the relative permittivity equation

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

Step 2: Rearrange for permittivity of the material ε

$$\varepsilon = \varepsilon_r \varepsilon_0$$

Step 3: Substitute in the values

$$\varepsilon = (4.5 \times 10^{11}) \times (8.85 \times 10^{-12}) = 3.9825 = 4 \text{ F m}^{-1}$$

Dielectric Material Effects on Capacitance

- A dielectric material separates the two conductive metal plates of a capacitor
  - The dielectric itself is an insulator
- When the polar molecules in a **dielectric** align with the applied electric field from the plates, they each produce their own electric field
  - This electric field **opposes** the electric field from the plates

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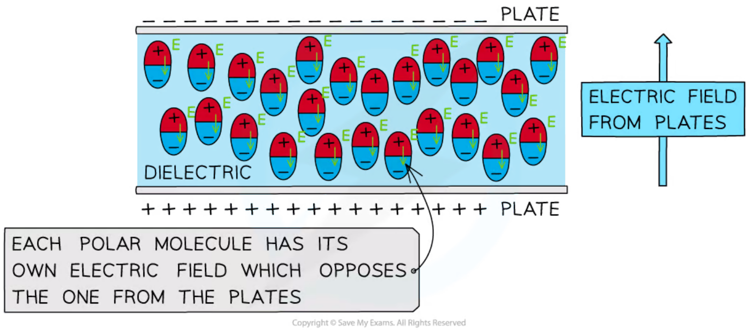

The electric field of the polar molecules opposes that of the electric field produced by the parallel plates

- The **larger** the opposing electric field from the polar molecules in the dielectric, the **larger** the permittivity
  - In other words, the permittivity is how well the polar molecules in a dielectric align with an applied electric field
- The opposing electric field **reduces** the **overall** electric field
- For an **isolated** capacitor, this **decreases** the potential difference between the plates
  - Therefore, the **charge** remains constant and the **capacitance** of the plates **increases**
- For a capacitor attached to a **power supply**, the potential difference between the plates is **unchanged** (with a dielectric)
  - Therefore, the **charge** on one of the plates **increases** and the **capacitance** of the plates also **increase**
- The capacitance of a capacitor can also be written in terms of the relative permittivity:

$$C = \frac{A\varepsilon_0\varepsilon_r}{d}$$

- Where:
  - C = capacitance (F)
  - A = cross-sectional area of the plates (m^2)
  - d = separation of the plates (m)
  - ε_r = relative permittivity of the dielectric between the plates
  - ε_0 = permittivity of free space (F m^{-1})
- When the electrical permittivity of the dielectric is known, a simpler form of this equation can be used:

$$C = \varepsilon\frac{A}{d}$$

- Where:
  - C = Capacitance (F)
  - A = cross-sectional area of the plates (m^2)
  - d = separation of the plates (m)
  - ε = permittivity of the dielectric between the plates (F m^{-1})

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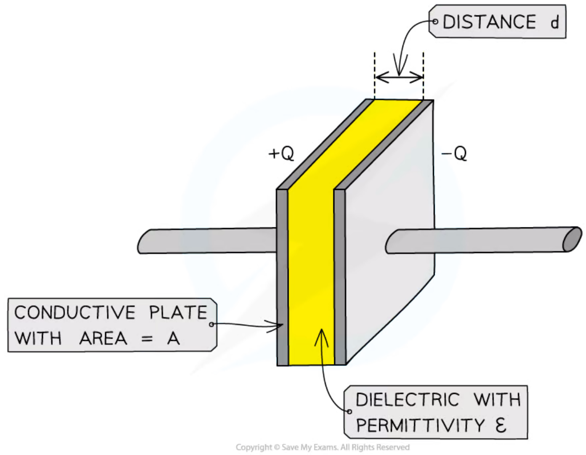

A parallel plate capacitor consists of conductive plates each with area A, a distance d apart and a dielectric ε between them

- Capacitor plates are generally square, therefore if they have a length L on all sides then their cross-sectional area is L^2

Worked Example

A parallel-plate capacitor has square plates of length L separated by distance d and is filled with a dielectric. A second capacitor has square plates of length $3L$ separated by distance $3d$ and has air as its dielectric. Both capacitors have the same capacitance.

Determine the relative permittivity of the dielectric in the first capacitor.

Step 1: Write down the capacitance equation with the relativity permittivity

$$C = \frac{A\varepsilon_0\varepsilon_r}{d}$$

Step 2: Write the known values for each capacitor

Capacitor 1:	Capacitor 2:
$C = C$	$C = C$
$A = L^2$	$A = (3L)^2 = 9L^2$
$\varepsilon_r = \varepsilon_r$	$\varepsilon_r = 1$
$\varepsilon_0 = \varepsilon_0$	$\varepsilon_0 = \varepsilon_0$
$d = d$	$d = 3d$

Since the dielectric for capacitor 2 is air, and air has a permittivity of ε_0

therefore using the equation:

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = \frac{\varepsilon_0}{\varepsilon_0} = 1$$

for capacitor 2

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Step 3: Substitute them into the capacitance equation

$$C_1 = \frac{L^2\varepsilon_0\varepsilon_r}{d}$$

$$C_2 = \frac{9L^2\varepsilon_0}{3d}$$

Step 4: Equate the capacitances

Both capacitors have the same capacitance

$$\frac{L^2\varepsilon_0\varepsilon_r}{d} = \frac{9L^2\varepsilon_0}{3d}$$

Step 5: Cancel out d, L^2 and ε_0 from both sides to find the value of ε_r

$$\varepsilon_r = \frac{9}{3} = \mathbf{3}$$

Exam Tip

Remember that *A*, the cross-sectional area, is only for **one** of the parallel plates. Don't multiply this by 2 for both the plates for the capacitance equation!

