1.9 Further Complex Numbers

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.9 Further Complex Numbers
Difficulty	Hard

Time allowed: 110

Score: /84

Percentage: /100

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Question 1a

Consider the equation $z^2 + pz - 2p - 1 = 0$, where $z \in \mathbb{C},\ p \in \mathbb{R}$.

(a)
Find the value of p for which one of the two distinct roots is $z_1 = 2 + \sqrt{3}\,\mathrm{i}$.

[4 marks]

Question 1b

(b)
Find the range of values of p for which the equation has two distinct, real roots.

[4 marks]

Question 2a

Consider the complex number $\omega = -1 + 4\mathrm{i}$.

(a)
Show that ω is a root of the cubic equation

$$z^3 + 5z^2 + 23z + 51 = 0$$

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[4 marks]

Question 2b

(b)
Find the other two roots of the cubic equation in part (a).

[4 marks]

Question 3

Consider $z = \text{cis}\ \theta$ where $z \in \mathbb{C}$, $z \neq 1$.

Show that $\text{Re}\left(\frac{1+z}{1-z}\right) = 0$.

[5 marks]

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Question 4a

Consider the equation $(z-2)^2 = \mathrm{i},\ z \in \mathbb{C}$.

a)
(i)

Verify that $\omega_1 = 2 + e^{\mathrm{i}\frac{\pi}{4}}$ is a root of this equation.

(ii)
Find the second root of the equation, expressing your answer in the form $\omega_2 = a + e^{\mathrm{i}\theta}$ where $a \in \mathbb{R}$ and $\theta > 0$.

[5 marks]

Question 4b

The roots ω_1 and ω_2 are represented by the points A and B respectively on an Argand diagram.

(b)
Find AB.

[3 marks]

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Question 5

Consider the equation $z^4 + (1 - 4i)z^2 - 4i = 0$, where $z \in \mathbb{C}$.

Find the four distinct roots of the equation, giving your answers in the form $a + bi$ where $a,\ b \in \mathbb{R}$.

[8 marks]

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Question 6a

Consider the complex numbers $w_1 = \frac{z_1}{z_2}$, $z_1 = \frac{\sqrt{2}e^{-\frac{\pi}{3}i}}{3}$ and $z_2 = 2 - 2\sqrt{3}i$.

(a)
Express
(i)
z_1 in the form $a + bi$
(ii)
z_2 in the form $r \operatorname{cis} \theta$, where $r > 0$ and $-\pi < \theta < \pi$.

[3 marks]

Question 6b

(b)
Find the exact value of w_1.

[2 marks]

Question 6c

(c)
Find $w_2 = z_1 z_2$, giving your answer in the form $r \operatorname{cis} \theta$, where $r > 0$ and $-\pi < \theta < \pi$.

[2 marks]

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Question 6d

(d)
Without drawing an Argand diagram, describe the geometrical relationship between z_1 and z_2.

[1 mark]

Question 7a

$$z = \frac{\sqrt{3}}{2}\mathrm{i} - \frac{1}{2}$$

(a)
Find all the powers z^n .

[5 marks]

Question 7b

(b)
Find the area of the shape made by the powers z^n when plotted on an Argand diagram.

[3 marks]

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Question 8a

Let $z = \cos\theta + \mathrm{i}\sin\theta$.

(a)
Write down the value of zz^*.

[2 marks]

Question 8b

Let $z_1 = r_1(\cos\theta_1 + \mathrm{i}\sin\theta_1)$ *and* $z_2 = r_2(\cos\theta_2 + \mathrm{i}\sin\theta_2)$.

(b)
Prove the results
(i)
$$\left|\frac{z_1}{z_2}\right| = \left|\frac{z_1}{z_2}\right|$$
(ii)
$$\arg\frac{z_1}{z_2} = \arg z_1 - \arg z_2$$

[5 marks]

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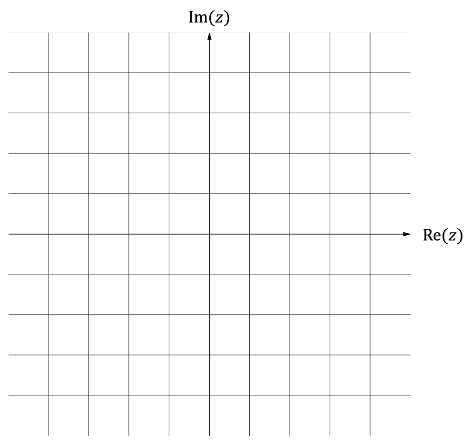
Question 8c

(c)
Using the results from part (b), describe fully the geometrical interpretation of dividing z_1 by z_2.

[1 mark]

Question 9a

(b)
Sketch ω_1, ω_2 and ω_3 on the Argand diagram below.



[3 marks]

Question 9b

ω_1, ω_2 and ω_3 represent the vertices of a triangle.

(c)
Find the area of the triangle.

[4 marks]

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Question 10a

The complex numbers $z_1 = a,\ z_2 = 3 - 2\mathrm{i}$ and z_3 are roots of the cubic equation $z^3 + pz^2 + qz - 26 = 0$, where $a,\ p,\ q \in \mathbb{R}$.

(a)
Find the values of $a,\ p$ and q.

[5 marks]

Question 10b

(b)
Express $z_1,\ z_2$ and z_3 in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $0 < \theta \leq 2\pi$.

[3 marks]

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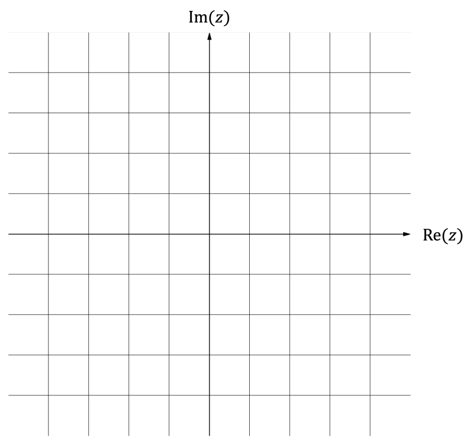
Question 11

Let $f(z) = z^4 + az^3 + 6z^2 + bz + 65$, where a and b are real constants.

Given that $z = 3 + 2\mathrm{i}$ is a root of the equation $f(z) = 0$, show the roots $f(z) = 0$ on the Argand diagram below.



[8 marks]

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