4.7 Further Probability Distributions

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.7 Further Probability Distributions
Difficulty	Very Hard

Time allowed: 110

Score: /85

Percentage: /100

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Question 1a

The continuous random variable X has the probability density function

$$f(x) = \begin{cases} kex, & 0 \leq x \leq a \\ k(e^2 - e^x), & a \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

where a and k are constants.

a)
Given that f is a continuous function, find the values of a and k.

[3 marks]

Question 1b

(b)
(i)
Find $\mathrm{P}(0.5 \leq X \leq 1.5)$
(ii)
Find $\mathrm{P}(X > 1)$

[5 marks]

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Question 1c

(c)
Find the median of X.

[3 marks]

Question 2a

The continuous random variable, X, follows a **uniform** distribution.

The probability density function is given by

$$f(x) = \begin{cases} k & a \le x \le b \\ 0 & \text{otherwise.} \end{cases}$$

(a)
(i)
Write down an expression for k in terms of a and b.

(ii)
Write down an expression for $\mathrm{E}(X)$ in terms of a and b.

(iii) Show that $\mathrm{Var}(X) = \dfrac{(b-a)^2}{12}$.

[5 marks]

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Question 2b

(b)

Given that $\mathrm{E}(X) = 8.5$ and $\mathrm{Var}(X) = 6.75$, find the values of a and b.

[3 marks]

Question 3a

A company meeting lasts T hours. The continuous random variable T can be modelled by the probability density function

$$\mathrm{f}(t) = \begin{cases} \frac{3}{4}t(t-2)^2 & 0 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$

(a)

Find the mean time of a meeting, giving your answer in minutes.

[3 marks]

Question 3b

(b)

Show that the median meeting time is greater than the modal meeting time. Fully explain your solution.

[3 marks]

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Question 4a

The continuous random variable X has probability density function

$$f(x) = \begin{cases} \dfrac{k}{x^3} & 2 \le x \le a \\ 0 & \text{otherwise} \end{cases}$$

a) Show that $k = \dfrac{8a^2}{a^2 - 4}$.

[3 marks]

Question 4b

(b) Given that $\mathrm{E}(X) = \dfrac{20}{7}$ show that $a = 5$ and hence find the value of k.

[5 marks]

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Question 4c

(c)
Find the exact value of $\text{Var}(X)$.

[2 marks]

Question 5a

Paul is travelling around France to try to find the perfect baguette. The continuous random variable C represents the cost, in euros, of a baguette. It can be modelled by the probability density function

$$f(c) = \begin{cases} \frac{1}{36}(14c - c^2 - 40) & 4 \le c \le 10 \\ 0 & \text{otherwise} \end{cases}$$

a)
Sketch the graph of $y = f(c)$.

[3 marks]

Question 5b

(b)
Explain why the mean cost of a baguette is €7.

[1 mark]

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Question 5c

(c)

Given that the probability that a randomly selected baguette costs less than €9 is $\frac{25}{27}$, find the probability that a randomly selected baguette costs between €5 and €9

[2 marks]

Question 6a

The amount of time, t years, it takes for an investment to return a profit is modelled by a continuous random variable, T, with probability density function

$$f(t) = \begin{cases} \frac{1}{24}\left(a - (t-b)^2\right) & 0 < t < 3 \\ 0 & \text{otherwise} \end{cases}$$

where a and b are integer constants.

(a)

Given that the mode of T is 1, find the values of a and b.

[4 marks]

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Question 6b

(b)
Find the expected length of time that a typical investor will have to wait before they make a profit. Give your answer in months, to the nearest month.

[2 marks]

Question 6c

(c)
A particular company owner will only commit to this investment if there is at least a 75% chance they will make a profit within the first 18 months. Determine whether the company owner will invest or not.

[2 marks]

Question 6d

(d)
Give a criticism of the model.

[1 mark]

Question 7

The continuous random variable, X, has probability density function given by

$$f(x) = \begin{cases} \frac{1}{6}(x-1), & 1 \le x \le 3 \\ \frac{1}{12}(7-x), & 3 \le x \le 7 \\ 0, & \text{otherwise} \end{cases}$$

Draw a box plot for the distribution of X. Mark the exact values of the quartiles on your diagram.

[6 marks]

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Question 8a

The discrete random variable, X, has the probability distribution given in the table below

x	1	2	3	4	5	6	7	8	9
$\mathrm{P}(X=x)$	$\frac{k-1}{27}$	$\frac{k}{27}$	$\frac{k+1}{27}$	$\frac{k-1}{27}$	$\frac{k}{27}$	$\frac{k+1}{27}$	$\frac{k-1}{27}$	$\frac{k}{27}$	$\frac{k+1}{27}$

where $k \in \mathbb{Z}^+$.

(a)
Show that $k=3$.

[1 mark]

Question 8b

(b)
Find
(i)
$\mathrm{P}(X>4 \mid X \leq 7)$
(ii)
$\mathrm{P}(X \text{ is prime} \mid X \leq 5)$.

[4 marks]

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Question 8c

(c)
Find
(i)
$\mathrm{E}(4X+5)$
(ii)
$\mathrm{Var}(6X+q)$, where q is a constant.

[4 marks]

Question 9a

A continuous random variable, X, has a probability distribution such that

$$P(X=x)=kx^3, \qquad 0\le x\le 2$$

(a)
Given that $\mathrm{P}(X\le 2)=1$, find the value of k.

[1 mark]

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Question 9b

(b)

Find

(i)

$P(X \leq 1)$

(ii)

$P(X \leq 1.5)$

(iii)

$P(1 \leq X \leq 1.5)$

[4 marks]

Question 9c

(c)

Find

(i)

$P(X \leq 1 | X \geq 0.5)$

(ii)

$P(X \geq 1.5 | X \geq 1)$

[4 marks]

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Question 10a

The function defined by $f(x) = \frac{1}{54}x(x^2 - 8x + 17)$ has a local maximum when $x = p$ and a local minimum when $x = q$.

(a)
Sketch the graph of f. State the values of p and q.

[3 marks]

Question 10b

The continuous random variable, X, has probability density function

$$g(x) = \begin{cases} f(x) & 0 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

(b)
Briefly explain how it can be verified that $g(x)$ is a suitable model for a probability density function.

[2 marks]

Question 10c

(c)
Write down the mode of X.

[2 marks]

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Question 10d

(d)

Find the probability that

(i)

X is such that $g(x) > g(p)$,

(ii)

X is such that $g(x) < g(q)$.

[4 marks]

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