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11.3.3 Capacitors in Series & Parallel

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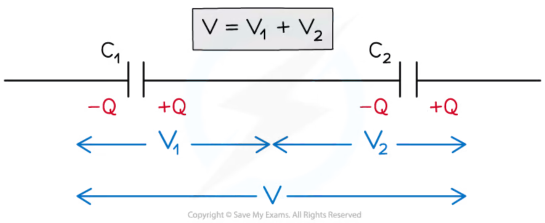
Capacitors in Series & Parallel

- Capacitors can be combined in series and parallel circuits
- The combined capacitance depends on whether the capacitors are connected in series or parallel

Capacitors in Series

- Consider two parallel plate capacitors C_1 and C_2 connected in series, with a potential difference (p.d) V across them



Capacitors connected in series have different p.d across them but have the same charge

- In a series circuit, p.d is **shared** between all the components in the circuit
  - Therefore, if the capacitors store the same charge on their plates but have different p.ds, the p.d across C_1 is V_1 and across C_2 is V_2
- The total potential difference V is the sum of V_1 and V_2

$$V = V_1 + V_2$$

- Rearranging the capacitance equation for the p.d V means V_1 and V_2 can be written as:

$$V_1 = \frac{Q}{C_1} \quad \text{and} \quad V_2 = \frac{Q}{C_2}$$

- Where the total p.d V is defined by the total capacitance

$$V = \frac{Q}{C_{total}}$$

- Substituting these into the equation $V = V_1 + V_2$ equals:

$$\frac{Q}{C_{total}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

- Since the current is the **same** through all components in a series circuit, the charge Q is the same through each capacitor and cancels out
- Therefore, the equation for combined capacitance of capacitors in **series** is:

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$$

Capacitors in Parallel

- Consider two parallel plate capacitors C_1 and C_2 connected in parallel, each with p.d V

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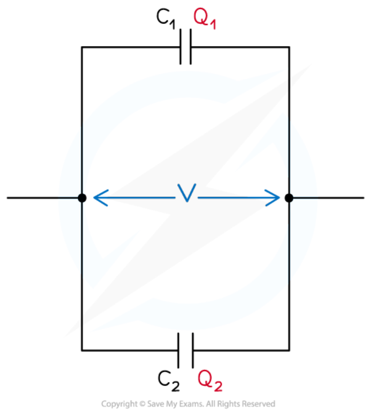

Capacitors connected in parallel have the same p.d across them, but different charge

- Since the current is **split** across each junction in a parallel circuit, the charge stored on each capacitor is **different**
- Therefore, the charge on capacitor C_1 is Q_1 and on C_2 is Q_2
- The total charge Q is the sum of Q_1 and Q_2

$$\mathbf{Q = Q_1 + Q_2}$$

- Rearranging the capacitance equation for the charge Q means Q_1 and Q_2 can be written as:

$$\mathbf{Q_1 = C_1V \quad and \quad Q_2 = C_2V}$$

- Where the total charge Q is defined by the total capacitance:

$$\mathbf{Q = C_{total}V}$$

- Substituting these into the $Q = Q_1 + Q_2$ equals:

$$\mathbf{C_{total}V = C_1V + C_2V = (C_1 + C_2)\,V}$$

- Since the p.d is the **same** through all components in each branch of a parallel circuit, the p.d V cancels out
- Therefore, the equation for combined capacitance of capacitors in **parallel** is:

$$\mathbf{C_{total} = C_1 + C_2 + C_3 \ldots}$$

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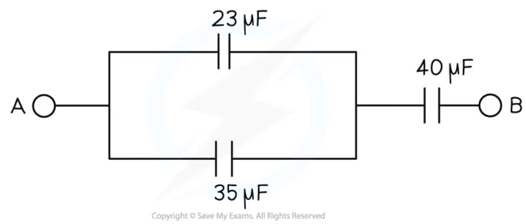
Worked Example

Three capacitors with a capacitance of 23 μF, 35 μF and 40 μF are connected as shown below.



Calculate the total capacitance between points **A** and **B**.

Step 1: Calculate the combined capacitance of the two capacitors in parallel

Capacitors in parallel: $C_{total} = C_1 + C_2 + C_3$ ...

