5.1 Differentiation

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.1 Differentiation
Difficulty	Medium

Time allowed: 90

Score: /71

Percentage: /100

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Question 1a

The equation of a curve is $y = \frac{3}{2}x^2 - 15x + 2$.

(a) Find $\frac{dy}{dx}$.

[2 marks]

Question 1b

The gradient of the tangent to the curve at point A is -3.

(b) Find

(i) the coordinates of A

(ii) the equation of the tangent to the curve at point A.
Give your answer in the form $y = mx + c$.

[4 marks]

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Question 2a

Consider the function $f(x) = 3x^7 - 12x$.

(a) Find $f'(x)$.

[1 mark]

Question 2b

(b) Find the gradient of the graph of f at $x = 0$.

[2 marks]

Question 2c

(c) Find the coordinates of the points at which the normal to the graph of f has a gradient of 4.

[3 marks]

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Question 3a

The equation of a curve is $y = 4 - \frac{4}{x}$.

(a) Find the equation of the tangent to the curve at $x = 2$.
Give your answer in the form $y = mx + c$.

[3 marks]

Question 3b

(b) Find the coordinates of the points on the curve where the gradient is 16.

[3 marks]

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Question 4a

Consider the function $f(x) = \frac{4}{x} + \frac{2x^4}{5} - \frac{2}{5}$, $x \neq 0$.

(a) Calculate

(i) $f(2)$

(ii) $f'(2)$.

[3 marks]

Question 4b

A line, l, is tangent to the graph of $y = f(x)$ at the point $x = 2$.

(b) Find the equation of l. Give your answer in the form $y = mx + c$.

[3 marks]

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Question 4c

The graph of $y = f(x)$ and l have a second intersection at point A.

(c) Use your graphic display calculator to find the coordinates of A.

[2 marks]

Question 5a

Consider the function $f(x) = x^2 - bx + c$.

(a) Find $f'(x)$.

[1 mark]

Question 5b

The equation of the tangent line to the graph $y = f(x)$ at $x = 2$ is $y = x - 1$.

(b) Calculate the value of b.

[2 marks]

Question 5c

(c) Calculate the value of c and write down the function $f(x)$.

[3 marks]

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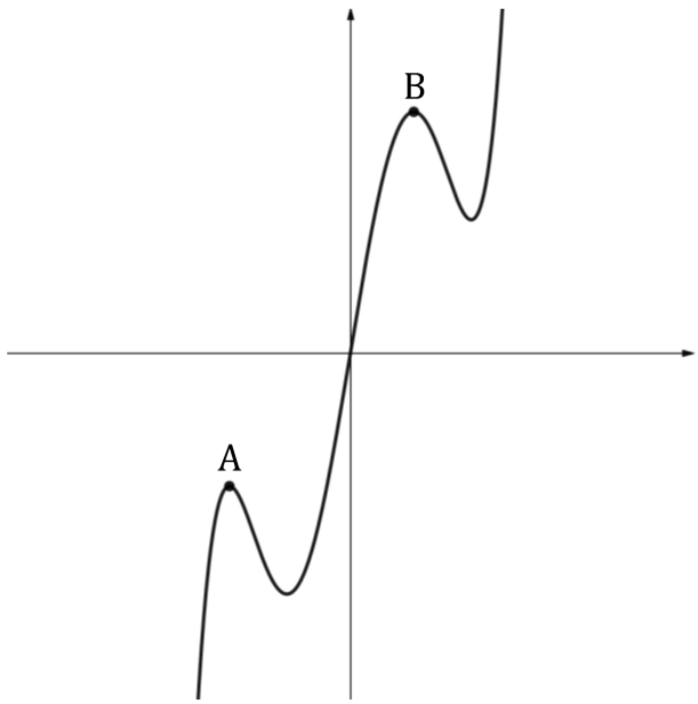
Question 6a

The equation of the curve C is $y = \frac{1}{35}x^5 - \frac{3}{4}x^3 + 6x$. A section of the curve C is shown on the diagram below.



(a) Find $\frac{dy}{dx}$.

[2 marks]

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Question 6b

Points A and B represent the local maximums on the diagram above.

(b) Write down the coordinates of

(i) A

(ii) B.

[4 marks]

Question 6c

There are two points, R and S, along the curve C at which the gradient of the normal to the curve C is equal to $-\frac{1}{10}$.

(c) Calculate the x-coordinates of points R and S.

[2 marks]

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Question 7a

The daily cost function of a company producing pairs of running shoes is modelled by the cubic function

$$C(x) = 1225 + 11x - 0.009x^2 - 0.0001x^3, \qquad 0 \leq x < 160$$

where x is the number of pairs of running shoes produced and C the cost in USD.

