3.1 Geometry Toolkit

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.1 Geometry Toolkit
Difficulty	Hard

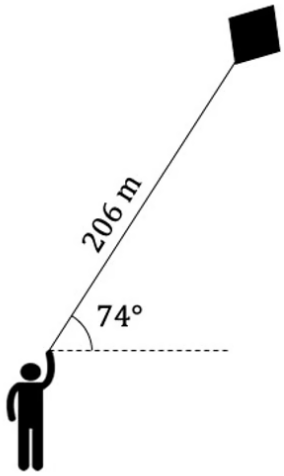
Time allowed: 110

Score: /90

Percentage: /100

Question 1a

A competitor is flying their kite in a competition. The kite is on a string of length 206 m and has an angle of elevation of 74° from the competitor as shown in the diagram below.

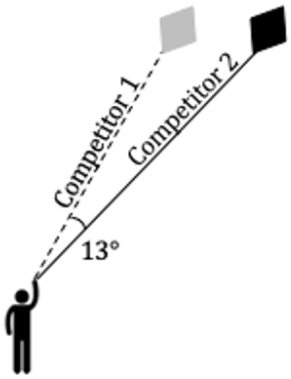


(a) Calculate the vertical height, in metres, that the kite is flying at above the point the competitor is holding it.

[2 marks]

Question 1b

A second competitor raises their kite to the same vertical height from the same position as the first competitor. The angle between the two kites is 13° as shown in the diagram below.

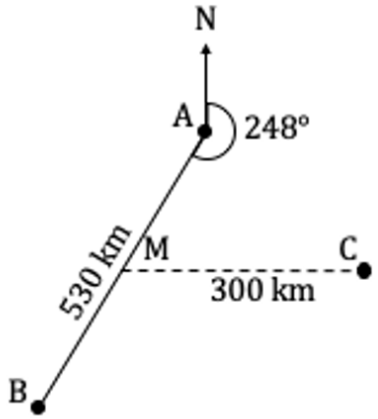


(b) Calculate the length of the string for the kite flown by the second competitor.

[3 marks]

Question 2a

A small airline operates between three locations A, B and C, in one particular country. B is located 530 km from A on a bearing of 248°. C is located 300 km due East from the midpoint of AB. This information is shown in the diagram below.



(a) Calculate the distance AC.

[3 marks]

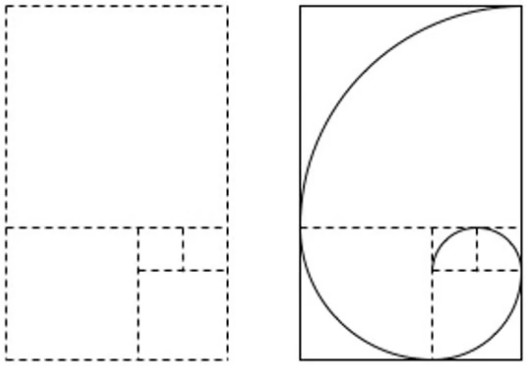
Question 2b

(b) Calculate the bearing that an aeroplane would need to fly on if it were travelling from C to B.

[4 marks]

Question 3a

An artist has been commissioned to create a sculpture for the Mathematics department of a University. She decides to approximate a Fibonacci spiral from a 5 m length of copper wire by putting together a series of squares of increasing size with an arc of a quarter circle in each square. The centre of each arc is at a vertex of the square and the radius is the same as the side length of the square. The copper wire is to be used for the spiral and the edges of the outer rectangle only as shown in the diagrams below.

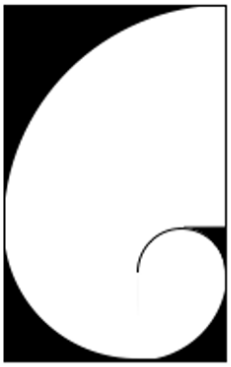


(a) Calculate the length and width of the rectangle that encases the spiral.

[6 marks]

Question 3b

In order to make the spiral stand out, various sections of the sculpture are to be filled in with coloured glass as shown below.