$C_{parallel} = 23 + 35 = 58$ μF

Step 2: Connect this combined capacitance with the final capacitor in series

Capacitors in series: $$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$$

$$\frac{1}{C_{total}} = \frac{1}{58} + \frac{1}{40} = \frac{49}{1160}$$

Step 3: Rearrange for the total capacitance

$$C_{total} = \frac{1160}{49} = 23.673\ldots = \mathbf{24\ \mu F}\ (2\ s.f)$$

Exam Tip

You will be expected to remember these derivations for your exam, therefore, make sure you understand each step. You should especially make sure to revise how the current and potential difference varies in a series and parallel circuit.

Both the combined capacitance equations look similar to the equations for combined resistance in series and parallel circuits. However, take note that they are the **opposite way** around to each other!

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11.3.4 The Time Constant

The Time Constant

- The time constant of a capacitor discharging through a resistor is a measure of how long it takes for the capacitor to discharge
- The definition of the time constant is:

  The time taken for the charge, current or voltage of a discharging capacitor to decrease to 37% of its original value

- Alternatively, for a **charging** capacitor:

  The time taken for the charge or voltage of a charging capacitor to rise to 63% of its maximum value

- 37% is 0.37 or $\frac{1}{e}$ (where e is the exponential function) multiplied by the original value (I_0, Q_0 or V_0)
  - This is represented by the Greek letter tau, τ, and measured in units of **seconds** (s)
- The time constant provides an easy way to compare the rate of change of similar quantities eg. charge, current and p.d.
- It is defined by the equation:

$$\tau = RC$$

- Where:
  - τ = time constant (s)
  - R = resistance of the resistor (Ω)
  - C = capacitance of the capacitor (F)

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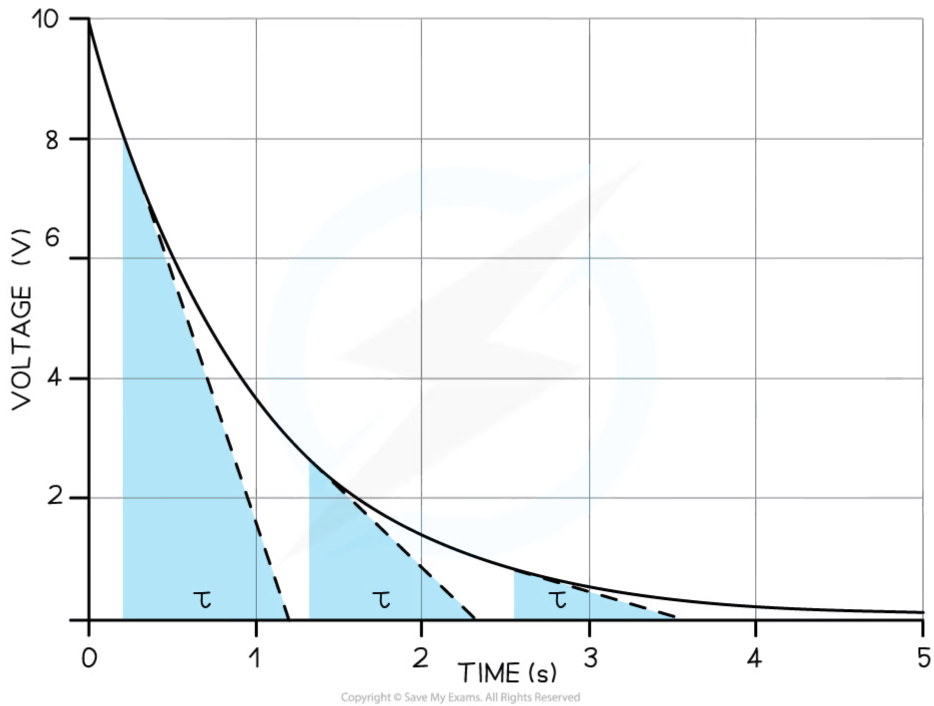

The graph of voltage-time for a discharging capacitor showing the positions of the first three time constants

- The time to half, $t_{1/2}$ (half-life) for a discharging capacitor is:

  The time taken for the charge, current or voltage of a discharging capacitor to reach half of its initial value

- This can also be written in terms of the time constant, τ:

$$t_{1/2} = \ln(2)\,\tau \approx 0.69\,\tau = 0.69\,RC$$

Exam Tip

Note that the time constant is **not** the same as half-life. Half-life is how long it takes for the current, charge or voltage to halve whilst the time constant is to 37% of its original value (not 50%).

Although the time constant is given on the datasheet, you will be expected to remember the half-life equation $t_{1/2} = 0.69RC$

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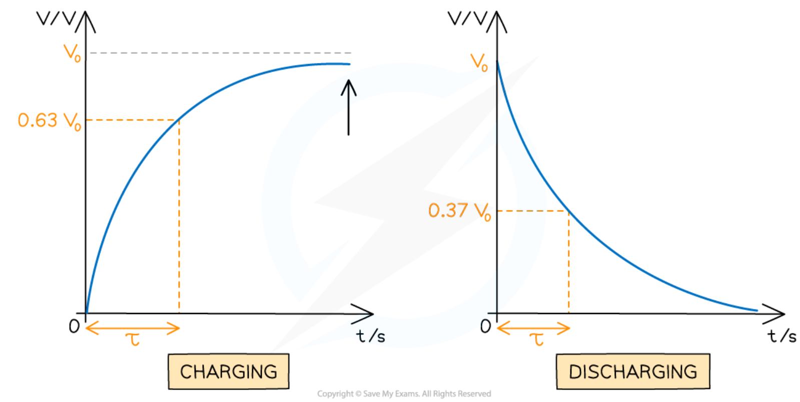
Problems Involving the Time Constant

