2.9 Further Functions & Graphs

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.9 Further Functions & Graphs
Difficulty	Very Hard

Time allowed: 120

Score: /93

Percentage: /100

Question 1a

(a)

By considering graph transformations of an appropriate quadratic function, sketch the graph of $y = (x+4)^2 - 2|x+4| - 3$.

Clearly indicate any x-intercepts and any y-intercepts.

[5 marks]

Question 1b

(b)

Hence, find the values of k such that the equation $(x+4)^2 - 2|x+4| - 3 = k$ has exactly 4 distinct real solutions.

[4 marks]

Question 2a

A function f is defined by $f(x) = (x-1)(x+2)(x-3), \ x \in R.$

(a)
(i)
Sketch the graph of $y = |f(x)|$.
(ii)
Hence find the set of values of x for which $f(x) < |f(x)|$

[4 marks]

Question 2b

(b)
Solve the equation $f(x) = |x^2 - 4x + 3|$.

[5 marks]

Question 3a

A function f is defined by $f(x) = x^2 - 8x + 15,\ x \in \mathbb{R}$.

(a)

Sketch the graph of $y = \frac{1}{f(x)}$. Clearly indicate any intercepts with the coordinate axes and state the equations of any asymptotes. Find the coordinates of any turning points.

[6 marks]

Question 3b

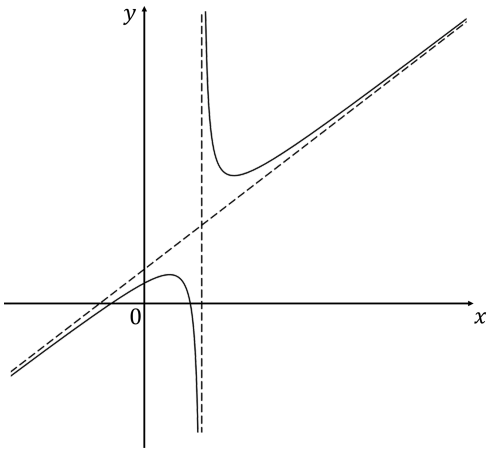
(b)

By sketching the graphs of $y = f(x)$ and y on the same axes, find the values of x for which $f(x) \leq \frac{1}{f(x)}$.

[6 marks]

Question 4a

Consider the function $f(x) = \dfrac{(x+3)(x-4)}{x-5}$, $x \in \mathbb{R}$, $x \neq 5$. The graph of f is shown below.



(a)
Find the equation of the oblique asymptote of the graph of f.

[4 marks]

Question 4b

(b)

Sketch the graph of $y = |f(x)|$. Clearly indicate the points where the graph crosses the coordinates axes and state the equations of the asymptotes.

[5 marks]

Question 4c

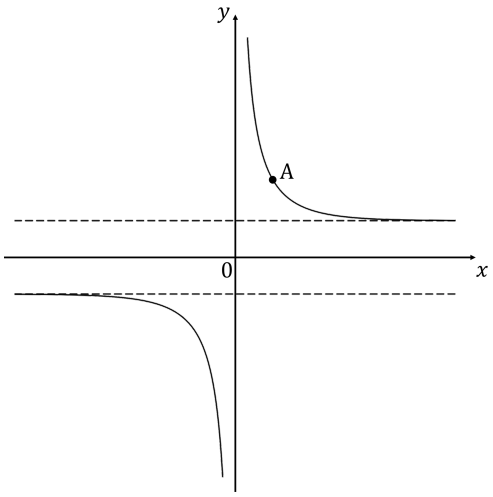
(c)

Sketch the graph of $y = f(|x|)$. Clearly indicate the points where the graph crosses the coordinates axes and state the equations of the asymptotes.

[4 marks]

Question 5a

The graph of a function f is shown below. The equations of the asymptotes are $x = 0$, y=-1 and $y = 1$. The point A(1, 2) lies on the graph.



