

# 5.3 Integration

## Question Paper

Course	DPIB Maths
Section	5. Calculus
Topic	5.3 Integration
Difficulty	Hard

**Time allowed:** 80  
**Score:** /62  
**Percentage:** /100

**Question 1a**

A function  $f$  is defined by the equation  $f(x) = -3x + 35$ .

(a) Sketch the graph of  $y = f(x)$  in the interval  $0 \leq x \leq 10$ .

[2 marks]

**Question 1b**

(b) Use your sketch from part (a), along with relevant area formulae, to work out the value of the integral

$$\int_1^9 (-3x + 35) dx$$

[3 marks]

**Question 2**

The derivative of the function  $f$  is given by

$$f'(x) = \frac{9}{2}x^2 + 7x - 2$$

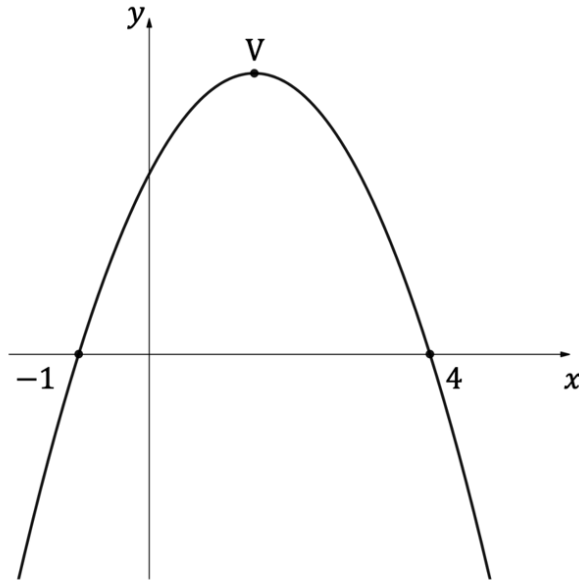
and the curve  $y = f(x)$  passes through the point  $\left(-3, -\frac{11}{2}\right)$ .

Find an expression for  $f$ .

[6 marks]

**Question 3a**

A curve  $y = f(x)$  has the gradient function  $f'(x) = 4ax + 6$ , where  $a \in \mathbb{R}$  is a constant. The diagram below shows part of the curve, with the  $x$  and  $y$  intercepts labelled and where  $V$  represents the vertex of the curve.



(a) Find

- (i) the value of  $a$
- (ii) the equation of the curve  $y = f(x)$
- (iii) the coordinates of  $V$ .

[5 marks]

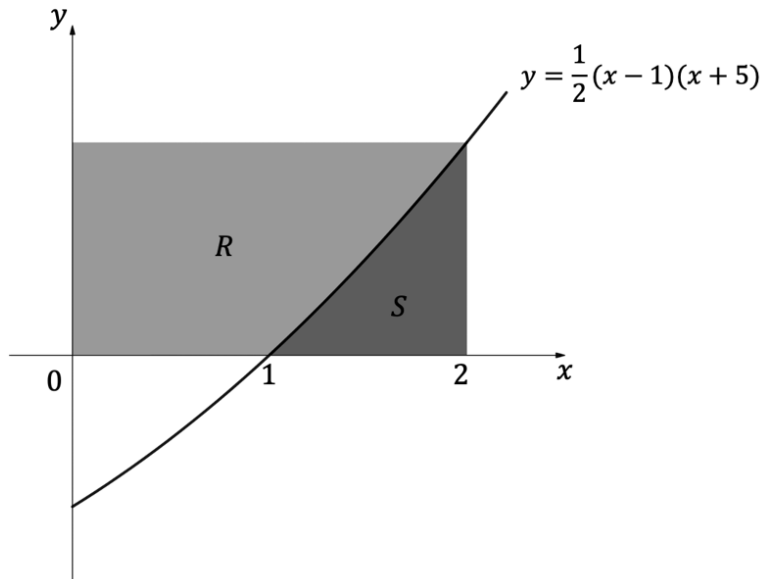
**Question 3b**

(b) Find the area between the curve and the  $x$ -axis.

[3 marks]

**Question 4a**

A section of the curve with equation  $y = \frac{1}{2}(x - 1)(x + 5)$  is shown below:



The shaded region  $S$  in the diagram is bounded by the curve, the  $x$ -axis and the line  $x = 2$ .

- (a) (i) Write down an integral for the area of the shaded region  $S$ .
- (ii) Find the area of  $S$ . Give your answer as a fraction.