- Problems involving the time constant tend to involve
  - Calculations
  - Determining the time constant from a graph
- To find the time constant from a voltage-time graph, calculate $0.37V_0$ and determine the corresponding time for that value



The time constant shown on a charging and discharging capacitor

Worked Example

A capacitor of 7 nF is discharged through a resistor of resistance R. The time constant of the discharge is 5.6×10^{-3} s.

Calculate the value of *R*.

Step 1: Write the known quantities

Capacitance, $C = 7 \text{ nF} = 7 \times 10^{-9}$ F

Time constant, $\tau = 5.6 \times 10^{-3}$ s

Step 2: Write down the time constant equation

$$\tau = RC$$

Step 3: Rearrange for R

$$R = \frac{\tau}{C}$$

Step 4: Substitute in values and calculate

$$R = \frac{5.6 \times 10^{-3}}{7 \times 10^{-9}} = \mathbf{800\ k\Omega}$$

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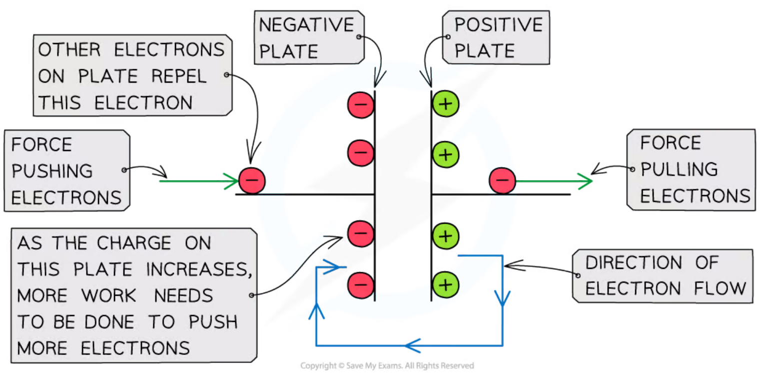
Energy Stored in a Capacitor

- When charging a capacitor, the power supply pushes electrons from the positive to the negative plate
  - It therefore does **work** on the electrons and **electrical energy** becomes stored on the plates
- At first, a small amount of charge is pushed from the positive to the negative plate, then gradually, this builds up
  - Adding more electrons to the negative plate at first is relatively easy since there is little repulsion
- As the charge of the negative plate increases ie. becomes more negatively charged, the **force of repulsion** between the electrons on the plate and the new electrons being pushed onto it **increases**
- This means a **greater** amount of **work** must be done to increase the charge on the negative plate or in other words:

The potential difference across the capacitor increases as the amount of charge increases



As the charge on the negative plate builds up, more work needs to be done to add more charge

- The charge Q on the capacitor is **directly proportional** to its potential difference *V*
- The graph of charge against potential difference is therefore a straight line graph through the origin
- The electrical (potential) energy stored in the capacitor can be determined from the **area under the potential-charge graph** which is equal to the **area** of a right-angled triangle:

$$\text{Area} = 0.5 \times \text{base} \times \text{height}$$

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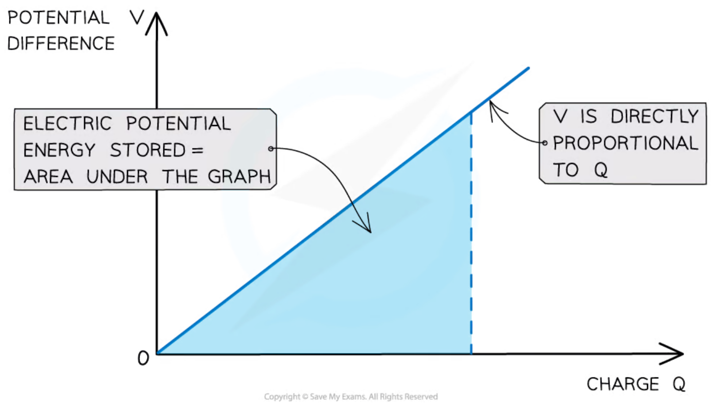

The electric energy stored in the capacitor is the area under the potential-charge graph

