1.8 Complex Numbers

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.8 Complex Numbers
Difficulty	Very Hard

Time allowed: 90

Score: /75

Percentage: /100

Question 1a

Consider the complex numbers $z_1 = \sqrt{3} + 2\text{i}$ and $z_2 = \text{i} - 3\sqrt{3}$.

(a)
Find
(i)
$u = z_1 z_2$
(ii)
$v = \frac{z_1}{z_2}$

[4 marks]

Question 1b

The complex numbers u and v are represented by the points A and B respectively on an Argand diagram with origin O.

(b)
Determine whether the angle made by OA with the positive horizontal axis is greater than or less than the angle made by OB with the positive horizontal axis. Give a reason for your answer.

[3 marks]

Question 2a

Consider the complex number $z = -a + \frac{3}{4}\mathrm{i}$.

(a)
Write down, in terms of a,
(i)
$\mathrm{Re}(z^2)$
(ii)
$\mathrm{Im}(z^3)$

[4 marks]

Question 2b

(b)
In the case where $a = 2$, find the modulus and argument of z^3.

[4 marks]

Question 3a

Consider the complex numbers $z_1 = \mathrm{i} - \frac{1}{2}$ and $z_2 = \frac{1}{2} - \frac{3}{\mathrm{i}}$.

(a)
Express z_2 in the form $a + b\mathrm{i}$, where $a, b \in \mathbb{R}$.

[3 marks]

Question 3b

(b)
Find
(i)
$z_1{}^* z_2$
(ii)
$\frac{z_2}{z_1}$
(iii)
$\left|\frac{z_2}{z_1}\right|$, giving your answer as an exact value.

[6 marks]

Question 4

Consider a general complex number $z = x + \mathrm{i}y$, where $x, y \in \mathbb{R}$, $z \in \mathbb{C}$ and $z \neq 0$.

Show that

(i)

$$\mathrm{Re}\left(\frac{1}{z} + \frac{1}{z^*}\right) = \frac{2x}{x^2 + y^2}$$

(ii)

$$\mathrm{Im}\left(\frac{1}{z} + \frac{1}{z^*}\right) = 0$$

(iii)

$$zz^* = |z|^2$$

[6 marks]

Question 5a

Consider the equation $zw - w + iz + 1 = 0$, where $w,\ z \in \mathbb{C}$, $w = x + iy$.

(a)
Find an expression in terms of x and y for $\mathrm{Re}(z)$.

[4 marks]

Question 5b

(b)
Find in terms of x given that z is purely real.

[4 marks]

Question 6a

Consider the complex numbers $z_1 = \frac{3 - \mathrm{i}}{1 - 2\mathrm{i}}$ and $z_2 = -3\mathrm{i} + 1$.

(a)

Find the modulus of $\frac{z_1}{z_2{}^*}$ giving your answer as an exact value.

[5 marks]

Question 6b

(b)

The argument of $\frac{z_1}{z_2{}^*}$ is given as $\theta = \tan^{-1} x$, where $0 < \theta < 2\pi$. Find the value of x.

[2 marks]

Question 7a

Consider the complex numbers $z = \frac{v}{w}, v = 1 - pi \text{ and } \mathrm{w} = 3\mathrm{i} - 2$

(a)

Express z in the form $a + b\mathrm{i}$, where $a,\ b,\ p \in \mathbb{R}.$.

[3 marks]

Question 7b

(b)
In the case where z is purely imaginary, represent $v,\ w$ and z on an Argand diagram.

[4 marks]

Question 8a

Consider the complex numbers $z = \frac{a - 3\mathrm{i}}{2 + \mathrm{i}}$, $w = a + b\mathrm{i}$ and $\frac{z}{w} = 1 + 2\mathrm{i}$ where $a,\ b \in \mathbb{R}$.

(a)
Find the values of a and b.

[4 marks]

Question 8b

(b)

Find the modulus of $\frac{w}{z}$, giving your answer as an exact value.

[2 marks]

Question 8c

(c)

Find the argument of $\frac{w}{z}$, giving your answer in the range $-\pi \le \arg\frac{w}{z} \le \pi$.

[2 marks]

Question 9

Consider the complex numbers $a - w = 2z - \mathrm{i}$ and $w - 2z = b\mathrm{i} - 1$.

Find the values of a and b such that $\mathrm{Re}(w) = \mathrm{Im}(z)$ and $\mathrm{Re}(w) = \mathrm{Re}(z) + 1$.

[7 marks]

Question 10a

Consider the complex numbers $z_1 = 5 + p\text{i}$, $z_2 = a + b\text{i}$ and $\frac{z_1}{z_2} = -1 + \text{i}$, where $z \in \mathbb{C}$ and $a,\ b \in \mathbb{R}$.

(a)
Find the values of a and b in terms of p.

[3 marks]

Question 10b

(b)
Given that $|z_2| = \sqrt{73}$, find the possible values of p.

[3 marks]

Question 10c

(c)
Given additionally that $\arg(z_2) = 2.78$ radians correct to 2 decimal places, determine the exact value of $\text{Im}(z_2)$.

[2 marks]