[3 marks]

**Question 4b**

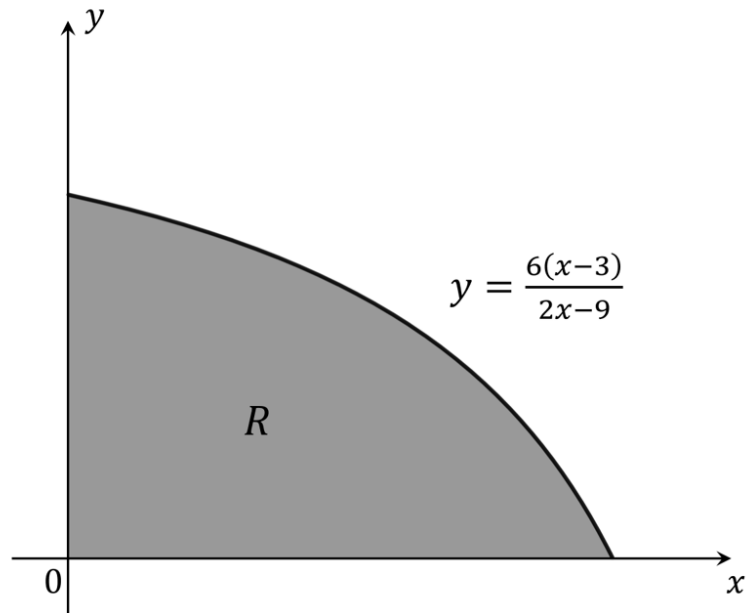
The shaded region  $R$  in the diagram is bounded on three sides by the curve, the  $x$ -axis and the  $y$ -axis. The boundary on the fourth side is a straight line parallel to the  $x$ -axis, and that line, the curve and the line  $x = 2$  all intersect at a single point.

(b) Find the area of region  $R$ . Give your answer as a fraction.

[3 marks]

**Question 5a**

A company is designing a plastic piece for a new game. The piece is to be in the form of a prism, with a cross-sectional area as indicated by the shaded region  $R$  in the following diagram:



Region  $R$  is bounded, as shown, by the positive  $x$ - and  $y$ -axes and the curve with equation  $y = \frac{6(x-3)}{2x-9}$ . All units are in centimetres.

- (a) Using technology, or otherwise, find the coordinates of the points of intersection of the curve with the  $x$ - and  $y$ -axes.

[2 marks]



**Question 5b**

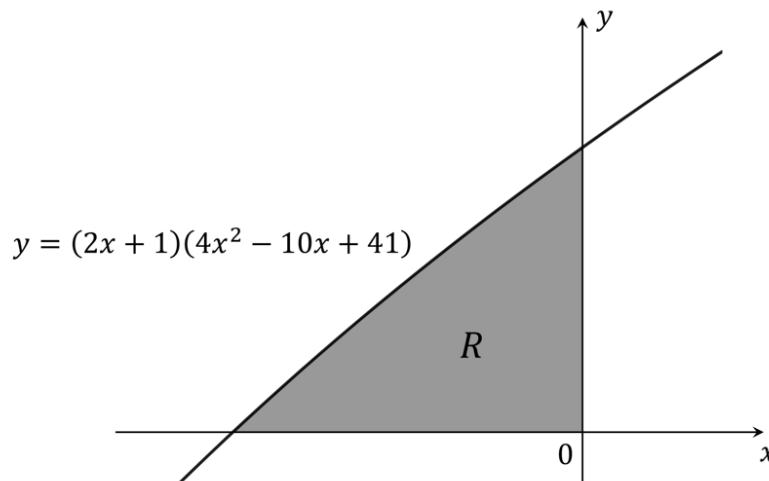
The volume of the puzzle piece is to be  $30 \text{ cm}^3$ .

(b) Find the length of the puzzle piece, giving your answer correct to 3 significant figures.

[4 marks]

**Question 6a**

The following diagram shows part of the graph of  $f(x) = (2x + 1)(4x^2 - 10x + 41)$ ,  $x \in \mathbb{R}$ . The shaded region  $R$  is bounded by the  $x$ -axis, the  $y$ -axis and the graph of  $f$ .

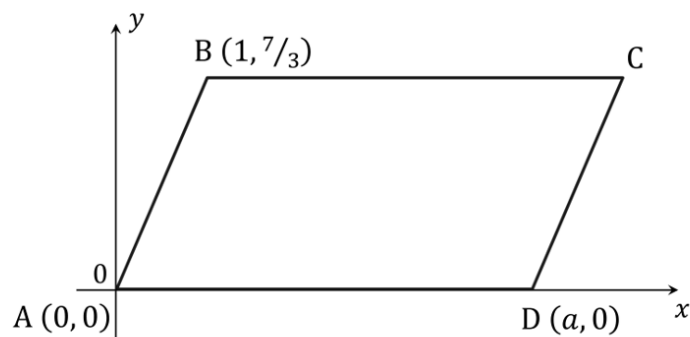


- (a) (i) Write down an integral for the area of region  $R$ .  
 (ii) Find the area of region  $R$ .