- Therefore the work done, or **energy stored** in a capacitor is defined by the equation:

$$E = \frac{1}{2}QV$$

- Where:
  - E = work done or energy stored (J)
  - Q = charge (C)
  - V = potential difference (V)
- Substituting the charge with the **capacitance** equation $\boldsymbol{Q = CV}$, the energy stored can also be defined as:

$$E = \frac{1}{2}CV^2$$

- By substituting the potential V, the energy stored can also be defined in terms of just the charge, Q and the capacitance, C:

$$E = \frac{Q^2}{2C}$$

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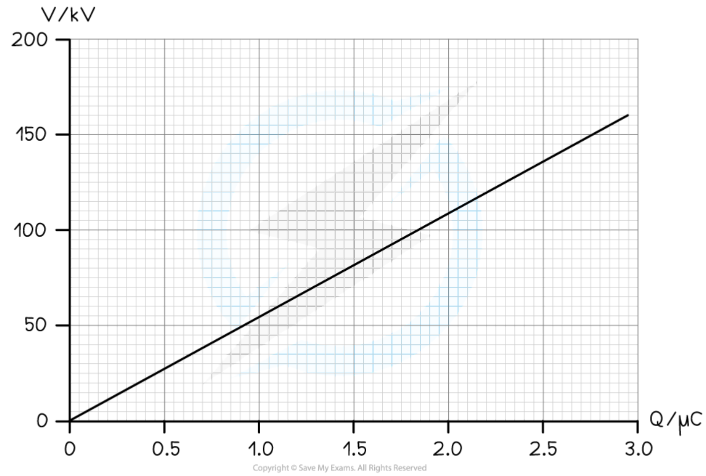
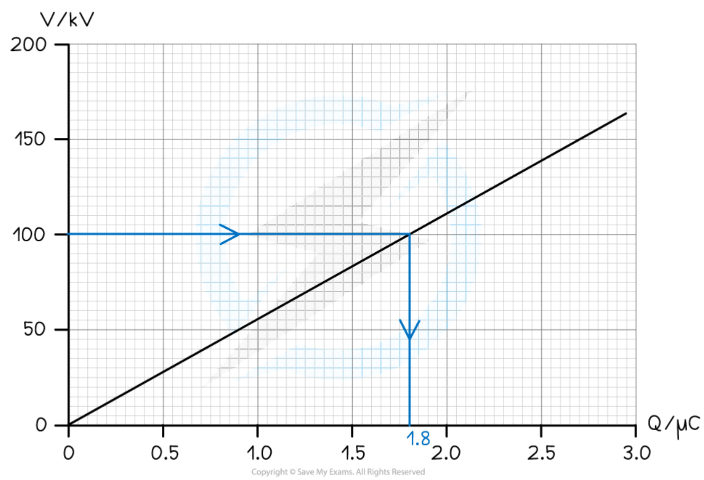
Worked Example

The variation of the potential *V* of a charged isolated metal sphere with surface charge Q is shown on the graph below.



Using the graph, determine the electric potential energy stored on the sphere when charged to a potential of 100 kV.

Step 1: Determine the charge on the sphere at the potential of 100 kV



- From the graph, the charge on the sphere at 100 kV is **1.8 μC**

Step 2: Calculate the electric potential energy stored

- The energy stored is equal to the area under the graph at 100 kV
- The area is equal to a right-angled triangle, so, can be calculated with the equation:

$$\text{Area} = 0.5 \times \text{base} \times \text{height}$$

$$\text{Area} = 0.5 \times 1.8\ \mu\text{C} \times 100\ \text{kV}$$

$$\text{Energy } E = 0.5 \times (1.8 \times 10^{-6}) \times (100 \times 10^{3}) = \mathbf{0.09\ J}$$

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Worked Example

Calculate the change in the energy stored in a capacitor of capacitance 1500 μF when the potential difference across the capacitor changes from 10 V to 30 V.

Step 1: Write down the equation for energy stored in terms of capacitance C and p.d V and list the known values

$$E = \frac{1}{2}CV^2$$

Capacitance, C = 1500 μF

Final p.d, V_2 = 30 V

Initial p.d, V_1 = 10 V

Step 2: The change in energy stored is proportional to the change in p.d

$$\Delta E = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}C(V_2 - V_1)^2$$

Step 3: Substitute in values

$$\Delta E = \frac{1}{2} \times 1500 \times 10^{-6} \times (30 - 10)^2 = 0.3 \text{ J}$$

Exam Tip

Only one equation for the energy stored will be given on your data booklet. Therefore, the derivation or use of others must be memorized.

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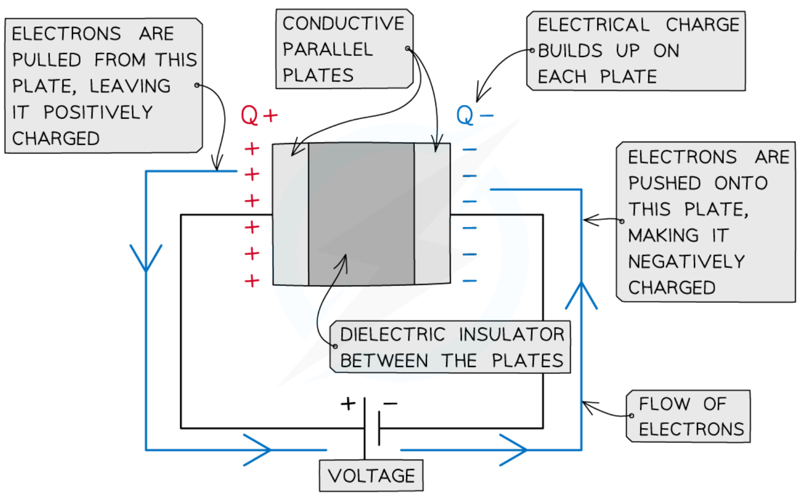
Exponential Discharge of a Capacitor

