2.3 Modelling with Functions

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.3 Modelling with Functions
Difficulty	Very Hard

Time allowed: 90

Score: /68

Percentage: /100

© 2015-2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 1a

Phillip is kayaking to an island located 7 km from where he starts. The distance, D, in kilometres, that Phillip has travelled from his original position can be modelled by the exponential function

$$D(t) = a + b(k^{-t}), \qquad t \geq 0,$$

where t is the time in minutes since Phillip started kayaking.

(a) State the value of a and explain what the value represents in the context of this question.

[2 marks]

Question 1b

(b) Find the value of b.

[2 marks]

Question 1c

After 15 minutes Phillip has travelled 5.5 km from his original position.

(c) Find the value of k.

[2 marks]

© 2015-2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 1d

When Phillip is less than 20 m away from the island he can stand up and walk his kayak ashore.

(d) Calculate the time it takes Phillip before he stands up and walks ashore. Give your answer to the nearest minute.

[2 marks]

Question 2a

The average temperature of a city, C, in degrees Celsius, fluctuates throughout a year and can be modelled by the function

$$C(t) = a\sin(kt) + b,$$

where t is the elapsed time, in weeks, since the start of the year.

The average temperature of the city in week 4 is 27 degrees Celsius and in week 28 it is 12 degrees Celsius.

(a) Find the value of k, assuming there are 52 weeks in a year.

[2 marks]

Question 2b

(b) Write down two equations connecting a and b and find their values.

[3 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 2c

A restaurant stores food in a storage unit outside and when the average temperature of the city gets below 0 degrees Celsius they have to be careful about some things freezing.

(c) Calculate how many weeks of the year that they have to be careful about the food freezing. Give your answer to the nearest integer.

[3 marks]

Question 3a

Algae in a lake can grow exponentially until the lake is fully covered in algae.

(a) Find the number of days it takes for a lake to be fully covered in algae when $\frac{1}{2048}$ of the lake is covered today and the covered area doubles once every five days.

[4 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 3b

(b) Find the number of days it takes for a lake to be fully covered in algae when 99.9% of the lake is uncovered and the covered area increases by a factor of 10 within each 10-day period.

[4 marks]

Question 4

A fence of length L, in metres, is made to form a rectangle around a house that borders a forest on one side. The fence does not run along the side next to the forest. The cost of the fence is $22.20 per metre. The total cost of the fence is $2250.

Calculate

(i) the maximal area of the rectangle.

(ii) the side lengths for the maximal area.

(iii) the total length L of the fence.

[6 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 5a

A potato top pie is removed from the oven and is left to cool. The pie's temperature, T, in °C, can be modelled by the function

$$T(t) = a + b(k^{-t}), \qquad t \geq 0,$$

where t is the time, in minutes, since the pie was removed from the oven.

The temperature of the kitchen is 24 °C.

(a) Write down the value of a and explain what it represents in the context of this question.

[2 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 5b

Initially the temperature of the pie is 200 °C

(b) Find the value of b.

[1 mark]

Question 5c

After five minutes the temperature of the pie is 107 °C.

(c) Find the value of k.

[1 mark]

Question 5d

Bacteria in the pie can grow rapidly when its temperature is in the “danger zone” which is between 5 °C and 60 °C. Food should never be left in the “danger zone” for more than 2 hours. Hence, the pie is put in the fridge after it has been in the “danger zone” for an hour and 20 minutes.

(d) Calculate the total amount of time between the pie being removed from the oven and being put in the fridge. Give your answer to the nearest minute.

[4 marks]

© 2015-2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 6a

Matt throws a discus in a competition, and its flight can be modelled by the function

$$y = \frac{x(22-x)}{20} + 1.8, \qquad 0 \le x \le 23.5,$$

where x is the horizontal distance in metres from where the athlete threw the javelin and y is the height of the javelin above the ground in metres.

(a) In the context of the model, explain the significance of the 1.8.

[1 mark]

Question 6b

(b) Sketch a graph of the model, labelling any intersections with the coordinate axes and the maximum point.

[3 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 6c

Matt has been training for this competition for 6 months and his improvement can be modelled by the function

$$D(t) = a\ln(t+1) + b\,, \qquad 0 \leq t \leq 6,$$

where D is the distance of his personal best throw, in metres, and t is the time that has elapsed since he started training, in months and a and b are constants.

At the start of his training Matt's personal best throw is 14 m. Matt's throw in the competition was a new personal best.

(c) Find the values of a and b.

[3 marks]

Question 7a

A tunnel is being constructed and its opening can be modelled by the quadratic function

$$h(x) = ax(b-x), \qquad x \geq 0,$$

where h is the height of the tunnel, in metres, and x is the width of the tunnel, in metres.

It is given that $h(10) = 10$ and $h(20) = 15$.

(a) Find the values of a and b.

[3 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 7b

The height required for a lane of traffic is 5 m and each lane requires a width of 2.8 m.

(b) Find the number of lanes of traffic the tunnel can fit.

[4 marks]

Question 8a

A company sells 55 cars per month for a sale price of $2000, whilst incurring costs for supplies, production and delivery of $890 per car. Reliable market research shows that for each increase (or decrease) of the sale price by $50 the company will sell 5 cars less (or more) and vice versa.

(a) Find an expression for the total profit, P, in terms of the sale price, x.

[3 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 8b

(b) Find the values of x when $\mathrm{P}(x) = 0$ and explain their significance in the context of the question.

[2 marks]

Question 8c

(c) Calculate

(i) the maximum monthly profit, giving your answer to the nearest dollar.

(ii) the sale price needed to generate the maximum monthly profit.

(iii) the number of cars sold to generate the maximum monthly profit.

[3 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 9a

A company sells L litres of water per month and their total monthly profit, P, can be modelled by the function

$$P(x) = (x - 0.45) \times N(x),$$

where x is the sale price of each litre sold, in dollars, at and N is the linear function for the number of litres the company can sell per month at each given sale price.

(a) In the context of the question, explain the significance of the 0.45.

[1 mark]

Question 9b

It is given that $N(0.5) = 400$ and $N(1.25) = 250$.

(b) Write down the function of N, in the form $N(x) = mx + c$, where m and c are constants.

[2 marks]

Question 9c

(c) Find the values of x when $\mathrm{P}(x) = 0$ and explain their significance in the context of the question.

[2 marks]

© 2015-2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers

Question 9d

(d) Calculate

(i) the maximum monthly profit.

(ii) the sale price needed to generate the maximum monthly profit.

(iii) the number of litres sold to generate the maximum monthly profit.

[3 marks]

© 2015–2023 Save My Exams, Ltd. · Revision Notes, Topic Questions, Past Papers