On separate sets of axes, sketch the graphs defined below. For each sketch, clearly label any asymptotes or discontinuities and clearly show the coordinates of the point where A gets mapped to.

(a)

$y = \frac{1}{f(x)} + 2$.

[4 marks]

Question 5b

(b)

$y = [f(x) - 2]^2$.

[4 marks]

Question 5c

(c)

$$y = \frac{1}{[f(x)]^2}$$

[4 marks]

Question 6a

Let f be the function defined by $f(x) = (x-1)(x^2+x-26)$.

(a)

Sketch the graph of $y = |f(x)|$. Give the exact coordinates of the x-intercepts.

[4 marks]

Question 6b

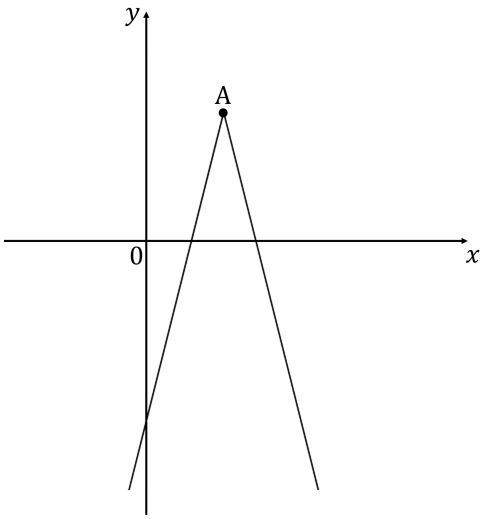
(b)

Use differentiation to find the set of values of k such that there are 4 points of intersection between the graph of $y = |f(x)|$ and the line $y = k$.

[6 marks]

Question 7a

The graph of the function $f(x) = a|x + p| + q$ is shown below, where $a, p, q \in \mathbb{R}$. The graph has a local maximum at the point A(3, 5) and intersects the y-axis at (0,-7).



(a)
Find the values of a, p and q. Hence solve $f(x) = 0$.

[4 marks]

Question 7b

(b)
Find the solutions to the equation $f(x) = |7 - 2x|$.

[4 marks]

Question 8a

Consider the function defined by $f(x) = x|x|, x \in R$.

(a)
Sketch the graph of f.

[2 marks]

Question 8b

(b)
By first sketching the graph of f^{-1} on the same set of axes as the graph of f, solve $f(x) = f^{-1}(x)$.

[3 marks]

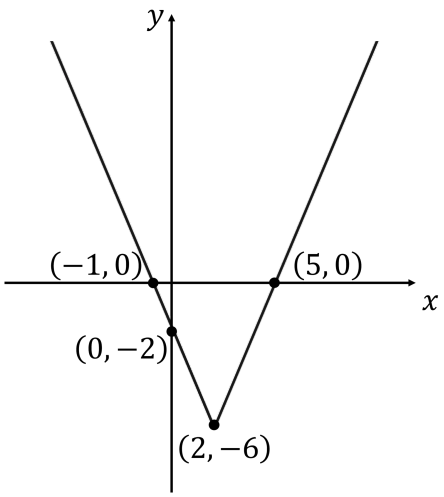
Question 8c

(c)
Given that k is a real constant, find an expression for $f^{-1}(k)$ in the case when:
(i)
$k \geq 0$,
(ii)
$k < 0$.

[3 marks]

Question 9a

The graph of a function f is shown below.



On separate sets of axes sketch the following graphs clearly showing any key points.

a)

$y = |f(x)| - f(x)$.

[2 marks]

Question 9b

(b)

$y = f(x)|f(x)|$

[4 marks]

Question 9c

(c)

$$y = \frac{f(x)}{|f(x)|}$$

[2 marks]

Question 10

$$f(x) = x^4 - 8x^3 - 40x^2 + 224x - 240.$$

Find the values of such that $|f(|x|)| = k$ has six distinct, real solutions.
Explain each stage of your solution in full.

[4 marks]