Charging

- Capacitors are charged by a **power supply** (eg. a battery)
- When charging, the electrons are pulled from the plate connected to the positive terminal of the power supply
  - Hence the plate nearest the positive terminal is **positively charged**
- They travel around the circuit and are pushed onto the plate connected to the negative terminal
  - Hence the plate nearest the negative terminal is **negatively charged**
- As the negative charge builds up, fewer electrons are pushed onto the plate due to electrostatic repulsion from the electrons already on the plate
- When no more electrons can be pushed onto the negative plate, the charging stops



A parallel plate capacitor is made up of two conductive plates with opposite charges building up on each plate

- At the start of charging, the current is large and gradually falls to zero as the electrons stop flowing through the circuit
  - The current decreases **exponentially**
  - This means the rate at which the current decreases is proportional to the amount of current it has left
- Since an equal but opposite charge builds up on each plate, the potential difference between the plates slowly increases until it is the same as that of the power supply
- Similarly, the charge of the plates slowly increases until it is at its maximum charge defined by the capacitance of the capacitor

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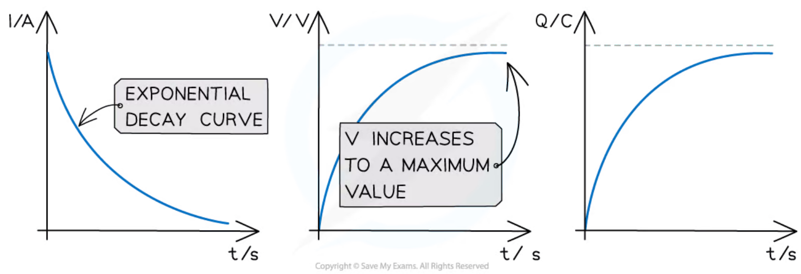
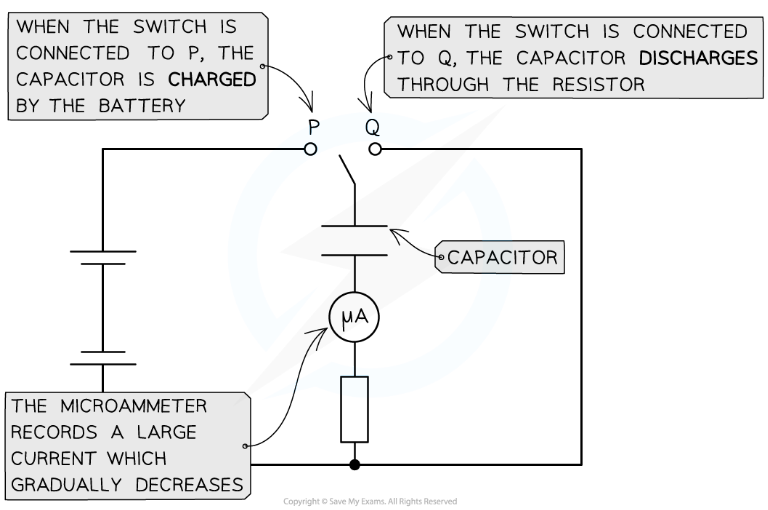


Graphs of variation of current, p.d and charge with time for a capacitor charging through a battery

- **The key features of the charging graphs are:**
  - The shapes of the p.d. and charge against time graphs are identical
  - The current against time graph is an **exponential decay** curve
  - The initial value of the current starts on the y axis and decreases exponentially
  - The initial value of the p.d and charge starts at 0 up to a maximum value

Discharging

- Capacitors are **discharged** through a resistor with **no** power supply present
- The electrons now flow back from the negative plate to the positive plate until there are equal numbers on each plate and no potential difference between them
- Charging and discharging is commonly achieved by moving a switch that connects the capacitor between a power supply and a resistor



The capacitor charges when connected to terminal P and discharges when connected to terminal Q

- At the start of discharge, the current is **large** (but in the opposite direction to when it was charging) and gradually falls to zero
- As a capacitor discharges, the current, p.d and charge all decrease **exponentially**
  - This means the rate at which the current, p.d or charge decreases is proportional to the amount of current, p.d or charge it has left

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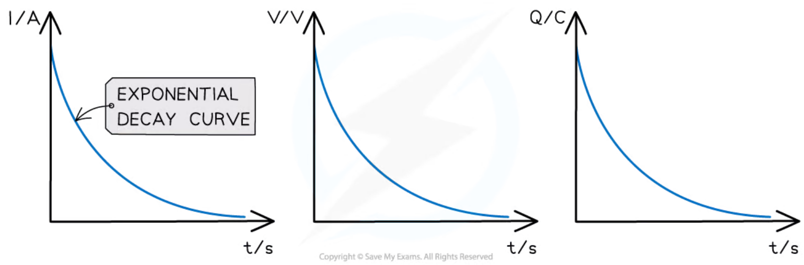
- The graphs of the variation with time of current, p.d and charge are all identical and follow a pattern of **exponential decay**