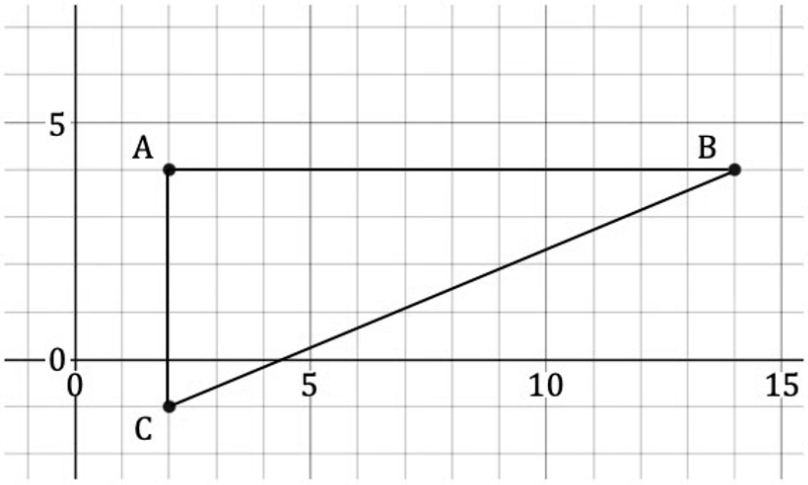


(b) Calculate the area of glass that is required to complete the sculpture.

[4 marks]

Question 4a

The points A, B and C, located on the grid below, form a triangle.



(a) Find the length of BC.

[2 marks]

Question 4b

(b) Find the coordinates of the midpoint, M, of [BC].

[2 marks]

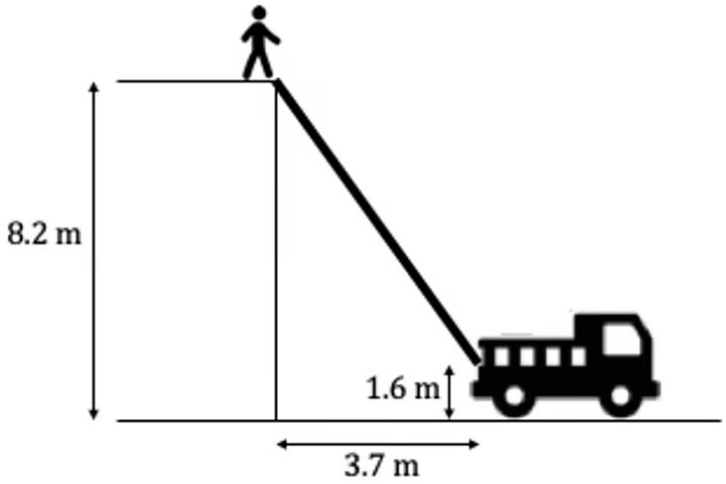
Question 4c

(c) Calculate the area of the triangle ABM.

[3 marks]

Question 5a

A person requires rescuing from the top of a building at a height of 8.2 m. A fire truck has an extendable ladder with its fixed end at a height of 1.6 m. It has been parked at a horizontal distance of 3.7 m from the tree, as shown in the diagram below.



(a) Calculate the length of the ladder required to reach the top of the building.

[2 marks]

Question 5b

For safety purposes, the angle made between the ladder and the horizontal surface it stands on should be between $70° - 80°$.

(b) Show that the ladder on the fire truck, in this situation, would not be safe.

[2 marks]

Question 5c

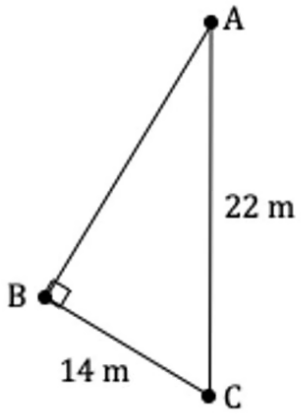
The fire truck is moved to a horizontal distance from the building that enables the optimal angle of 75° to be achieved.

(c) Calculate the length that the ladder now has to be extended to.

[2 marks]

Question 6a

A triangular piece of land has been marked out by placing string around 3 stakes at positions A, B and C as shown in the diagram below. The length AC is 22 m, BC is 14 m and $A\widehat{B}C$ is a right angle.