(a) Write down the daily cost to the company if no pairs of running shoes are produced.

[1 mark]

Question 7b

The marginal cost of production is the cost of producing one additional unit. This can be approximated by the gradient of the cost function.

(b) Find an expression for the marginal cost, $C'(x)$, of producing pairs of running shoes.

[2 marks]

Question 7c

(c) Find the marginal cost of producing

(i) 40 pairs of running shoes

(ii) 90 pairs of running shoes.

[2 marks]

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Question 7d

The optimum level of production is when marginal revenue, $R'(x)$, equals marginal cost, $C'(x)$. The marginal revenue, $R'(x)$, is equal to 4.5.

(d) Find the optimum level of production.

[3 marks]

Question 8a

A cyclist riding over a hill 3can be modelled by the function

$$h(t) = -\frac{1}{24}t^2 + 3t + 12, \qquad 0 \le t \le 70$$

where h is the cyclist's altitude above mean sea level, in metres, and t is the elapsed time, in seconds.

(a) Calculate the cyclist's altitude after a minute.

[2 marks]

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Question 8b

(b) Find $h'(t)$.

[2 marks]

Question 8c

(c) Calculate the cyclist's maximum altitude and the time it takes to reach this altitude.

[3 marks]

Question 9a

A company produces and sells cricket bats. The company's daily cost, C, in Australian dollars (AUD), changes based on the number of cricket bats they produce per day. The daily cost function of the company can be modelled by

$$C(x) = 6x^3 - 10x^2 + 10x + 4$$

where x hundred cricket bats is the number of cricket bats produced on a particular day.

(a) Find the cost to the company for any day zero cricket bats are produced.

[1 mark]

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Question 9b

The company's daily revenue, of AUD, from selling x hundred cricket bats is given by the function $R(x) = 42x$.

(b) Given that profit = revenue – cost, determine a function for the profit, $P(x)$, in hundreds of AUD from selling x hundred cricket bats.

[2 marks]

Question 9c

(c) Find $P'(x)$.

[2 marks]

Question 9d

The derivative of $P(x)$ gives the marginal profit. The production of bats will reach its profit maximising level when the marginal profit equals zero and $P(x)$ is positive.

(d) Find the profit maximising production level and the expected profit.

[3 marks]

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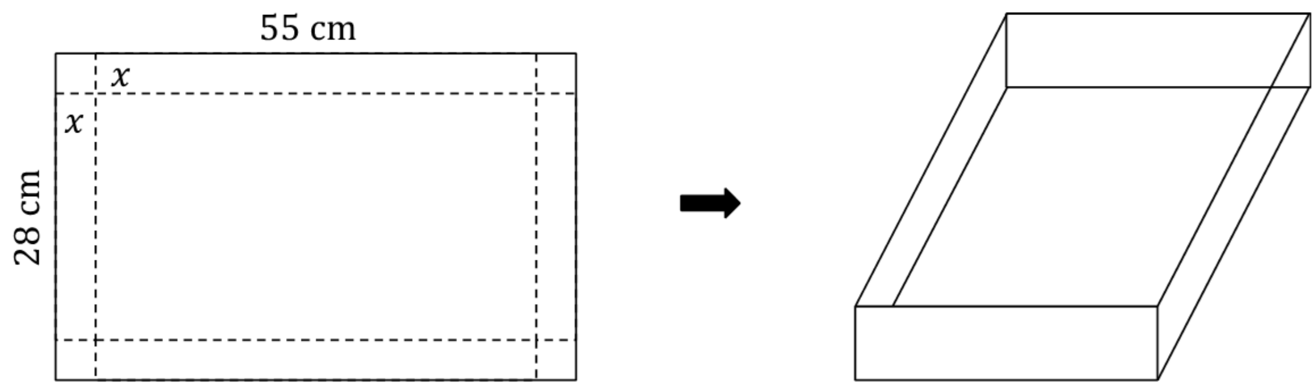
Question 10a

Dora decides to build a cardboard container for when she goes strawberry picking from a rectangular piece of cardboard, 55 cm $\times$ 28 cm. She cuts squares with side length x cm from each corner as shown in the diagram below.



(a) Show that the volume, V cm^3, of the container is given by

$$V = 4x^3 - 166x^2 + 1540x$$

[2 marks]

Question 10b

(b) Find $\frac{dV}{dx}$.

[2 marks]

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Question 10c

(c) Find

(i) the value of x that maximises the volume of the container

(ii) the maximum volume of the container. Give your answer in the form $a \times 10^k$, where $1 \leq a \leq 10$ and $k \in \mathbb{Z}$.

[4 marks]

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