Graphs of variation of current, p.d and charge with time for a capacitor discharging through a resistor

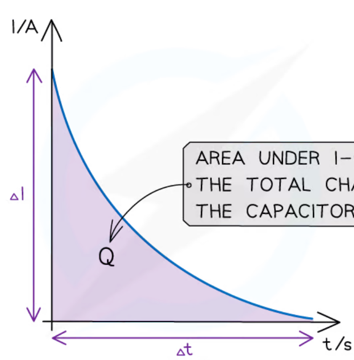
- **The key features of the discharge graphs are:**
  - The shape of the current, p.d. and charge against time graphs are identical
  - Each graph shows exponential decay curves with decreasing gradient
  - The initial values (typically called I_0, V_0 and Q_0 respectively) start on the y axis and decrease exponentially
- The rate at which a capacitor discharges depends on the **resistance** of the circuit
  - If the resistance is **high**, the current will decrease and charge will flow from the capacitor plates more slowly, meaning the capacitor will take longer to discharge
  - If the resistance is **low**, the current will increase and charge will flow from the capacitor plates quickly, meaning the capacitor will discharge faster

Properties of Capacitor Discharge Graphs

- From electricity, the charge is defined:

$$\Delta Q = I\Delta t$$

- Where:
  - I = current (A)
  - ΔQ = change in charge (C)
  - Δt = change in time (s)
- This means that the **area** under a current-time graph for a charging (or discharging) capacitor is the **charge stored** for a certain time interval



The area under the I-t graph is the total charge stored in the capacitor in the time interval Δt

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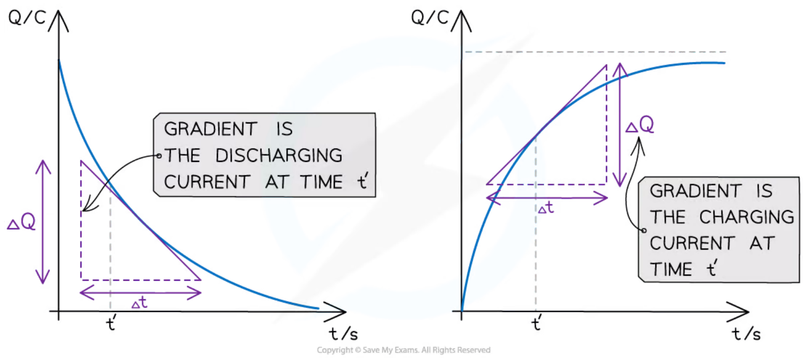
- Rearranging for the current:

$$I = \frac{\Delta Q}{\Delta t}$$

- This means that the **gradient** of the charge-time graph is the **current** at that time

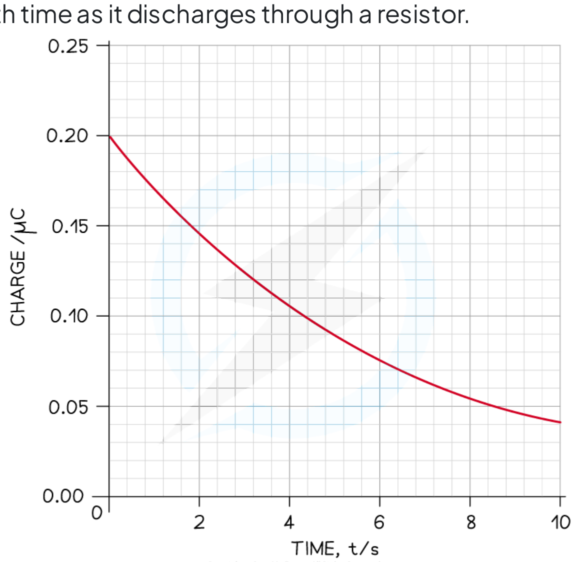


The gradient of a discharging and charging Q-t graph is the current

- In the **discharging** graph, this is the **discharging** current at that time
- In the **charging** graph, this is the **charging** current at that time
- To calculate the gradient of a curve, draw a tangent at that point and calculate the gradient of that tangent

Worked Example

The graph below shows how the charge stored on a capacitor with capacitance C varies with time as it discharges through a resistor.



Calculate the current through the circuit after 4 s.