(a) Calculate the total length of the string used.

[3 marks]

Question 6b

(b) Calculate the area of the piece of land.

[2 marks]

Question 6c

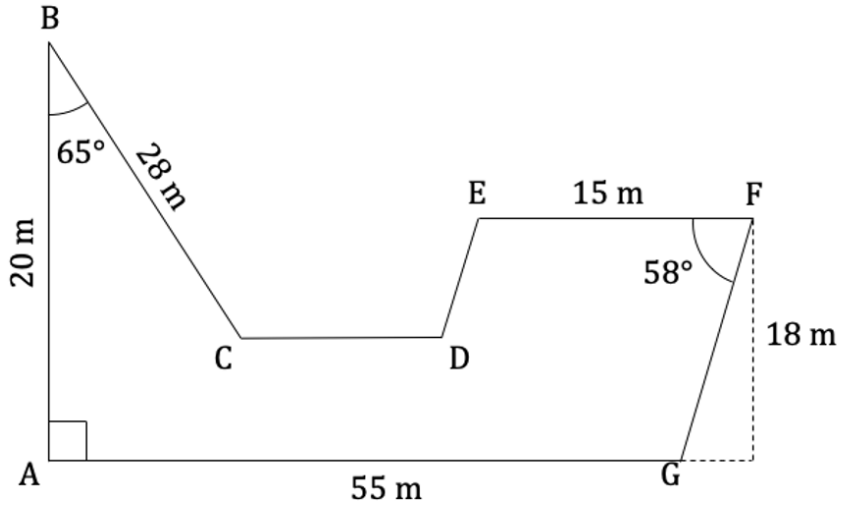
The section of land is to be adjusted. Point A and C remain fixed in position but point B is moved until angle $A\hat{C}B$ becomes 90°. The overall length of the string does not change.

(c) Calculate the new length of BC.

[4 marks]

Question 7a

The shape ABCDEFG, as seen in the diagram below, shows the footprint of a new building that is to be constructed. ED and FG are parallel, as are CD, AG and EF. BC = 28 m, AB = 20 m, AG = 55 m, EF = 15 m and the perpendicular height of FG is 18 m. Angle $B\hat{A}G$ is 90°, angle $A\hat{B}C$ is 65° and angle $E\hat{F}G$ is 58°.



(a) Calculate the area of the footprint of the building.

[5 marks]

Question 7b

An internal wall is to be constructed along the line DG.

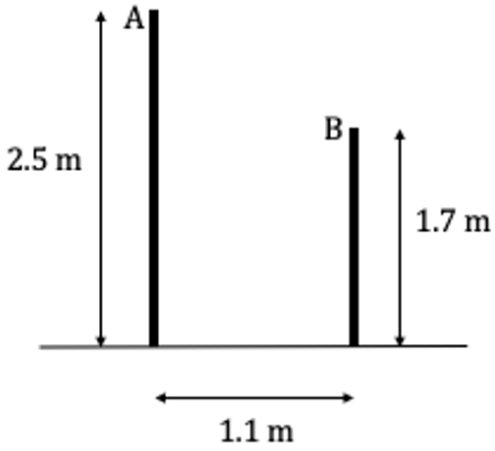
(b) Work out the length of the internal wall and the angle that it makes with FG.

[4 marks]

Question 8a

A gymnast is competing in the women's uneven bars event. The bars are held in place by vertical supports at points A and B, as shown in the diagram, where A and B are situated at heights of 2.5 m and 1.7 m above the ground respectively. The horizontal distance between the bars is 1.1 m. This information is shown in the diagram below.

It can be assumed that the gymnast travels in a straight line when moving between points A and B.



(a) Calculate the distance the gymnast travels in moving between points A and B.

[2 marks]

Question 8b

(b) Calculate the angle of depression from point A to point B.

[2 marks]

Question 8c

When the gymnast is hanging vertically from the higher bar with her arms fully extended, there is a distance of 0.6 m between point A and her eye level.

(c) Calculate the difference between the angle of depression calculated in part (b) and the angle of depression that the gymnast sees to point B.

[4 marks]

Question 9

A pizza is divided into slices with an angle of 30°. Each slice has an area of $\frac{3\pi}{16}$ cm^2.

