2.4 Other Functions & Graphs

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.4 Other Functions & Graphs
Difficulty	Hard

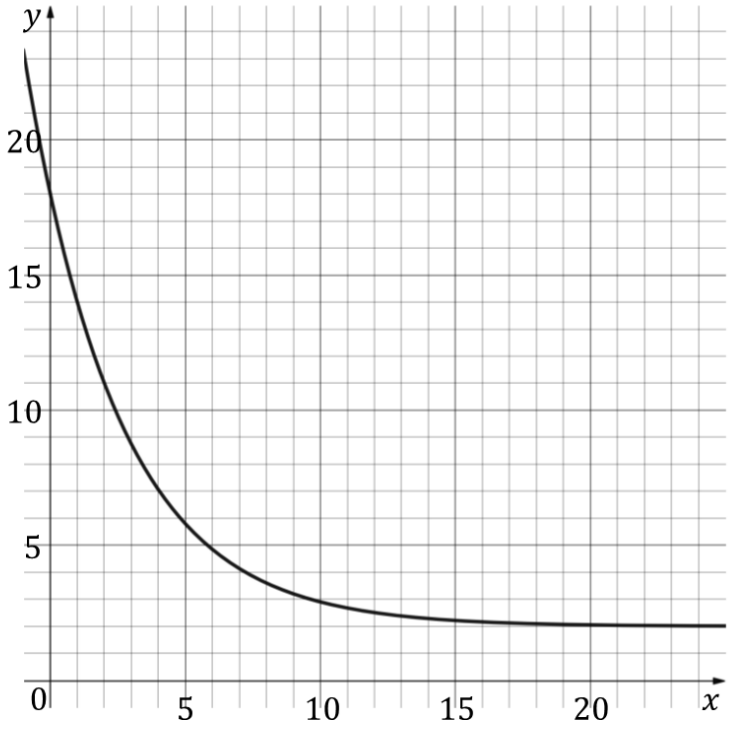
Time allowed: 60

Score: /48

Percentage: /100

Question 1a

Consider the function $f(x) = a(0.75)^x + b$ where a and b are constants. The graph of f passes through the points $(0, 18)$ and $(2, 11)$ and is shown below.



(a) Write down two equations relating a and b.

[2 marks]

Question 1b

(b) Find the value of a and b.

[2 marks]

Question 1c

(c) Write down the equation of the horizontal asymptote of the graph of f.

[2 marks]

Question 2a

The average fat-free mass, M, in kg, of footballers as a function of their age, a, in years, can be given by the logarithmic function:

$$\mathrm{M}(a) = 10\log(a - 15) + 50, \qquad 16 \leq a \leq 25.$$

(a) Calculate the average fat free mass of players aged:

(i) 16 years

(ii) 25 years.

[2 marks]

Question 2b

(b) Find an expression for a linear model using your answers to part (a) (i) and (ii).

[3 marks]

Question 3a

The number of bacteria, n, in a dish, after t minutes is given by $n = 5231e^{0.12t}$.

(a) Find the initial amount of bacteria.

[2 marks]

Question 3b

(b) Find the amount of bacteria after 12 minutes. Give your answer in the form $a \times 10^k$, where $1 \leq a < 10, k \in \mathbb{Z}$.

[3 marks]

Question 3c

(c) Find the value of t when $n = 2.7 \times 10^4$.

[2 marks]

Question 4a

Let $f(x) = e^{-x} + 1$ and $g(x) = 2x - m$, for $x \in \mathbb{R}$, where m is a constant.

(a) Find $(g \circ f)(x)$.

[2 marks]

Question 4b

(b) Given that $\lim_{x \to \infty} (g \circ f)(x) = -1$, find the value of m.

[3 marks]

Question 5a

Consider the functions $f(x) = 2\ln(5x - 1)$ and $g(x) = \ln(5x - 1)^2$ where the domain for each function is as large as possible.

(a) (i) Write down the domain for f and the domain for g.

(ii) Write down the set of values of x for which $f(x) = g(x)$.

[3 marks]

Question 5b

(b) (i) Find the inverse function of f.

(ii) Explain why g does not have an inverse.

[4 marks]

Question 5c

The function h is the same as function g but with its domain restricted to $x \leq k$, where $k \in \mathbb{Z}$, so that h has an inverse.

(c) (i) Write down the largest possible value of k.

(ii) Find the inverse function of h.

[4 marks]

Question 6a

Consider the function $f(x) = \frac{1}{2}(e^x + e^{-x}), x \in \mathbb{R}$.

(a) Sketch the graph of f and write down its range.

[3 marks]

Question 6b

(b) (i) For $k > 1$, show that $f(x) = k$ leads to the equation $e^{2x} - 2ke^x + 1 = 0$.

(ii) Find the two solutions in terms of k.

[4 marks]

Question 6c

The domain of f is now restricted to $x \leq p$ so that it has an inverse.

(c) (i) Write down the largest value of p.

(ii) Sketch the graph of f^{-1} and state its domain and range.

(iii) Use the solution to (b) to write down the inverse of f.

[3 marks]

Question 7

Show that the function f, defined by $f(x) = \ln\left(\frac{e^x+1}{e^x-1}\right)$, is a self-inverse function.

[4 marks]