Step 1: Draw a tangent at *t* = 4

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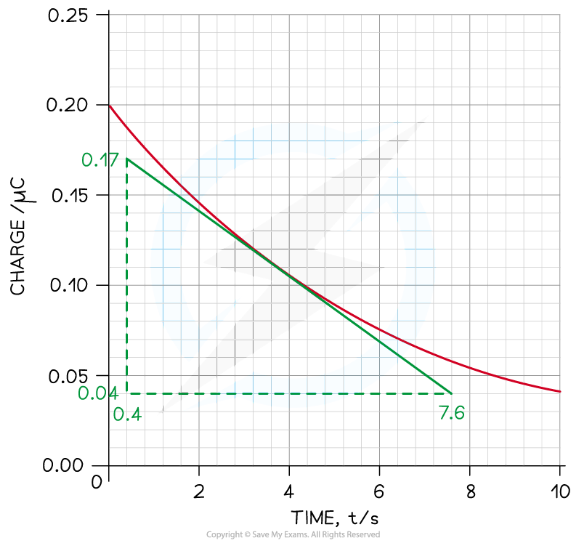

Step 2: Calculate the gradient of the tangent to find the current *I*

$$\text{gradient} = I = \frac{\Delta Q}{\Delta t} = \frac{(0.04 - 0.17) \times 10^{-6}}{(7.6 - 0.4)} = \frac{-0.13 \times 10^{-6}}{7.2}$$

$$I = \mathbf{-1.8 \times 10^{-8}\ A}$$

Exam Tip

Make sure you're comfortable with sketching and interpreting charging and discharging graphs, as these are common exam questions.

Remember that conventional current flow is in the **opposite** direction to the electron flow

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11.3.7 Discharge Calculations

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Capacitor Discharge Equation

- The time constant is used in the exponential decay equations for the current, charge or potential difference (p.d) for a capacitor discharging through a resistor
  - These can be used to determine the **amount** of current, charge or p.d left **after a certain amount of time** for a discharging capacitor

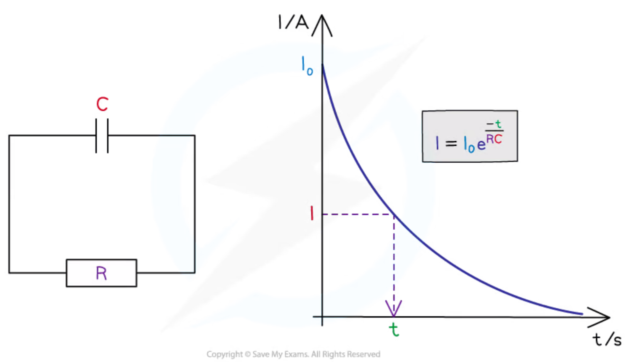
- This exponential decay means that no matter how much charge is initially on the plates, the amount of time it takes for that charge to **halve** is the **same**
- The exponential decay of current on a discharging capacitor is defined by the equation:

$$I = I_0 e^{-\frac{t}{RC}}$$

- Where:
  - I = current (A)
  - I_0 = initial current before discharge (A)
  - e = the exponential function
  - t = time (s)
  - RC = resistance (Ω) × capacitance (F) = the time constant τ (s)

- This equation shows that the **smaller** the time constant τ, the **quicker** the exponential decay of the current when discharging
- Also, how big the initial current is affects the **rate** of discharge
  - If I_0 is large, the capacitor will take **longer** to discharge

- **Note:** during capacitor discharge, I_0 is always larger than I, as the current I will always be decreasing



Values of the capacitor discharge equation on a graph and circuit

- The current at any time is directly proportional to the p.d across the capacitor and the charge across the parallel plates
- Therefore, this equation also describes the charge on the capacitor after a certain amount of time:

$$Q = Q_0 e^{-\frac{t}{RC}}$$

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- Where:
    - Q = charge on the capacitor plates (C)
    - Q_0 = initial charge on the capacitor plates (C)
- As well as the p.d after a certain amount of time:

$$V = V_0 e^{-\frac{t}{RC}}$$

- Where:
    - V = p.d across the capacitor (C)
    - V_0 = initial p.d across the capacitor (C)

The Exponential Function e

- The symbol e represents **the exponential constant**, a number which is approximately equal to e = 2.718...
- On a calculator, it is shown by the button e^x
- The inverse function of e^x is ln(y), known as the **natural logarithmic function**
    - This is because, if $e^x = y$, then x = ln (y)
- The 0.37 in the definition of the **time constant** arises as a result of the exponential constant, the true definition is:

The time taken for the charge of a capacitor to decrease to $\frac{1}{e}$ of its original value

- Where $\frac{1}{e}$ = 0.3678

Worked Example

The initial current through a circuit with a capacitor of 620 μF is 0.6 A. The capacitor is connected across the terminals of a 450 Ω resistor.

Calculate the time taken for the current to fall to 0.4 A.

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Step 1: Write out the known quantities

Initial current before discharge, I_0 = 0.6 A

Current, I = 0.4 A

Resistance, R = 450 Ω

Capacitance, C = 620 μF = 620 × 10^{-6} F

Step 2: Write down the equation for the exponential decay of current

$$I = I_0 e^{-\frac{t}{RC}}$$

Step 3: Rearrange for t

$$\frac{I}{I_0} = e^{-\frac{t}{RC}}$$

The exponential is removed by taking the natural log (ln) of both sides

$$\ln\left(\frac{I}{I_0}\right) = -\frac{t}{RC}$$

$$t = -RC\ln\left(\frac{I}{I_0}\right)$$

Step 4: Substitute in the values

$$t = -450 \times (620 \times 10^{-6}) \times \ln\left(\frac{0.4}{0.6}\right) = 0.1131 = \mathbf{0.1\ s}$$

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