Find the perimeter of each slice. Leave your answer in terms of π.

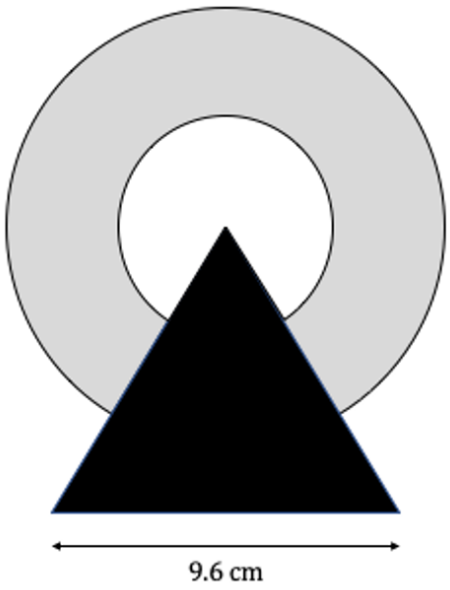
[5 marks]

Question 10

A new lamp has been designed that comprises an annulus containing the light bulbs on top of a stand in the shape of an equilateral triangle of length 9.6 cm. The supporting edges of the stand are divided into thirds by the inner and outer edges of the light disc connecting to it at equally spaced points. The top vertex of the triangular base is located at the centre of the of the two circles that define the annulus.

[An annulus is a ring shaped object made up of a circle with a concentric circle removed from the centre].

A diagram of the lamp is shown below.



Calculate the area of the section of the annulus that can be seen from the front.

[5 marks]

Question 11a

The points A, B, C and D have position vectors:

$$\mathbf{a} = 3\boldsymbol{i} + 2\boldsymbol{j} + 7\boldsymbol{k},$$
$$\mathbf{b} = \boldsymbol{i} + 5\boldsymbol{j} + \boldsymbol{k},$$
$$\mathbf{c} = 4\boldsymbol{i} - 4\boldsymbol{j} + 10\boldsymbol{k} \text{ and}$$
$$\mathbf{d} = 2\boldsymbol{j} - 2\boldsymbol{k}.$$

(a) (i) Find $\overrightarrow{CD}$.

(ii) Show that $\overrightarrow{AB} = \lambda\overrightarrow{CD}$ and state the value of λ.

[3 marks]

Question 11b

A fifth point $E(p, q, r)$ is located such that A, B and E lie on a straight line and $|BE| = 21$ units.

(b) Find the value of p, q and r.

[4 marks]

Question 12a

Points A, B and C have position vectors $\overrightarrow{OA} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 4 \\ 14 \end{pmatrix}$ respectively.

(a) Given that ABC is a right-angled triangle, find the area of ABC.

[3 marks]

Question 12b

A fourth point, D, is to be located such that $\overrightarrow{BD} = 5\overrightarrow{CA}$.

(b) Find the coordinates of point D.

[3 marks]

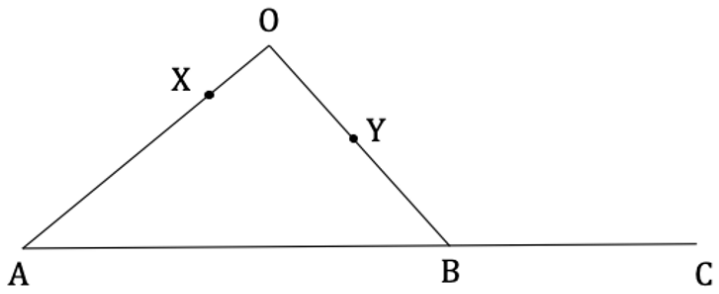
Question 13

In the diagram below [OXA], [OYB] and [ABC] are straight lines.

The position vectors of A and B relative to O are $6\mathbf{a}$ and $4\mathbf{b}$ respectively.

Y is the midpoint of [OB] and $\overrightarrow{AB} = 2\overrightarrow{BC}$.

$\frac{1}{k}\overrightarrow{OA} = \overrightarrow{XA}$.



Given that [XYC] is a straight line, find the value of k.

[6 marks]