[3 marks]

**Question 6b**

ABCD is a parallelogram with vertices  $A(0,0)$ ,  $B\left(1, \frac{7}{3}\right)$ , C and  $D(a, 0)$ , as shown in the diagram below. The area of ABCD is equal to the area of region R above.

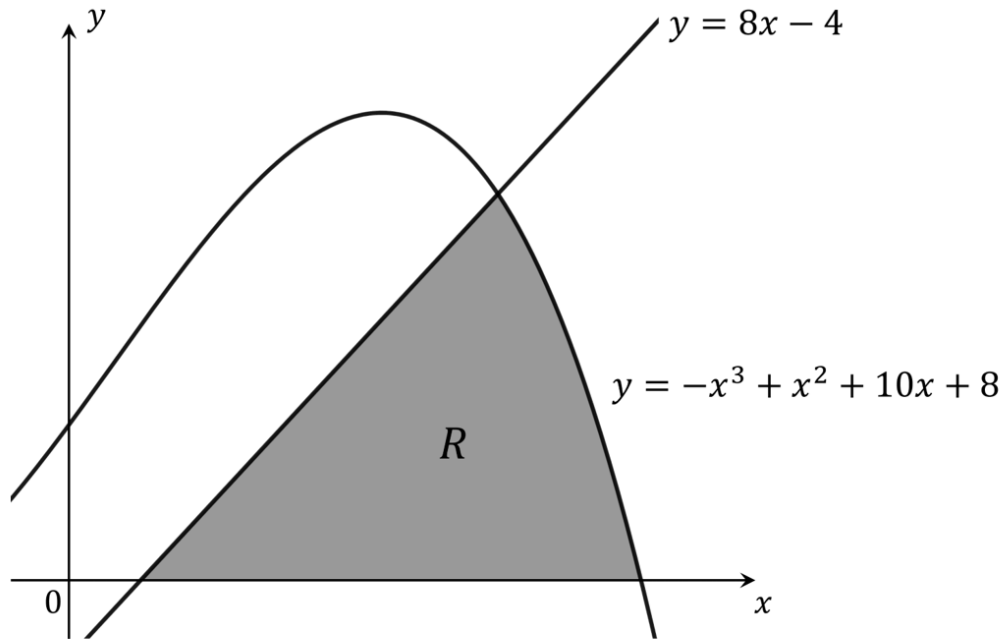


- (b) By first finding the value of  $a$ , the  $x$ -coordinate of point D, determine the coordinates of point C. The coordinates should be given as exact fractions.

[3 marks]

**Question 7a**

The shaded region  $R$  in the following diagram is bounded by the  $x$ -axis, the line  $y = 8x - 4$  and the curve  $y = -x^3 + x^2 + 10x + 8$ .



- (a) Using technology, or otherwise, find the coordinates of
- (i) the point of intersection between the curve and the line
  - (ii) the point of intersection between the line and the  $x$ -axis
  - (iii) the point of intersection between the curve and the  $x$ -axis that is shown in the diagram.

[3 marks]

**Question 7b**

(b) Show that the area of region  $R$  is equal to exactly  $\frac{439}{12}$  units<sup>2</sup>. Be sure to show all of your working.

[6 marks]

**Question 8a**

Consider the function  $f$  where  $f(x) = x(x^2 - 12) + 16$ ,  $x \in \mathbb{R}$ .

The turning points on the graph of  $f$  are A and B. The  $x$ -coordinates of points A and B are  $a$  and  $b$  respectively, where  $a < b$ .

- (a) (i) Determine an expression for the derivative of  $f$ .
- (ii) Hence find the values of  $a$  and  $b$ , and the coordinates of points A and B.

[4 marks]

**Question 8b**

Point C is the point on the graph with  $x$ -coordinate  $c$ , where  $c > 0$  and  $f(c) = 32$ .

(b) (i) Determine the value of  $c$ .

(ii) Sketch the graph of  $f$ , clearly indicating the locations of points A, B and C, along with all other points where the graph intersects one of the coordinate axes.

[3 marks]

**Question 8c**

Region  $R$  is the region enclosed by the graph of  $f$  and the line  $y = 32$ .

(c) Find the area of region  $R$ .

[4 marks]

**Question 9a**

For a particle  $P$  travelling in a straight line, the velocity,  $v$  m/s, of the particle at time  $t$  seconds is given by the equation

$$v(t) = 2t^2 - 8t + 9, \quad t \geq 0$$

(a) Sketch the graph of  $v(t)$  in the interval  $0 \leq t \leq 5$ .

[2 marks]

**Question 9b**

The distance travelled between times  $t_1$  and  $t_2$  by a particle moving in a straight line may be found by finding the area beneath the particle's velocity-time graph between those two times.

(b) Find the distance travelled by the particle  $P$  between the times  $t = 1$  and  $t = 4.5$ .

[3 marks]

